Wide-field Calibration and Imaging Issues

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Narrow-field Calibration & Imaging

Assume that calibrated visibilities sample the same 2D FFT of the sky

$$V_{jk}(t,\nu) = g_j(t,\nu)g_k^*(t,\nu)\,S_{jk}(t,\nu)\iint A(l,m)I(l,m)\,e^{-i2\pi(ul+vm)}dldm$$

Sky is fixed over the course of an observation

All antennas have the same primary beams, except for direction-independent gain terms

2D Fourier transform between sky and gridded visibilities

What are the wide-field issues?

Variable 3D phases

$$V_{jk}(t,\nu) = G_{jk}(t,\nu)S_{jk}(t,\nu)\iint I(l,m)\, e^{-i2\pi(ul+vm+w(n-1))}dldm$$

Visibility-sky relationship is not 2D

- Curved sky & 3D distribution of visibilities = "w-terms"
- Terms scale linearly with w (large arrays, long tracks, high ν, …)
- Terms scale quadratically with field of view width.

Variable primary beams

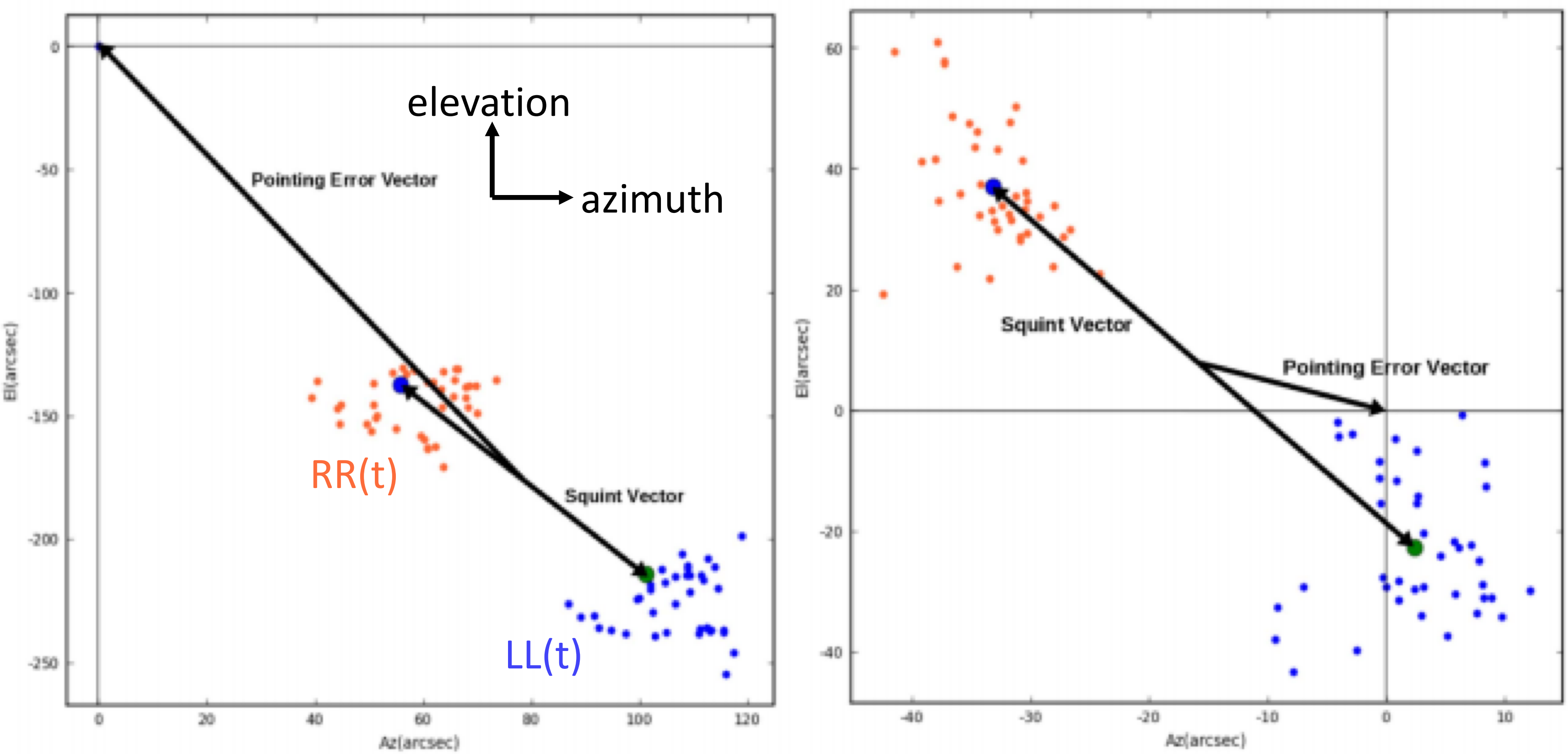
$$V_{jk}(t,\nu) = G_{jk}(t,\nu)S_{jk}(t,\nu)\iint A_{jk}^{inst}(l,m,t,\nu,p)\, I(l,m)\, e^{-i2\pi(ul+vm)} dldm$$

Variable in:

- time (e.g. rotating primary beams)
- frequency (e.g. beam width ∝ λ/D)
- antenna (e.g. pointing errors, beam-former variability)
- pointing direction (e.g. multi-beam or phase-array feed systems)
- polarisation (e.g. XX and YY beams when forming Stokes I)
- baseline (e.g. decorrelation)

JVLA Primary Beam Variability

Pointing offsets for two JVLA antennas



Bhatnagar & Cornwell 2017 (arXiv:1808.04516)

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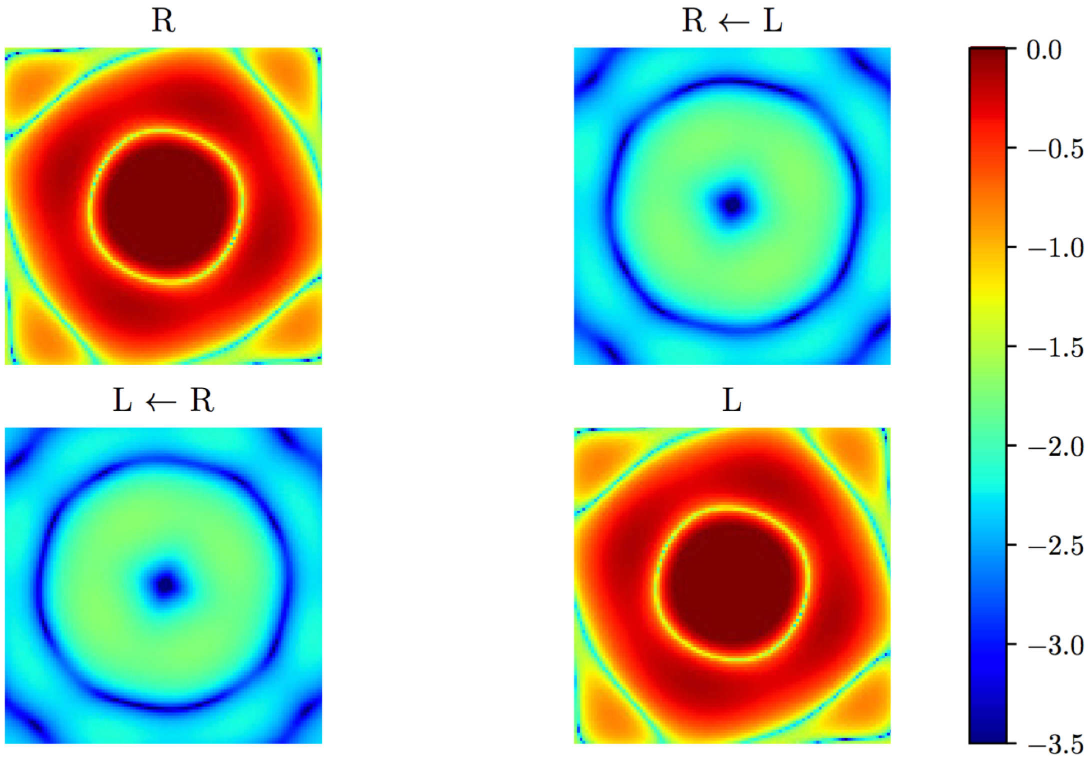
JVLA Primary Beam Variability

Azimuthal asymmetries rotate on the sky
when tracking a field with alt-az dishes

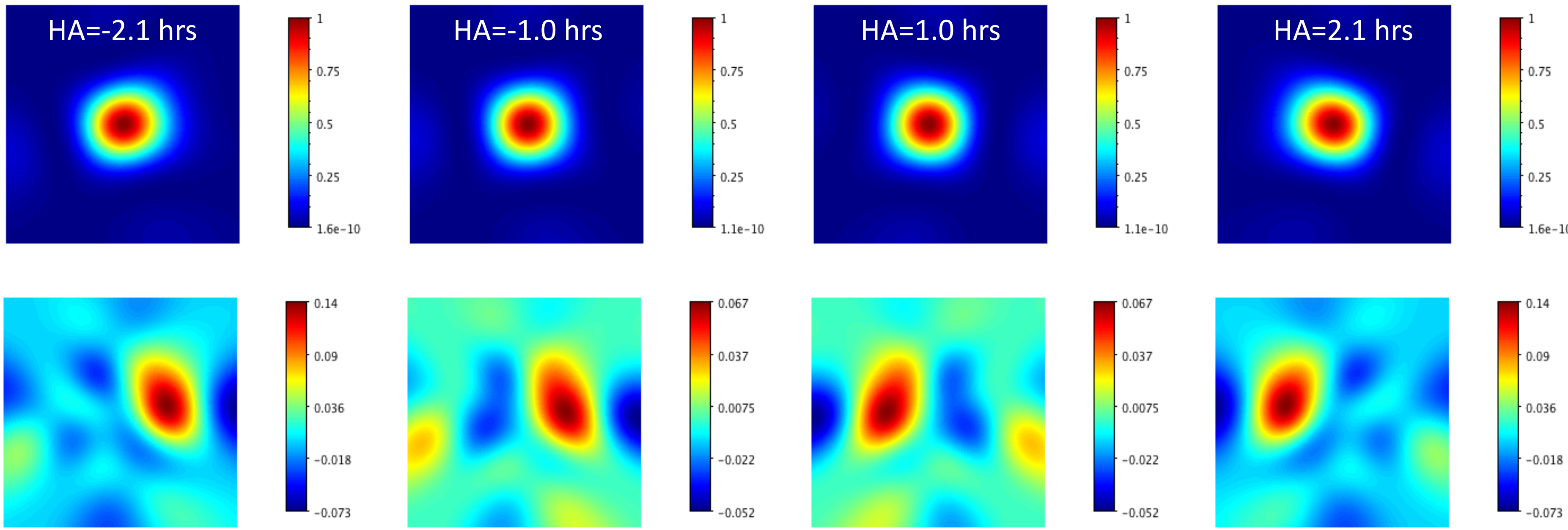


Jagannathan et al. 2017 (arXiv:1706.01501)

MWA Primary Beam Variability

Phased array beams from a tile or station of dipoles that are fixed to the ground can change shape as they track

Time-dependent tile response

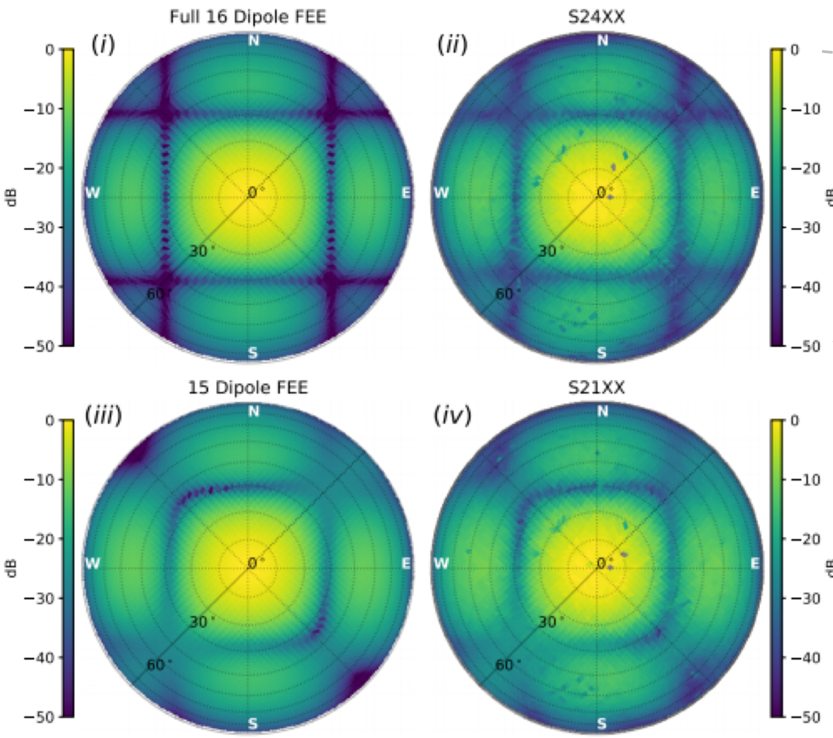


Time-dependent error

Simulated MWA primary beams

MWA Primary Beam Variability

And phased-array beams can change from station to station



Embedded Element beam models. Sokolowski et al. 2017 (arXiv:1710.07478)

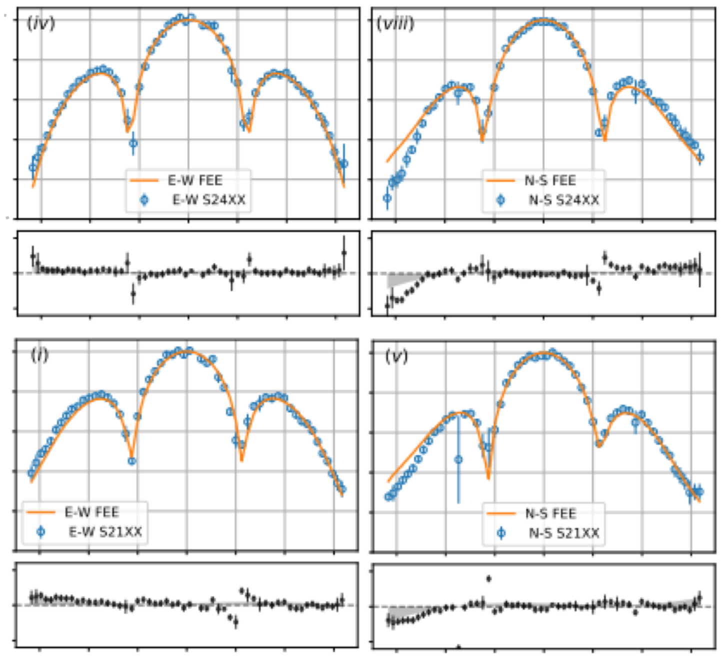
Measured primary beams

Full tile

One dead dipole

East-West slice

North-South slice



Beam measurements at ~ 138 MHz using ORBCOMM satellites
Line et al. 2018 (arXiv:1808.04516)

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Variable Polarised Primary Beams

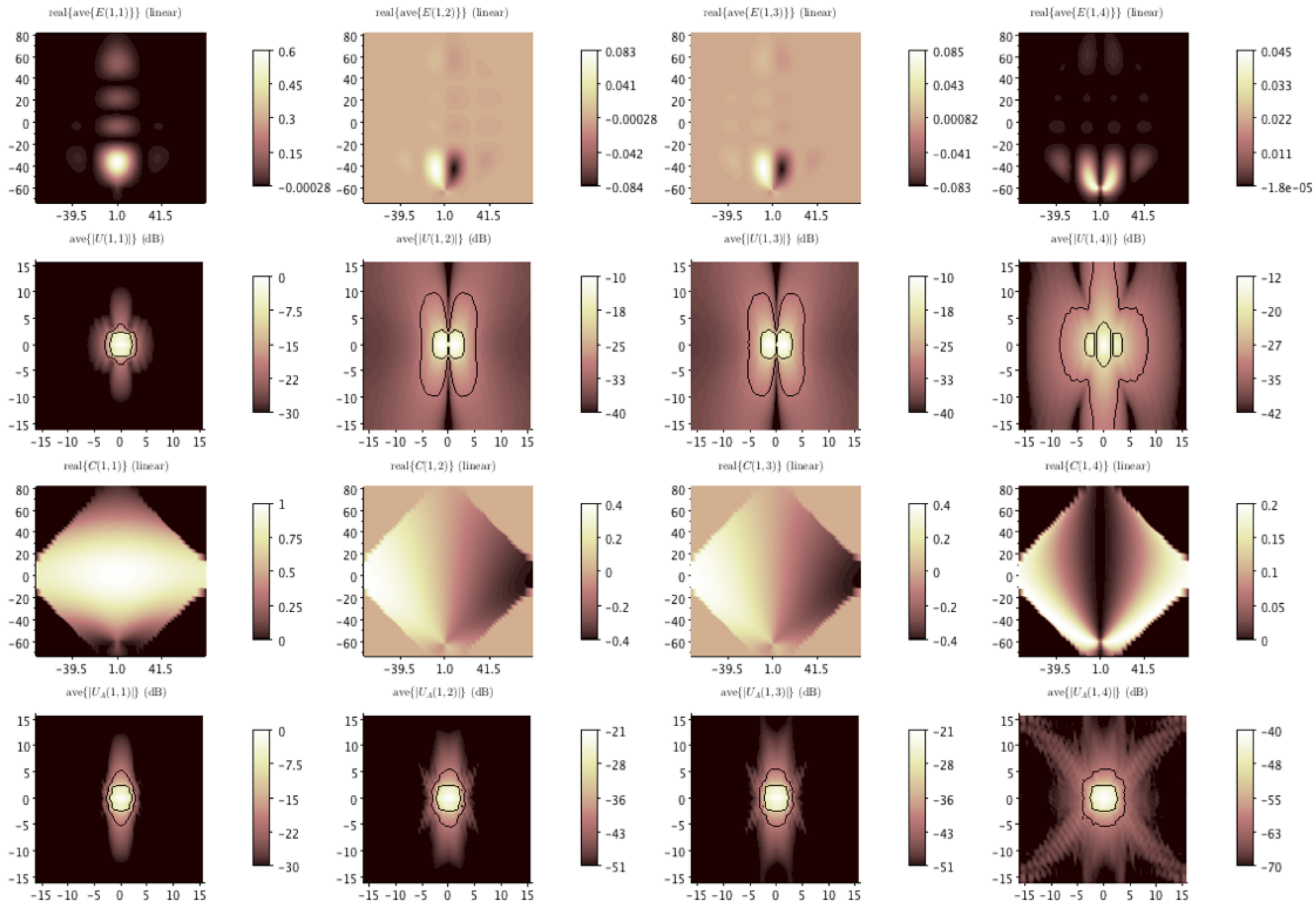
NN←xx NN←xy NN←yx NN←yy

All-sky polarised tile response

Fourier response

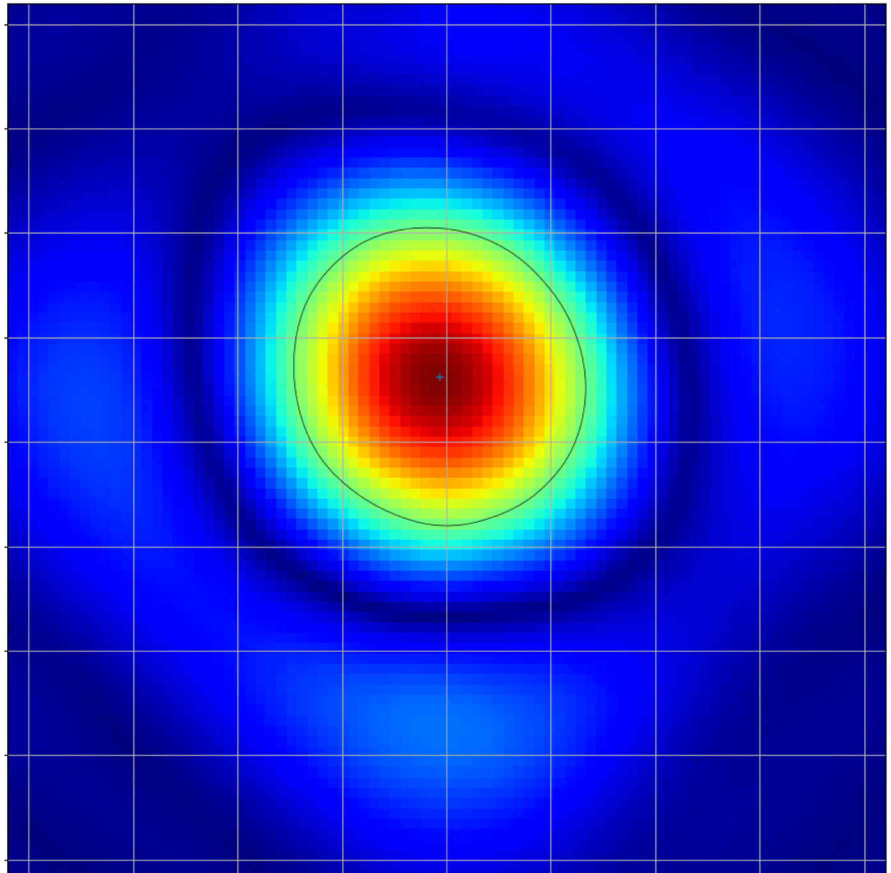
High frequency terms due to coordinate transformations

Instrument-only Fourier response

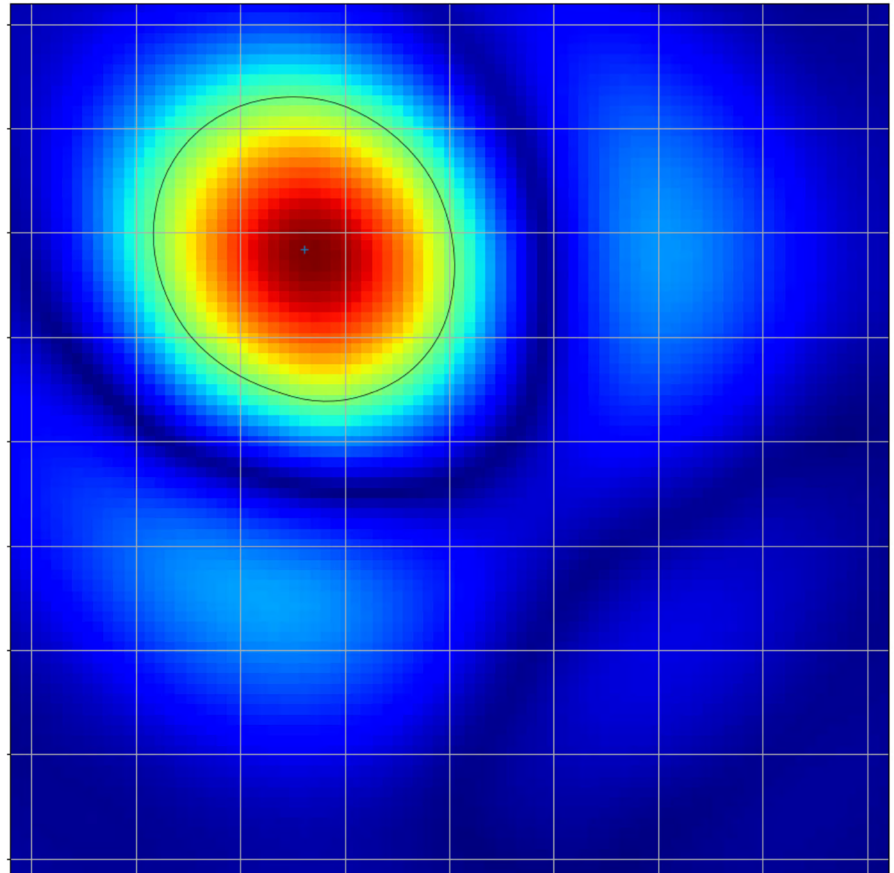


ASKAP Primary Beam Variability

AK05 beam 00 792MHz, XX



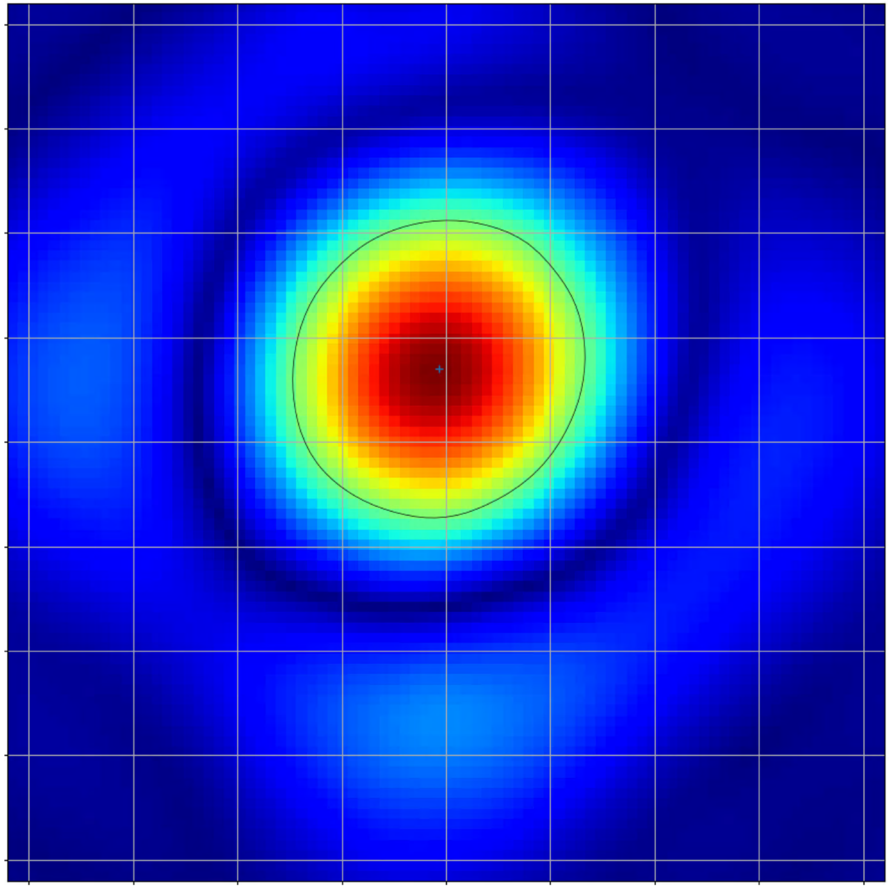
AK05 beam 18 792MHz, XX



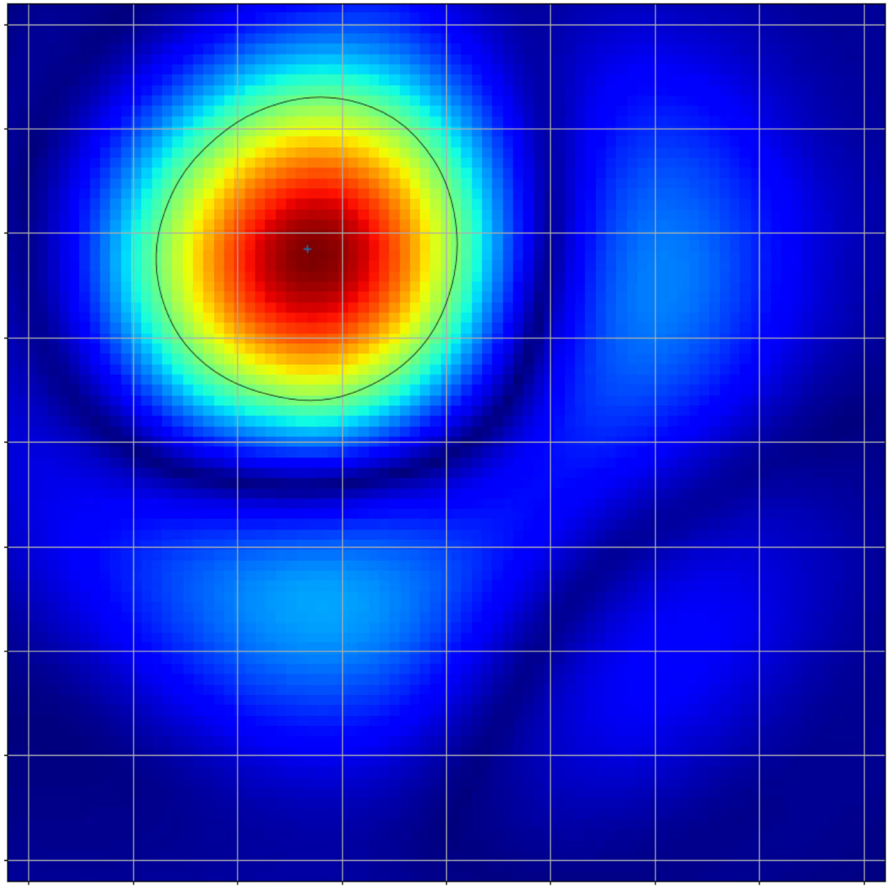
Thanks to Aidan Hotan

ASKAP Primary Beam Variability

AK05 beam 00 792MHz, YY

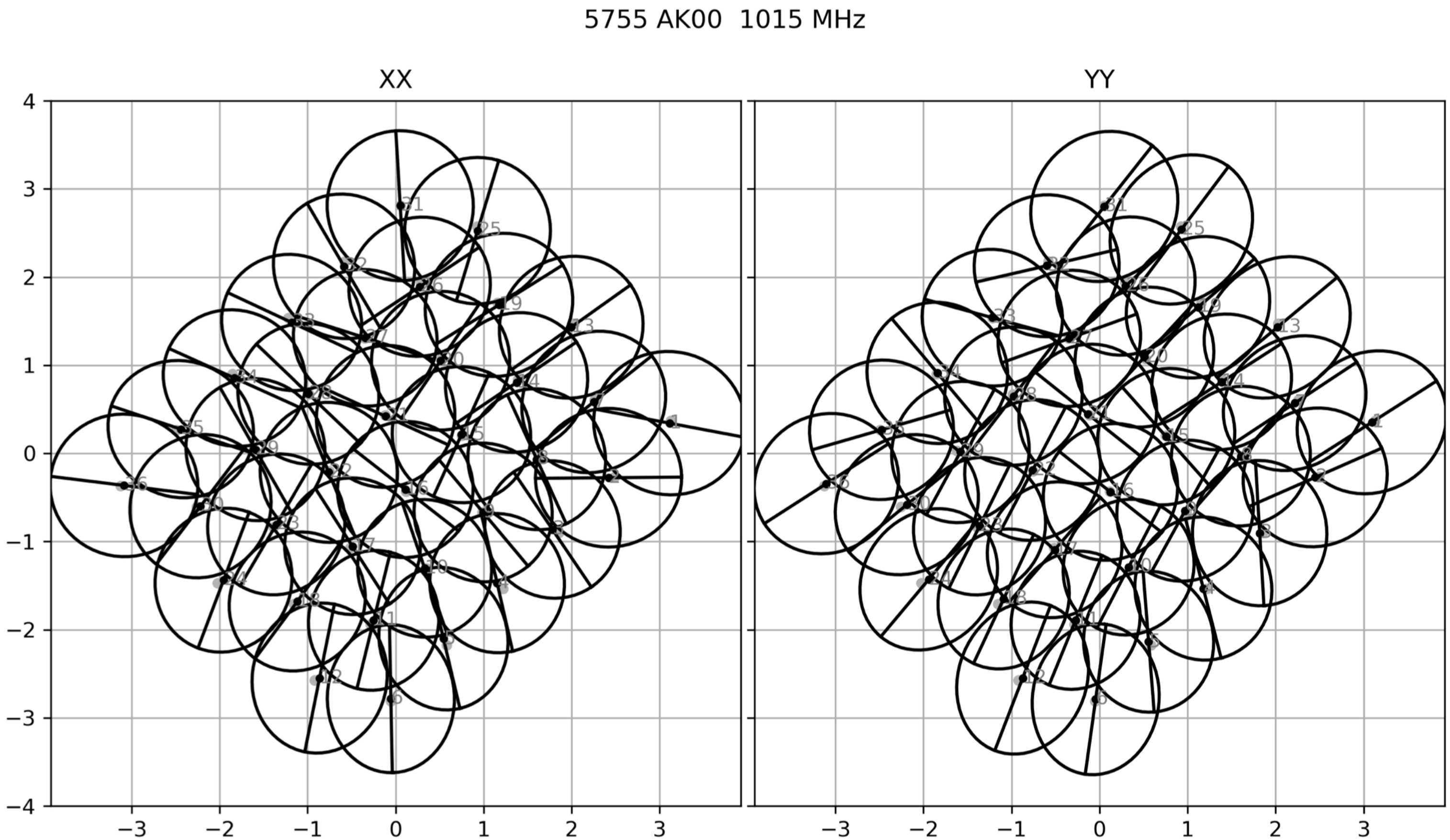


AK05 beam 18 792MHz, YY



Thanks to Aidan Hotan

ASKAP Primary Beam Variability



Thanks to Dave McConnell

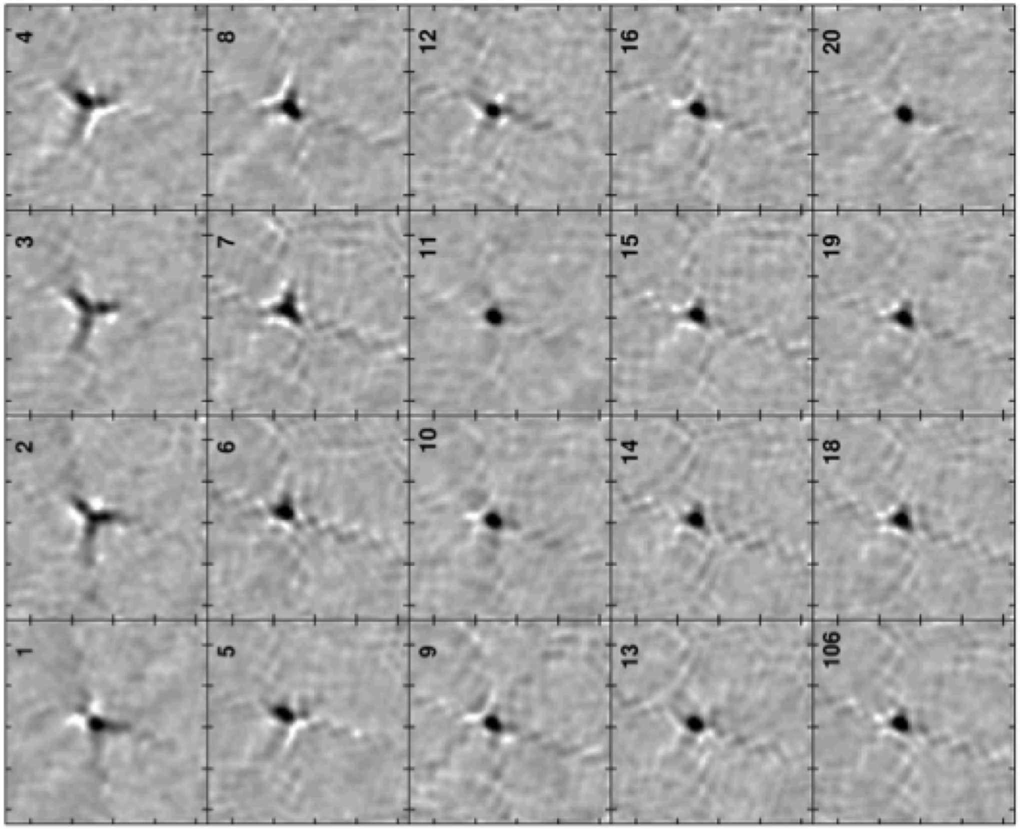
Variable atmospheric conditions

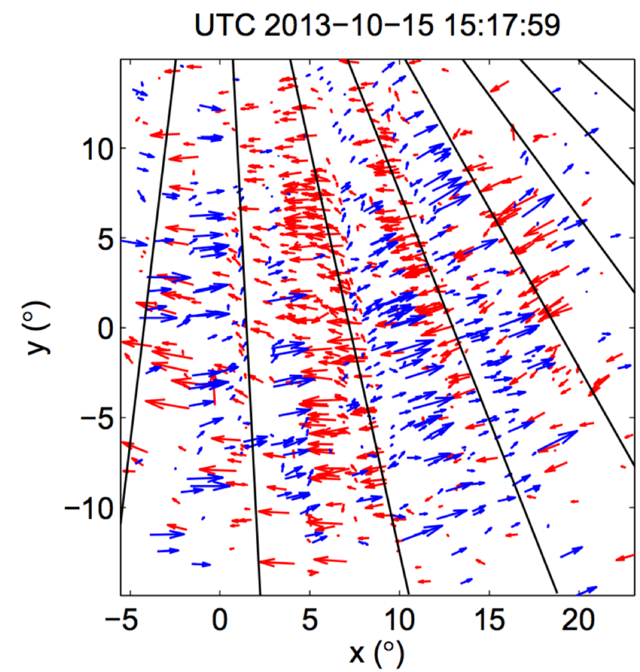
$$V_{jk}(t,\nu) = G_{jk}(t,\nu)S_{jk}(t,\nu)\iint A_{jk}^{atm}(l,m,t,\nu,p)I(l,m)\,e^{-i2\pi(ul+vm)}dldm$$

- Ionospheric refraction (lower frequencies)
- Ionospheric Faraday rotation
- Troposphere (higher frequencies)

Ionospheric Refraction Variability

Cotton (2004) ASP Conf. Series 345, 74 MHz, 1-min VLA snapshots

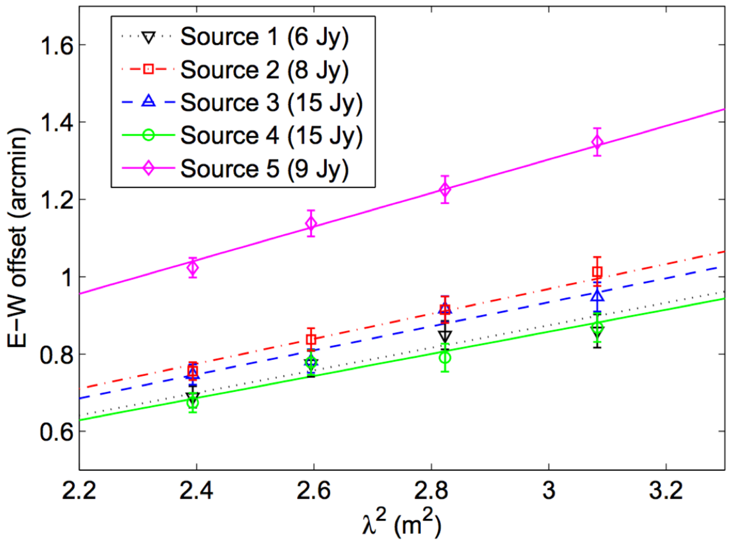




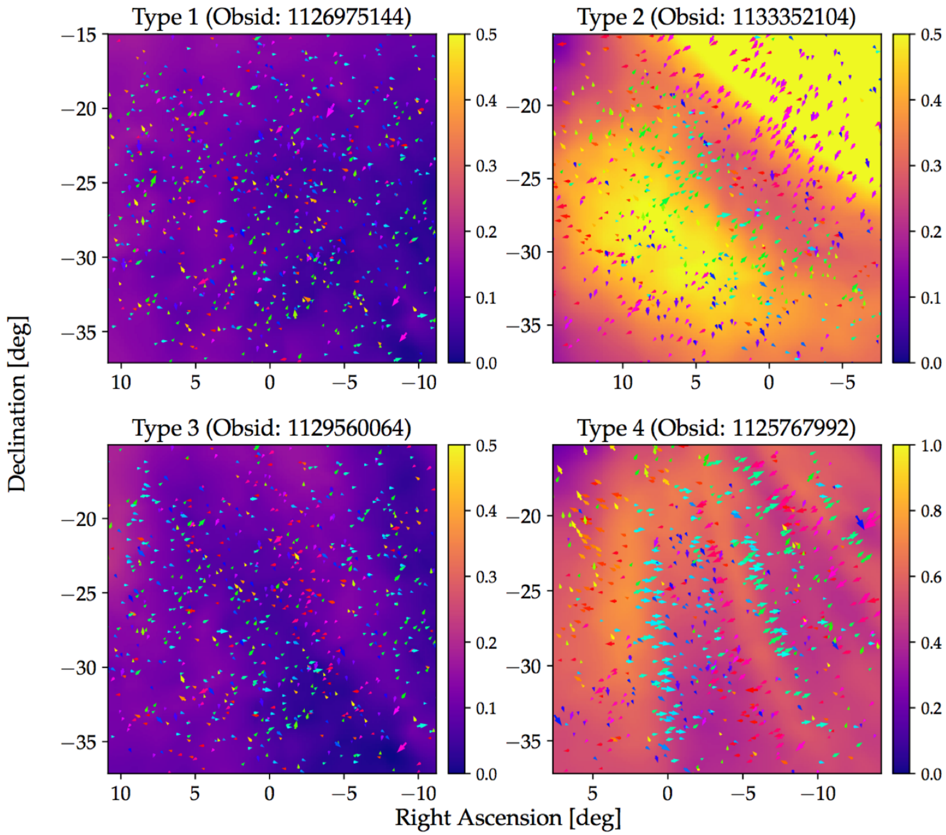
Loi et al. (2015) arXiv:1504.06470

Ionospheric Refraction Variability

Loi et al. 2015 (arXiv:1504.06470)



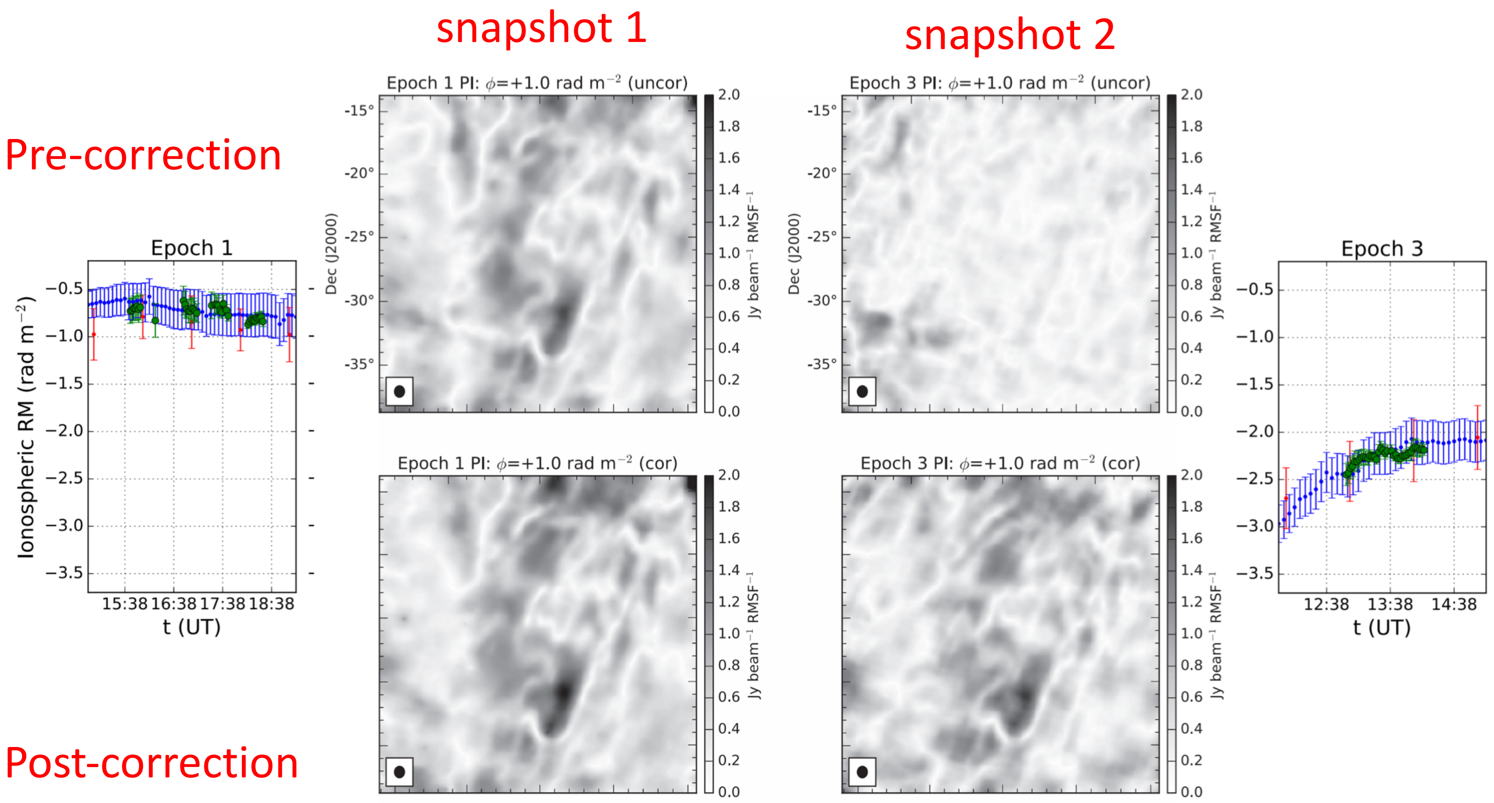
Jordan et al. 2017 (arXiv:1707.04978)



Ionospheric Faraday Rotation Variability

The ionosphere rotates the angle of linear polarisation as a function of λ^2

- Changes the position of features in Rotation Measure spectra (time, direction, antenna)



Lenc et al. 2017 (arXiv:1607.05779)

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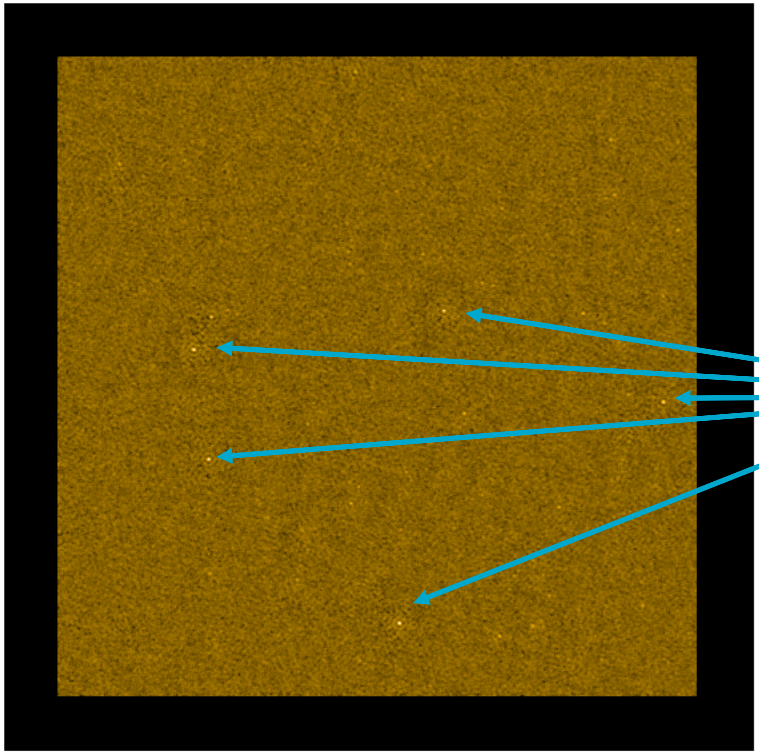
Wide-field calibration and imaging approaches

Direction-dependent gain calibration

- There are numerous approaches
- Many involve some form of peeling
- One form that approximates a joint non-linear solver is:
  - First subtract an initial sky model with an initial calibration model
  - Then phase up to the strong sources, one-by-one, starting with the strongest
    - Redo calibration
    - Redo subtraction
  - Iterate

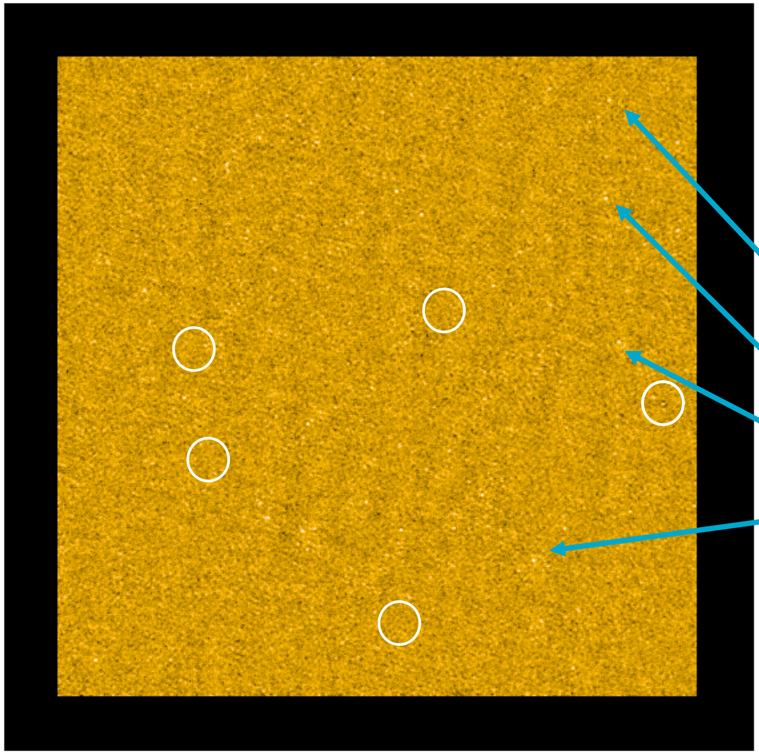
Peeling



Dirty image with direction-independent calibration

Five dominant sources

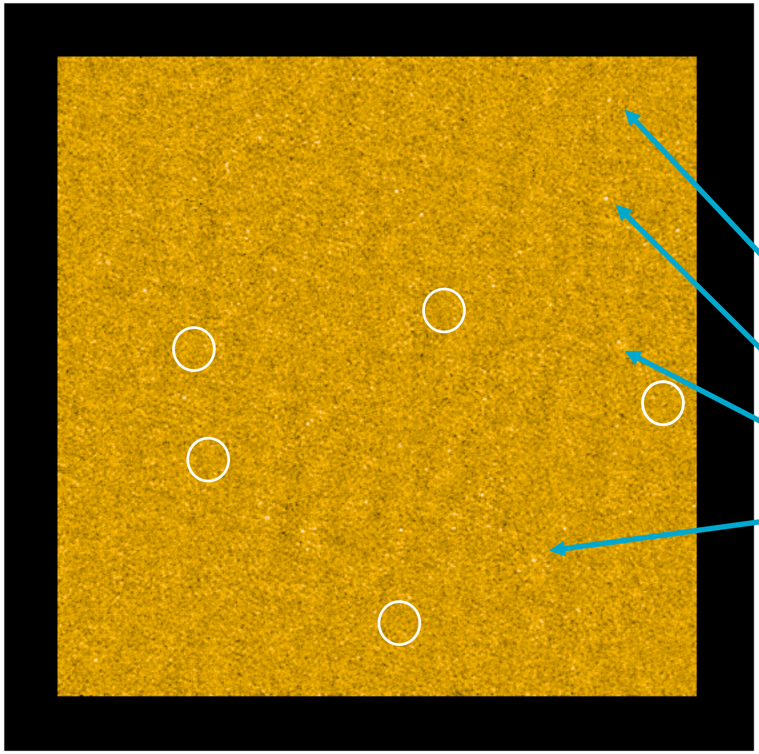
Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (but not peeled)

The residuals are dominated by weaker sources, not by subtraction artefacts.

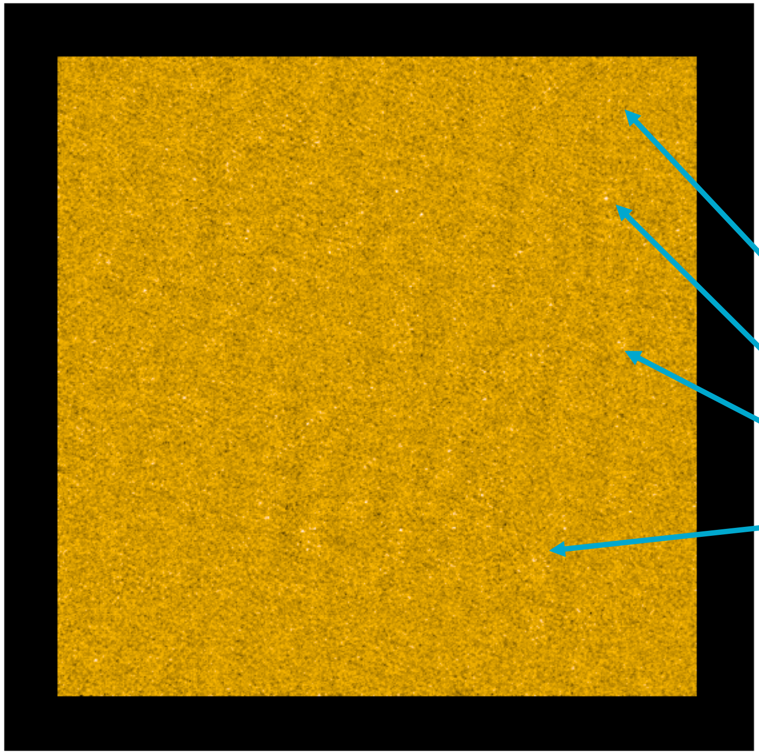
Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (5 of the 50 peeled)

Peeling the 5 brightest sources doesn't have too much of an effect on the residuals.

Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (10 of the 50 peeled)

But as more of the 50 sources are peeled the weaker sources and their sidelobes disappear!

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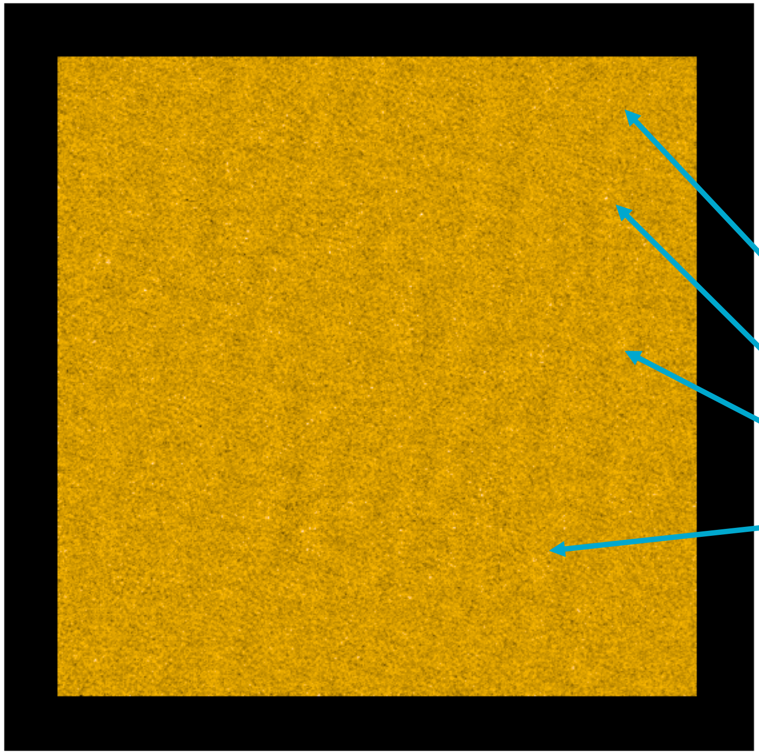
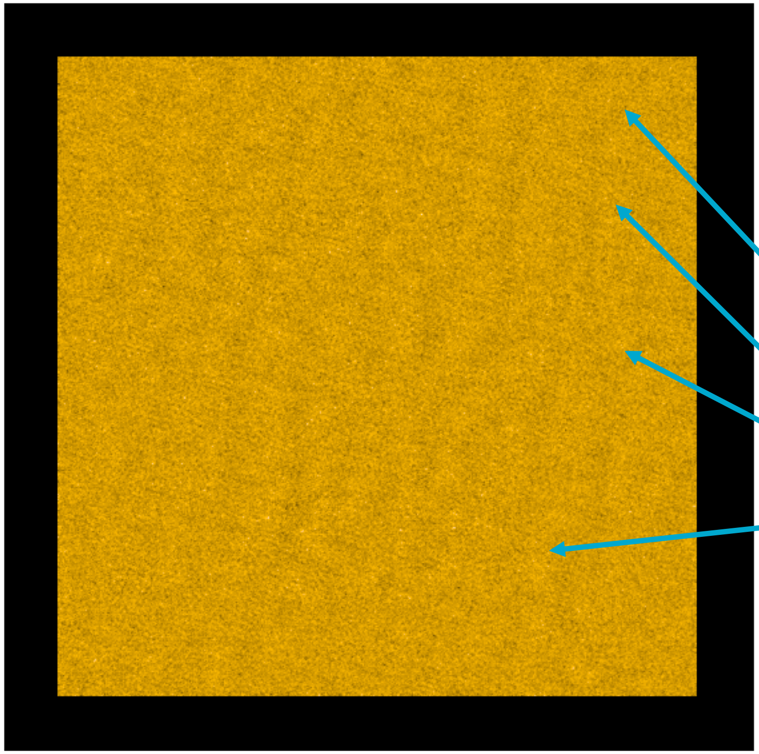
Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (20 of the 50 peeled)

But as more of the 50 sources are peeled the weaker sources and their sidelobes disappear!

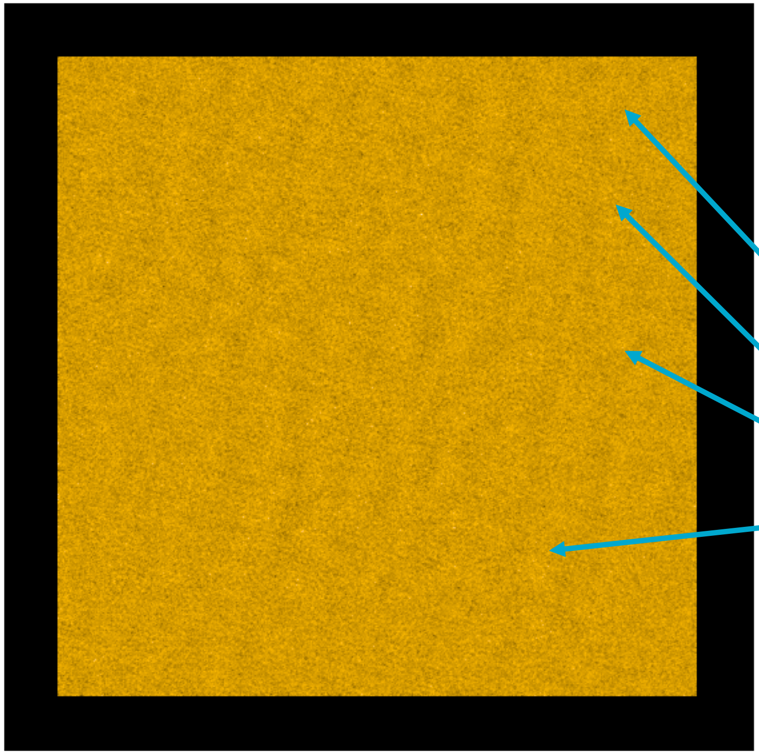
Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (30 of the 50 peeled)

But as more of the 50 sources are peeled the weaker sources and their sidelobes disappear!

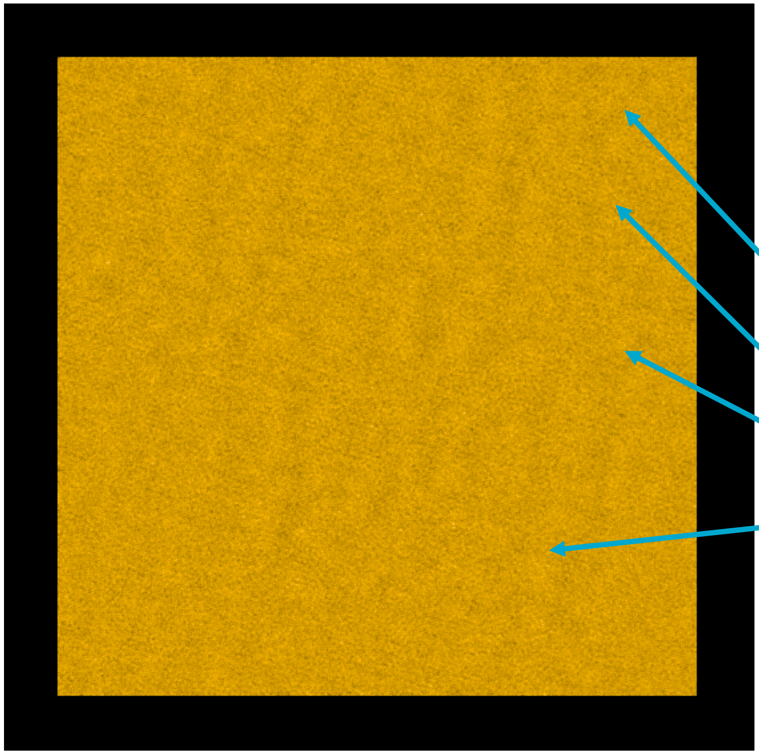
Peeling



Dirty image with direction-independent calibration and 50 sources subtracted (40 of the 50 peeled)

But as more of the 50 sources are peeled the weaker sources and their sidelobes disappear!

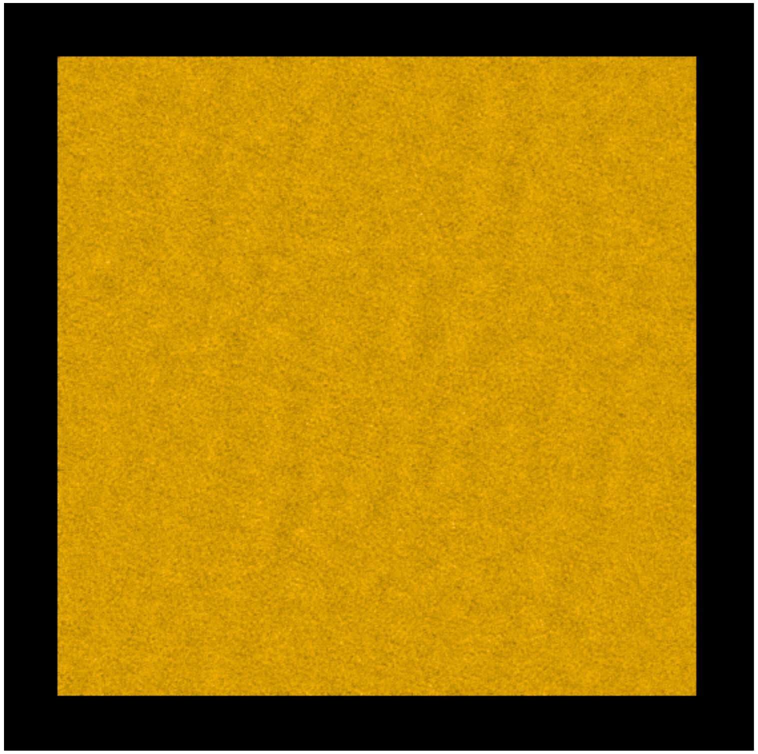
Peeling

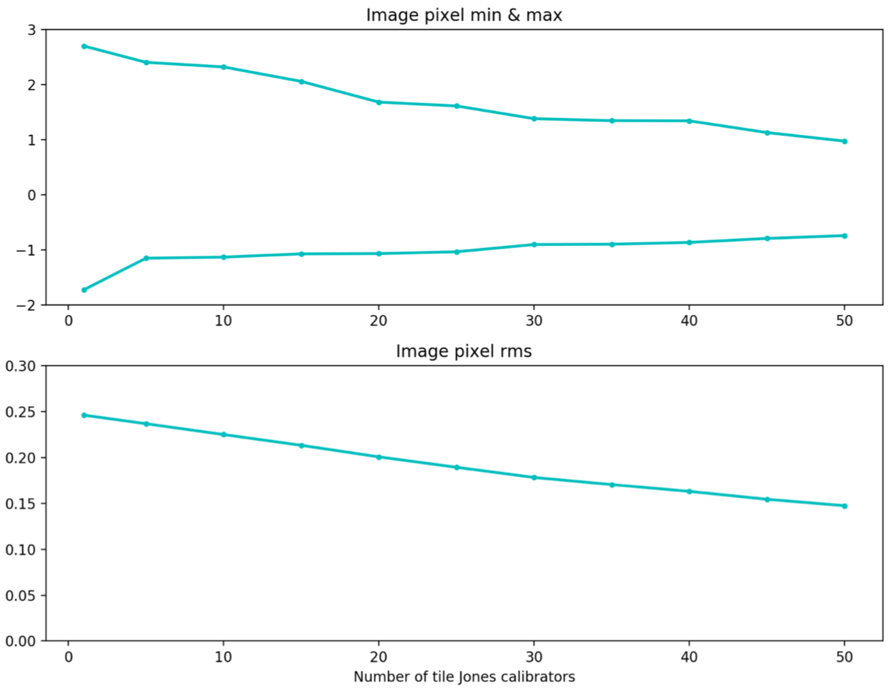


Dirty image with direction-independent calibration and 50 sources subtracted (all 50 peeled)

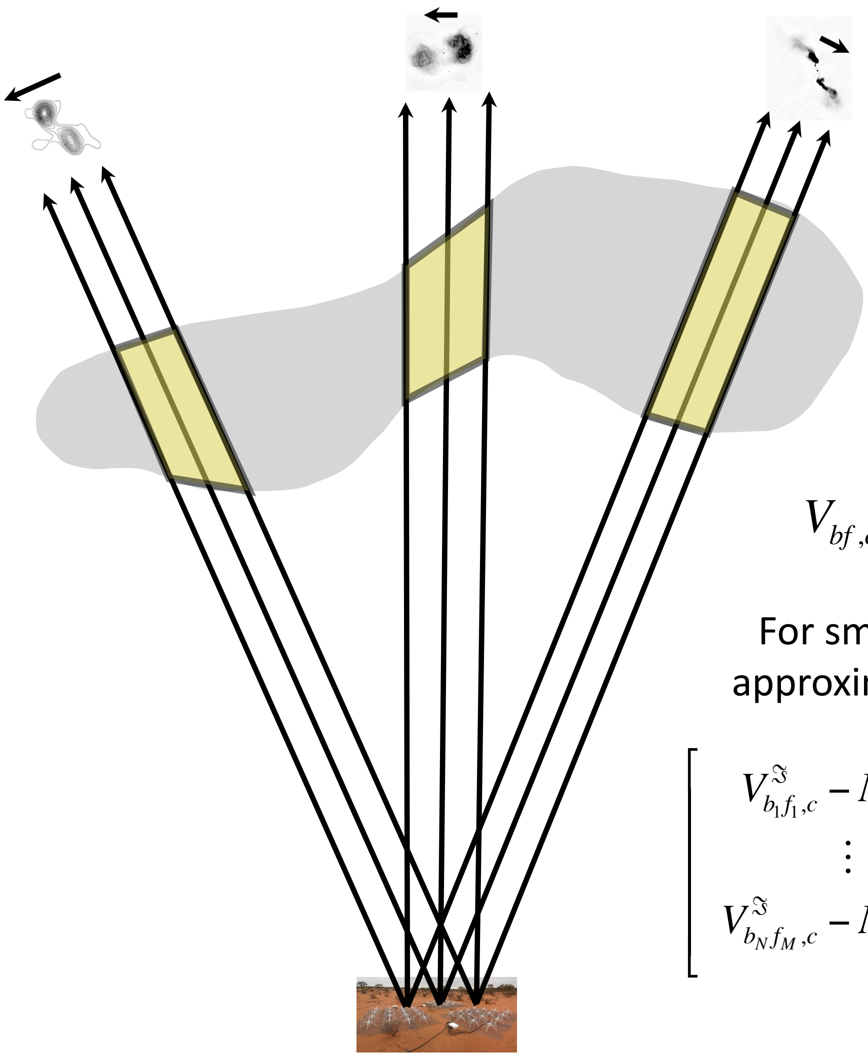
But as more of the 50 sources are peeled the weaker sources and their sidelobes disappear!

Peeling — over-peeling

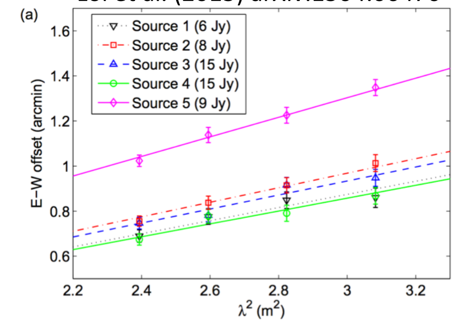




Constrained Peeling → linear phase model



Loi et al. (2015) arXiv:1504.06470

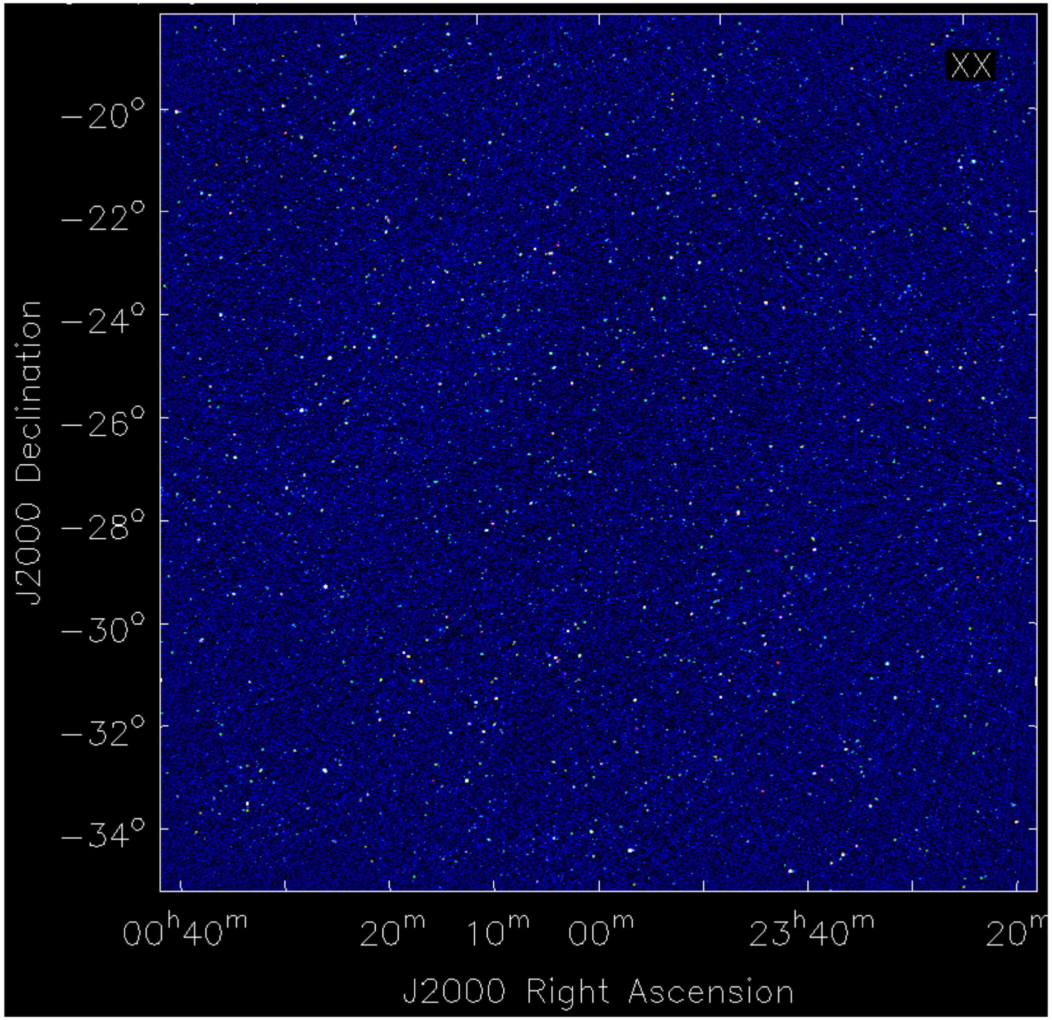


$$V_{bf,c} \approx N_{bf} + M_{bf,c} \exp\left\{-i2\pi\lambda_f^2 \left(u_{bf}\alpha_{l,c} + v_{bf}\alpha_{m,c}\right)\right\}$$

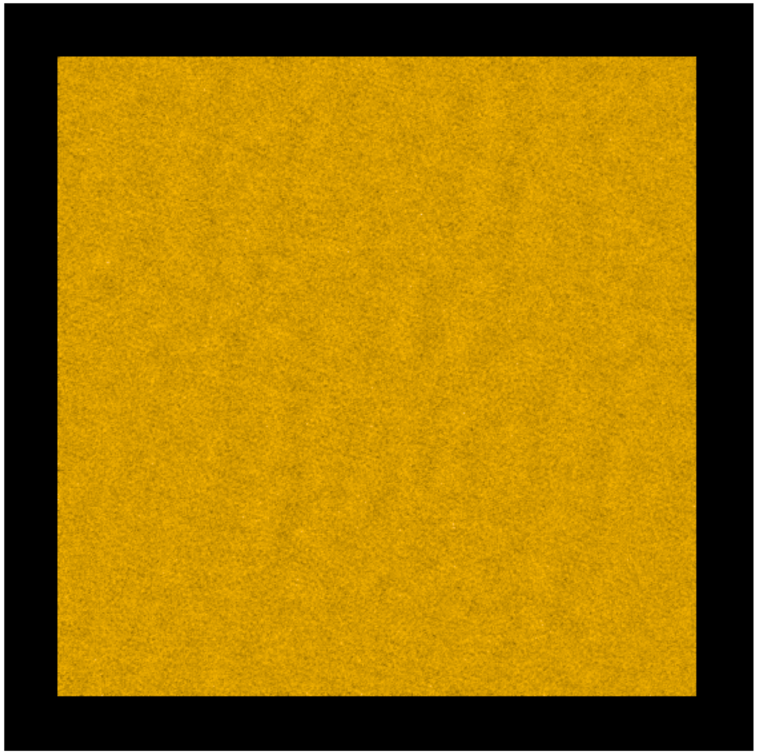
For small offsets ($\lambda^2\alpha$ << synthesised beam size) imag{$V_{bf,c}$} is approximately linear in the α parameters => linear least-squares.

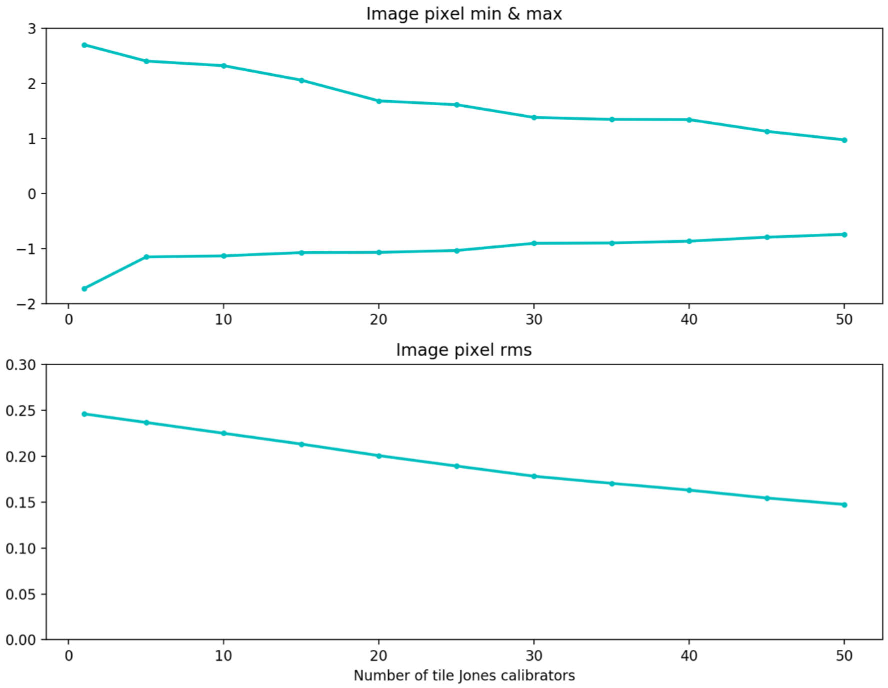
$$\begin{bmatrix} V^{\Im}_{b_1f_1,c} - M^{\Im}_{b_1f_1,c} \\ \vdots \\ V^{\Im}_{b_Nf_M,c} - M^{\Im}_{b_Nf_M,c} \end{bmatrix} \approx -2\pi \begin{bmatrix} \lambda^2_{f_1} M^{\Re}_{b_1f_1,c} u_{b_1f_1} & \lambda^2_{f_1} M^{\Re}_{b_1f_1,c} v_{b_1f_1} \\ \vdots & \vdots \\ \lambda^2_{f_M} M^{\Re}_{b_Nf_M,c} u_{b_Nf_M} & \lambda^2_{f_M} M^{\Re}_{b_Nf_M,c} v_{b_Nf_M} \end{bmatrix} \begin{bmatrix} \alpha_{l,c} \\ \alpha_{m,c} \end{bmatrix}$$

MWA data: 8 seconds, 31 MHz (182 MHz)

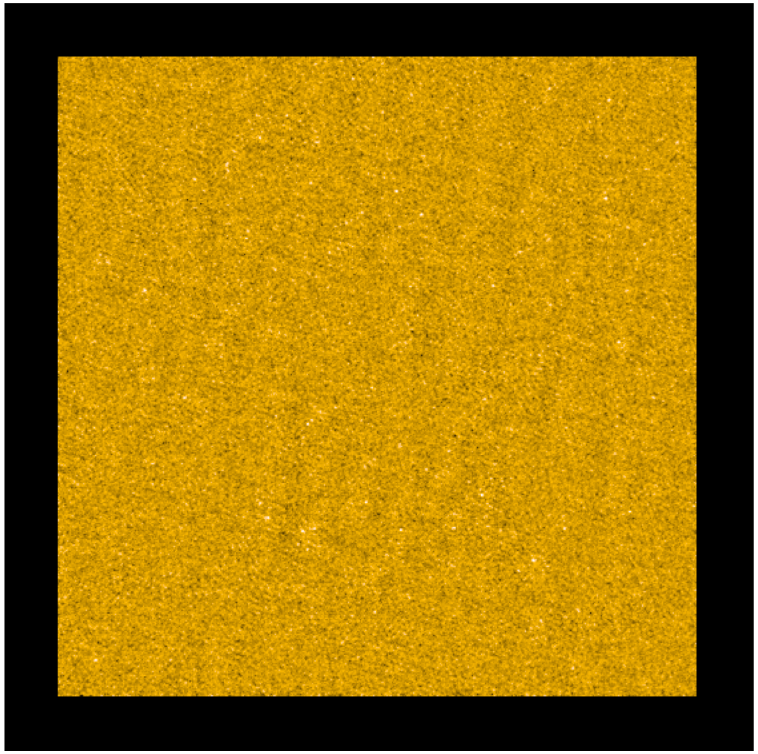


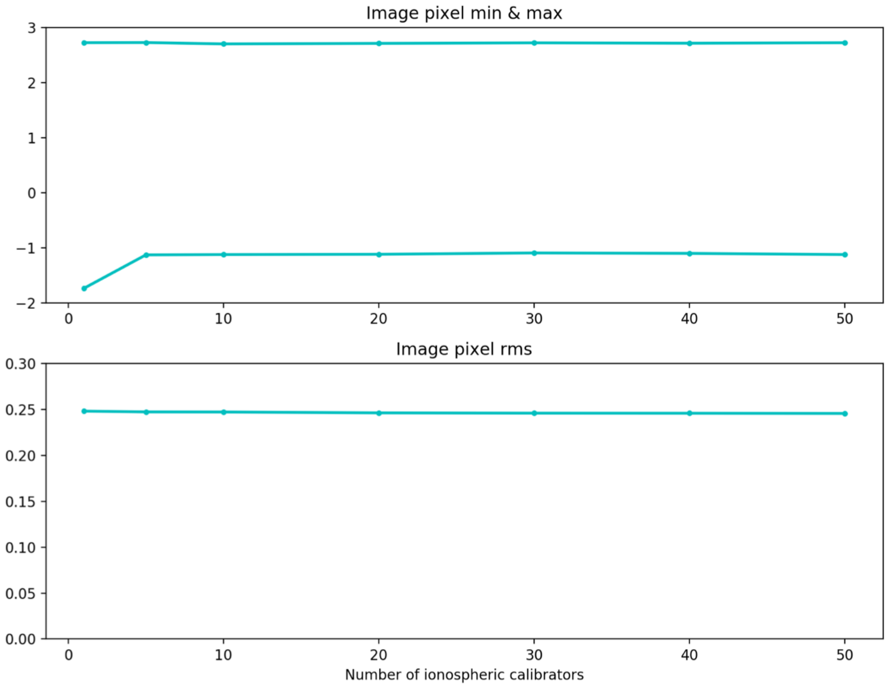
Peeling — over-peeling



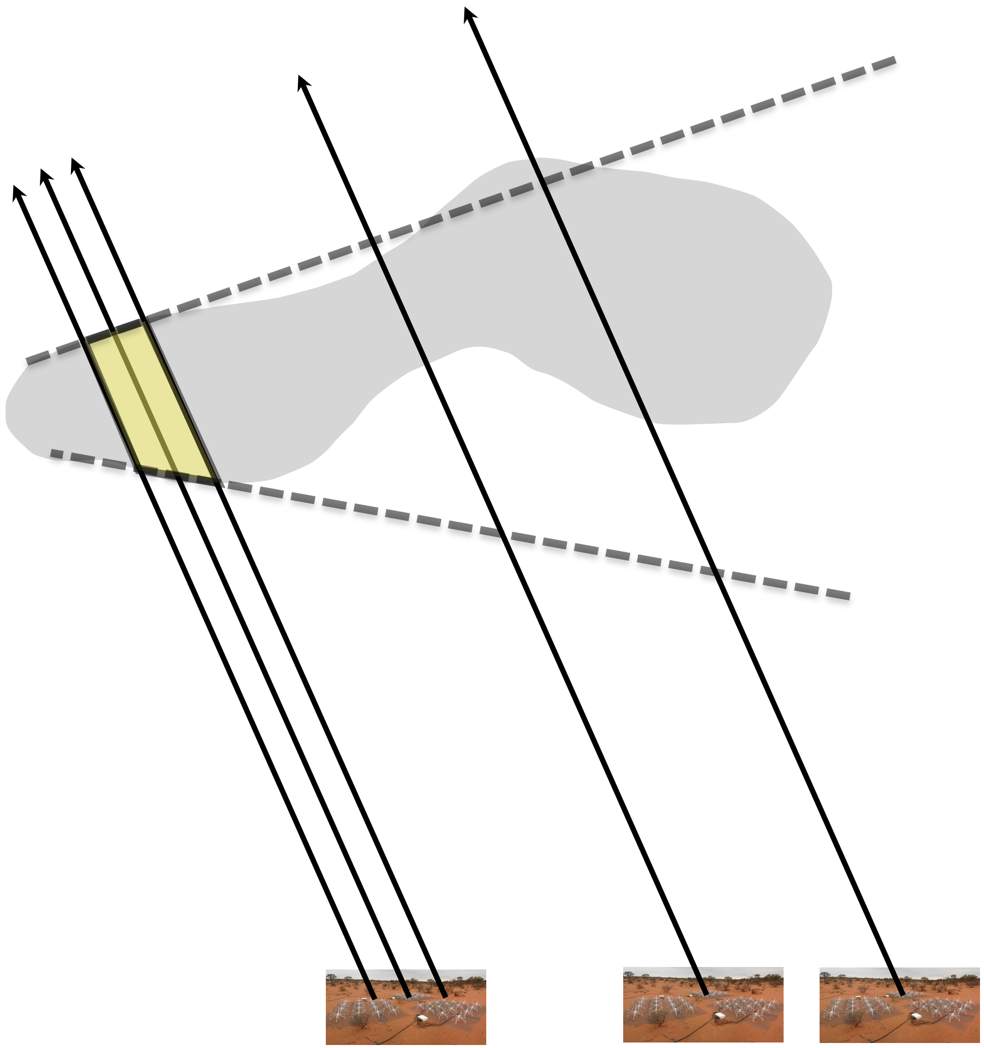


Peeling — ionospheric corrections

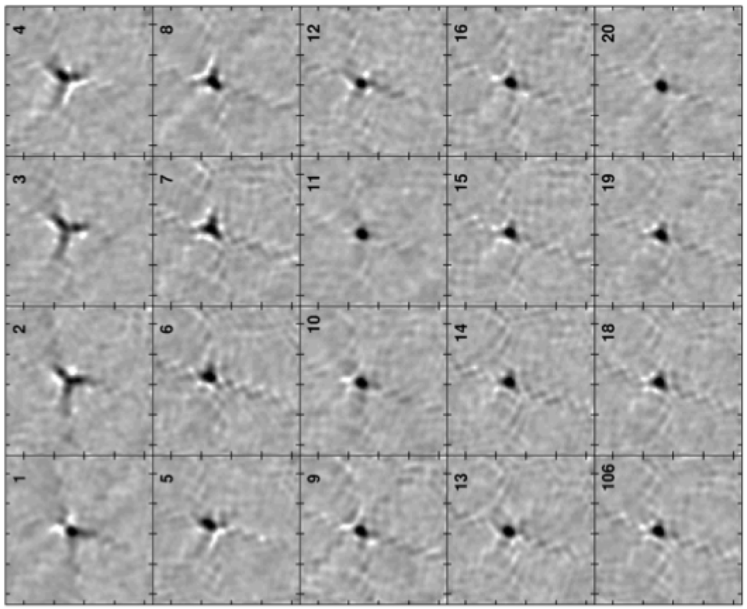


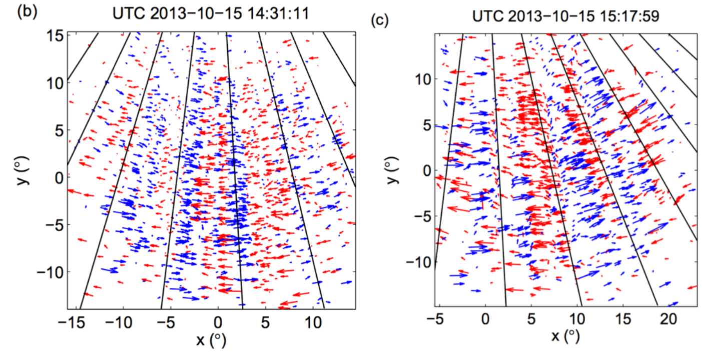


Non-linear ionospheric phases



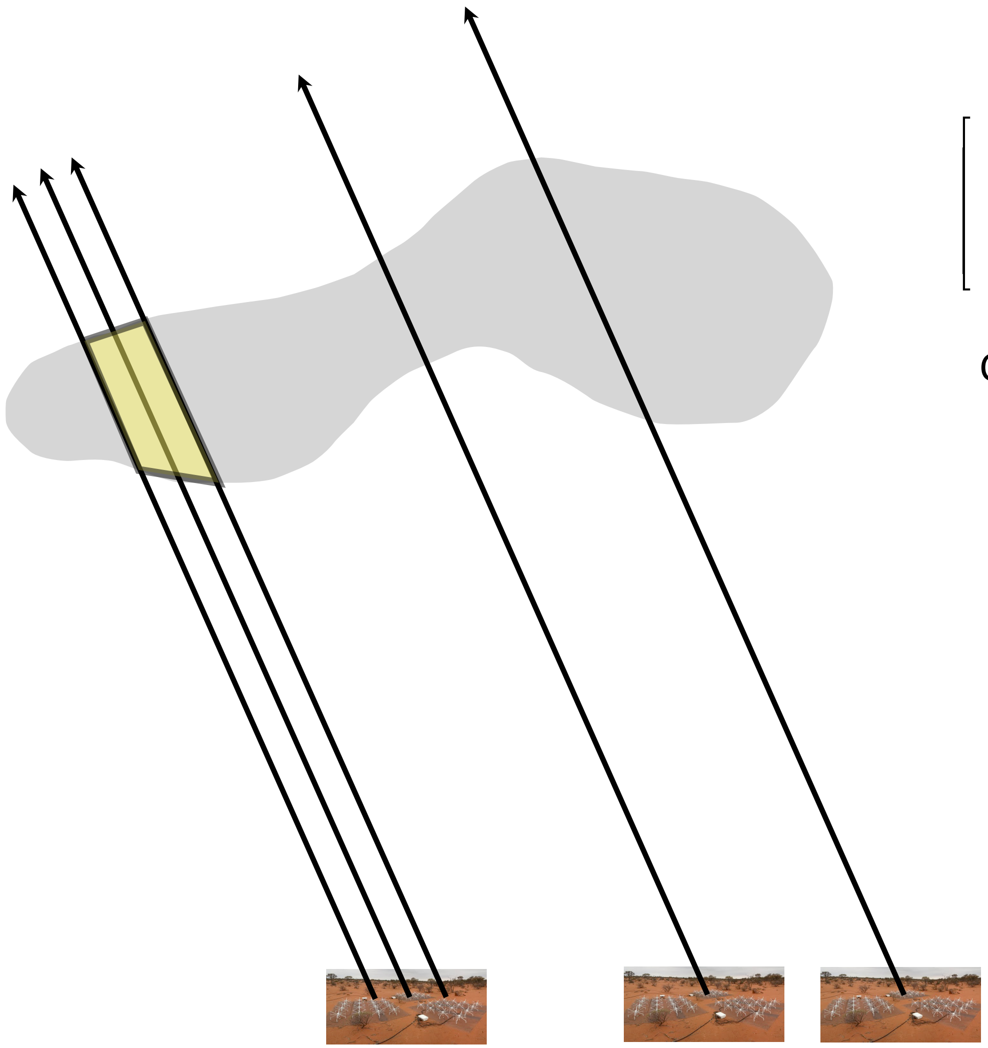
Cotton (2004) ASP Conf. Series 345, 74 MHz, 1-min VLA snapshots





Loi et al. (2015) arXiv:1504.06470

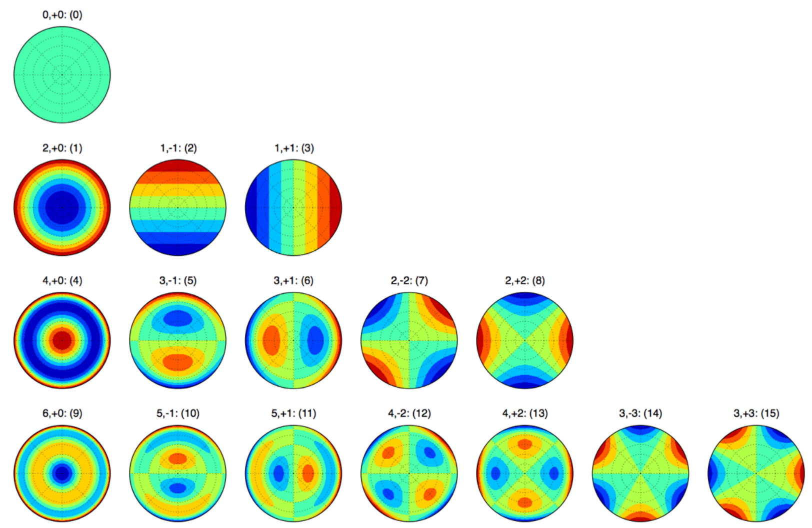
Non-linear ionospheric phases



For example, instead of u & v:

$$\begin{bmatrix} V^{\Im}_{b_1 f_1,c} - M^{\Im}_{b_1 f_1,c} \\ \vdots \\ V^{\Im}_{b_N f_M,c} - M^{\Im}_{b_N f_M,c} \end{bmatrix} \approx -2\pi \begin{bmatrix} \lambda^2_{f_1} M^{\Re}_{b_1 f_1,c} u_{b_1 f_1} & \lambda^2_{f_1} M^{\Re}_{b_1 f_1,c} v_{b_1 f_1} \\ \vdots & \vdots \\ \lambda^2_{f_M} M^{\Re}_{b_N f_M,c} u_{b_N f_M} & \lambda^2_{f_M} M^{\Re}_{b_N f_M,c} v_{b_N f_M} \end{bmatrix} \begin{bmatrix} \alpha_{l,c} \\ \alpha_{m,c} \end{bmatrix}$$

Could use Zernike polynomials:



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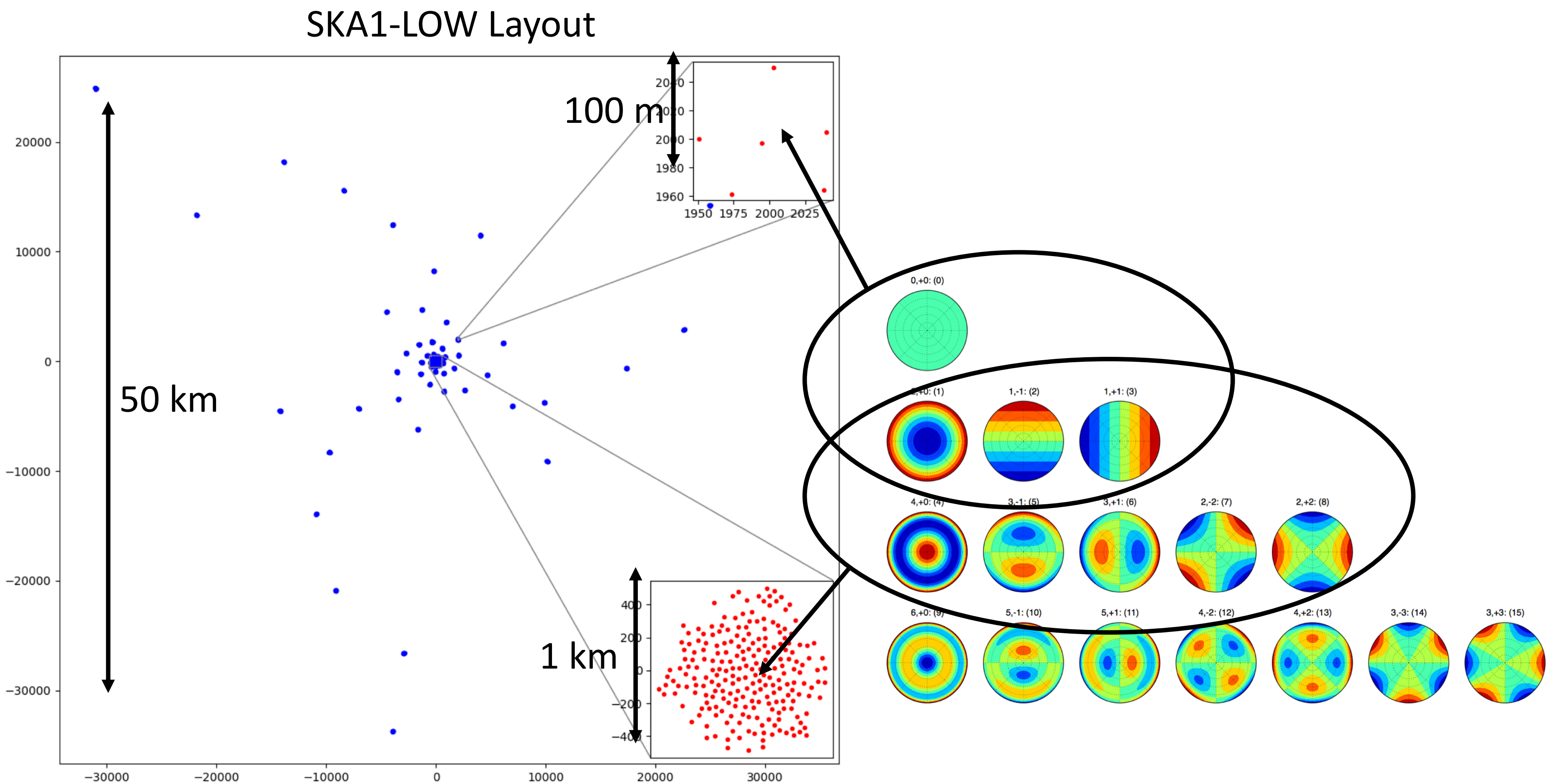
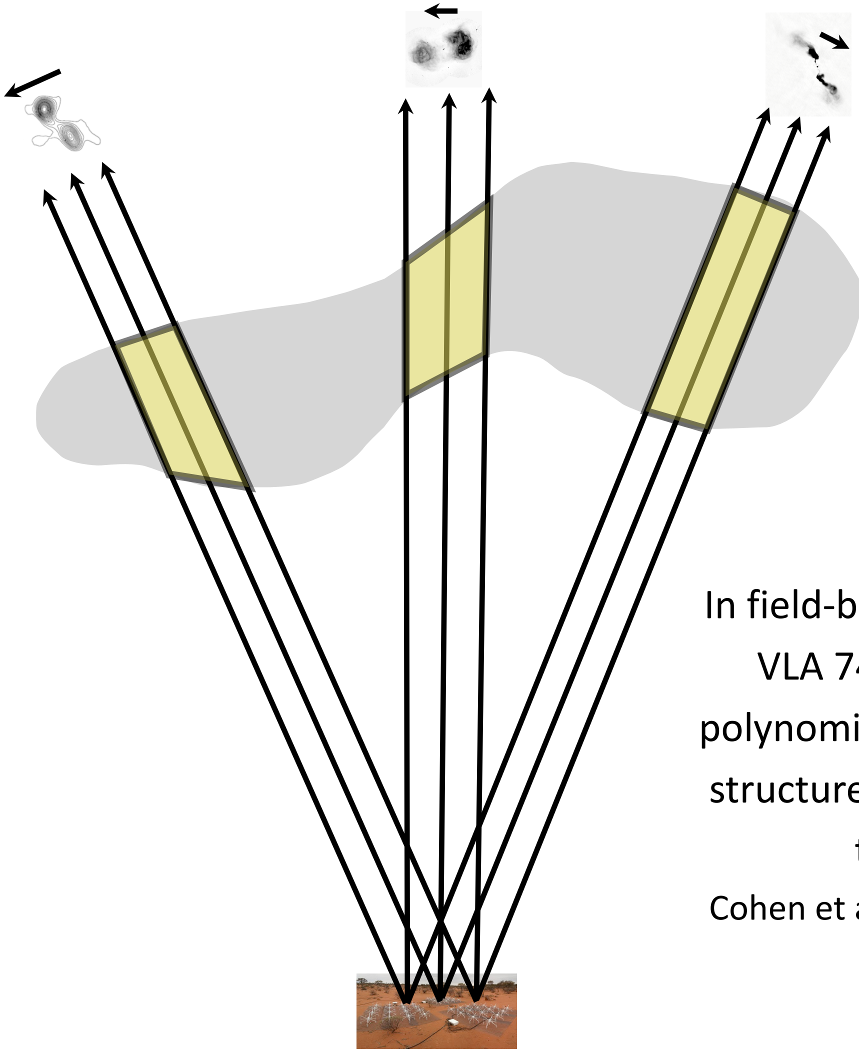
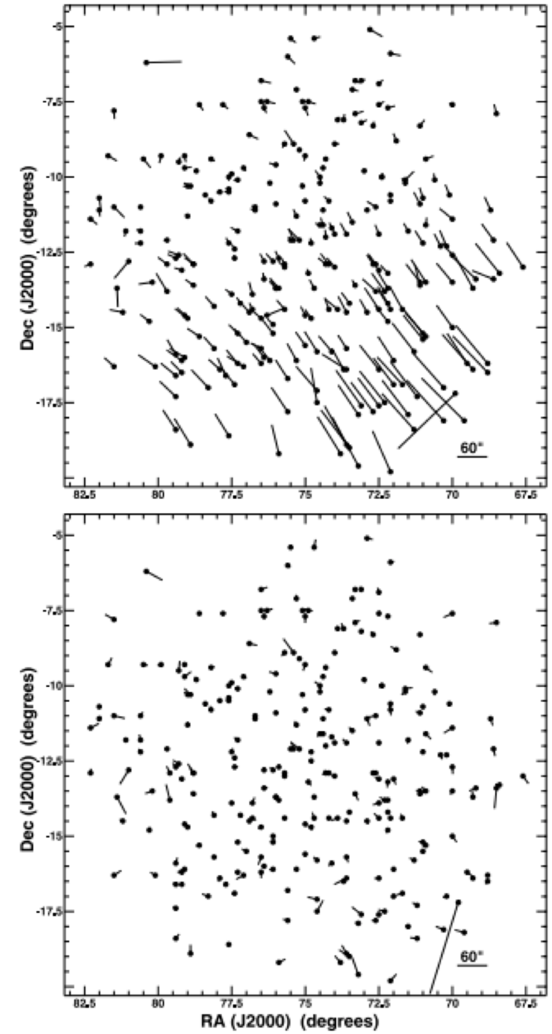
Non-linear ionospheric phases



Field-based calibration



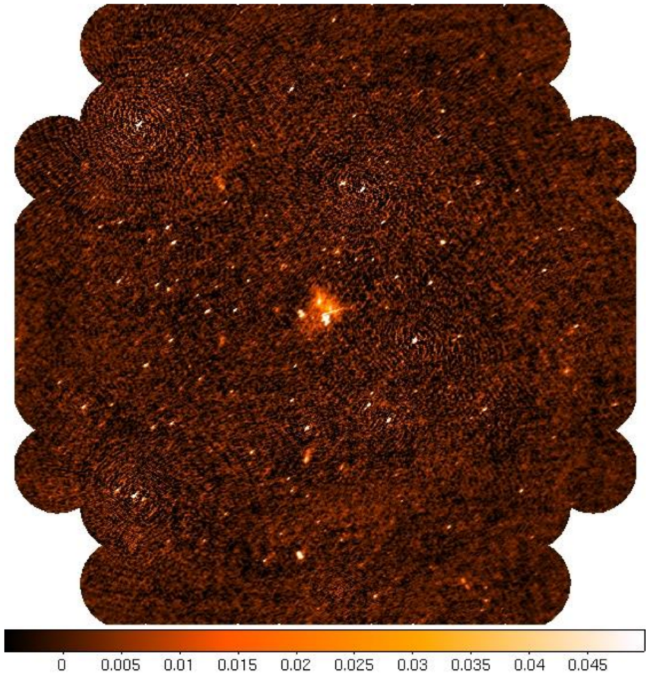


In field-based calibration for the VLA 74 MHz VLSS, Zernike polynomials were used to model structure across the FoV, rather than the array.

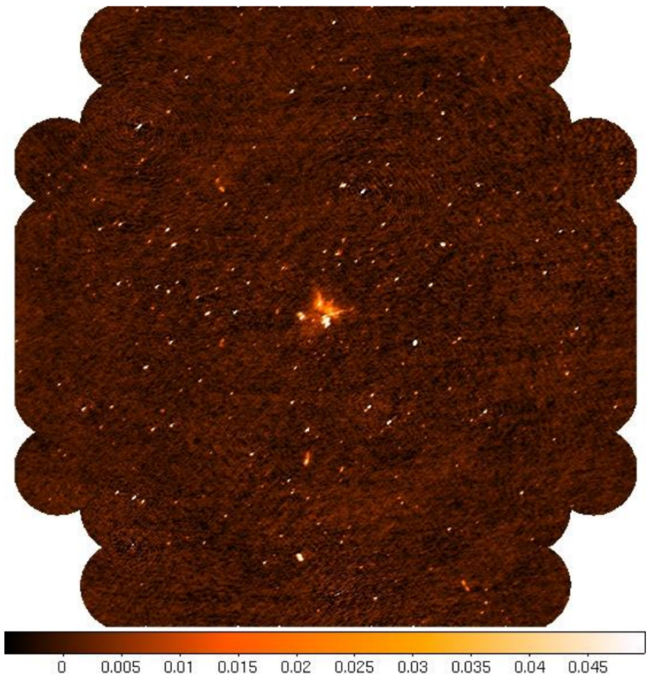
Cohen et al. (2007) arXiv:0706.1191

Model at the ionosphere

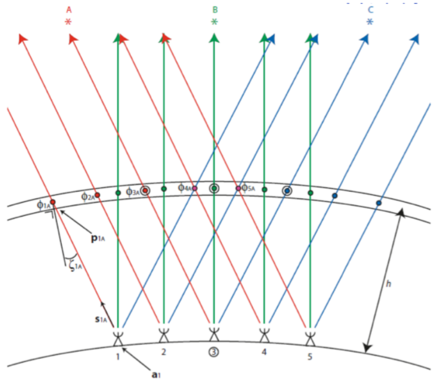
SPAM (Source Peeling and Atmospheric Modeling, arXiv:0904.3975)

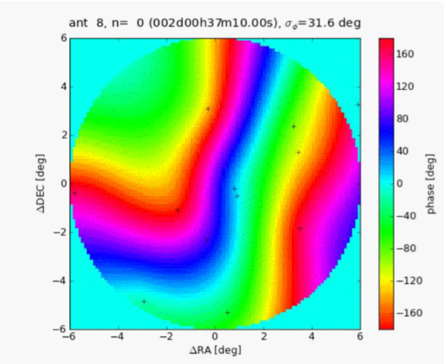


Self-calibration result on Abell 2256
GMRT 150 MHz



SPAM result on Abell 2256
GMRT 150 MHz

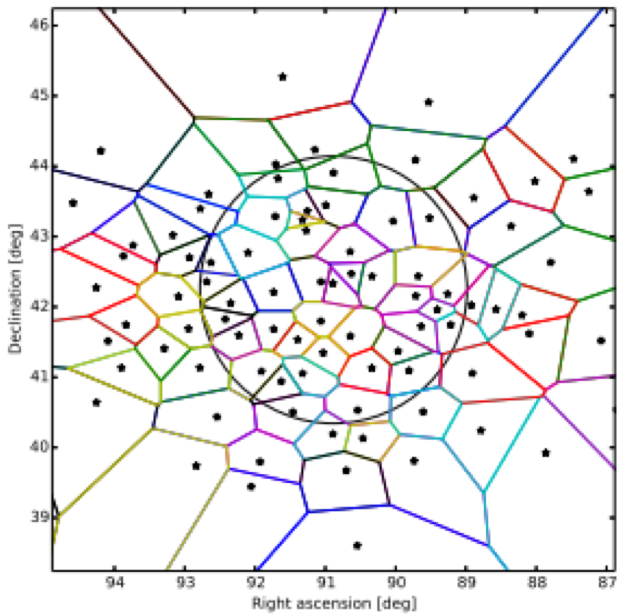




Thanks to Huib Intema

Wide-field imaging approaches

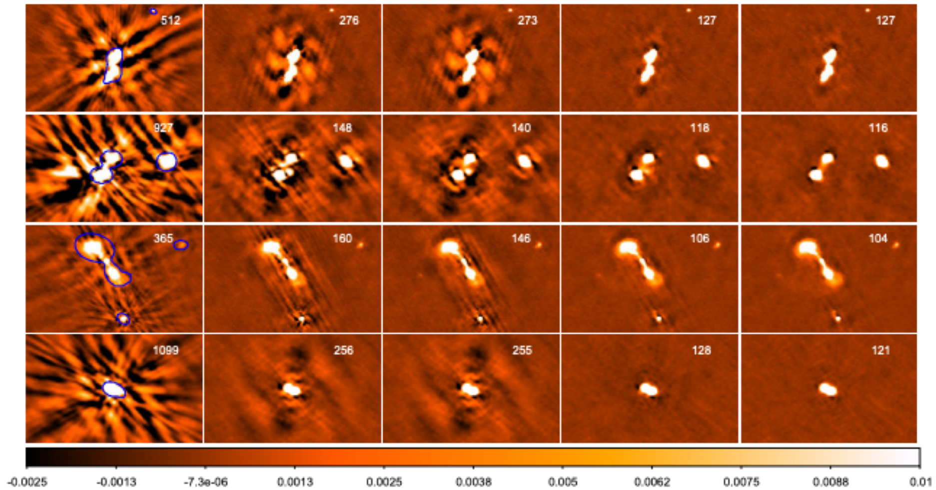
Image Corrections: Image Facets

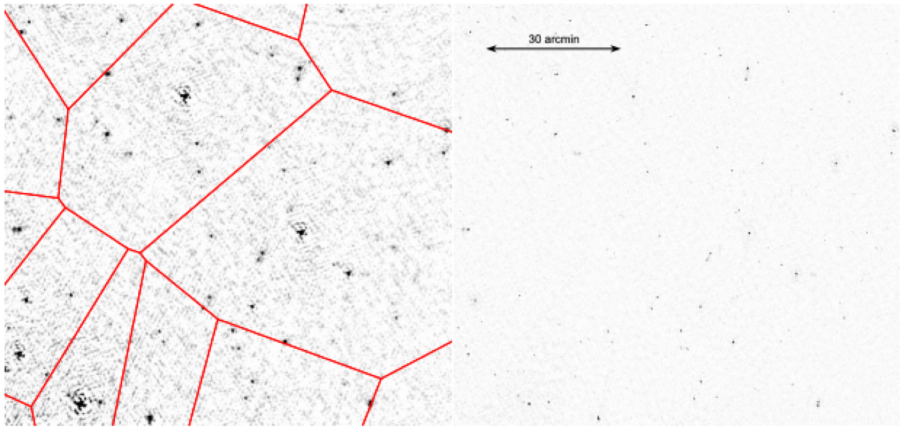


LOFAR Facet Calibration

van Weeren et al. (2016) arXiv:1601.05422

Calibration steps →





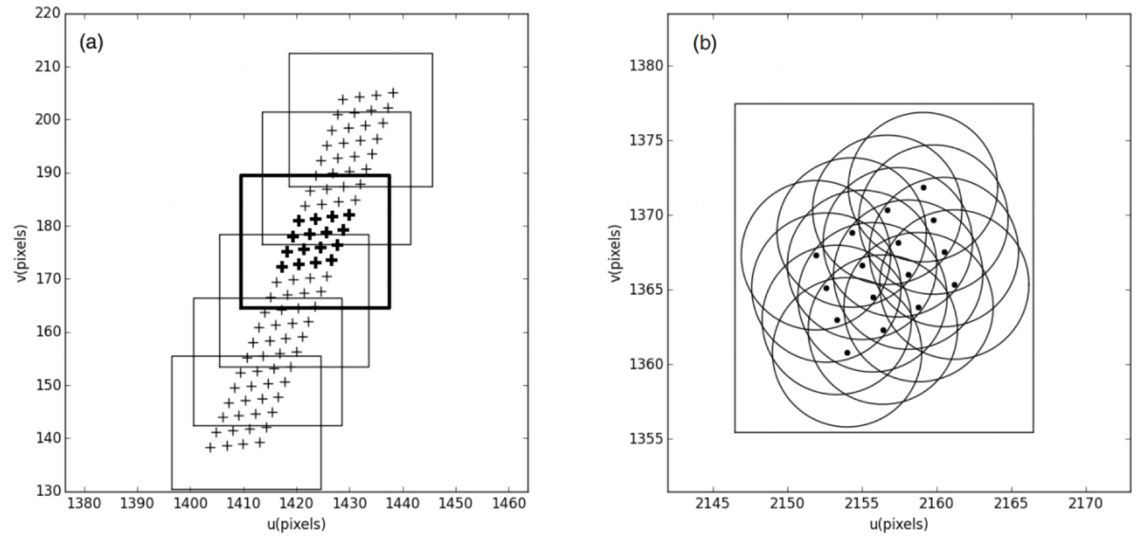
uncorrected corrected

Image Corrections: Visibility Gridding

- Use a convolution kernel with a desired image-domain response
  - e.g. a prolate spheroidal gridding kernel = prolate spheroidal window fn
- Can be different for each visibility:
  - W-projection for w-terms
  - A-projection for A-terms (primary beams, ionosphere, decorrelation, etc)
- Kernels can become very large for wide fields of view
  - Expensive to grid (# pixels $\propto \theta^4$)
  - Expensive to generate & cache ($\propto \theta^6$)
  - Algorithms can become limited by memory or memory-bandwidth.

Image Corrections: Visibility Gridding

Image-domain gridding

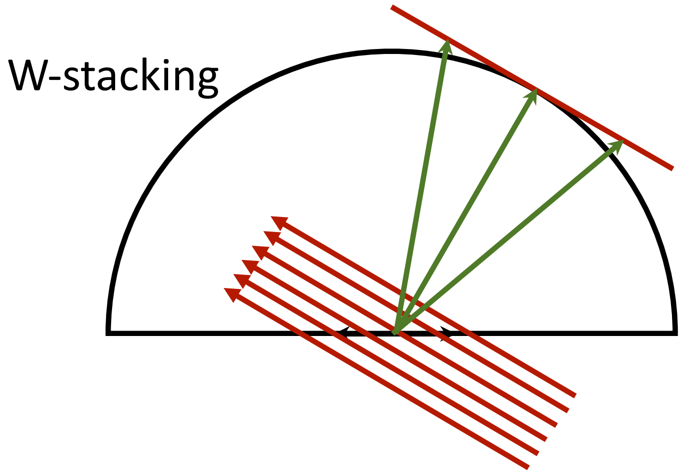


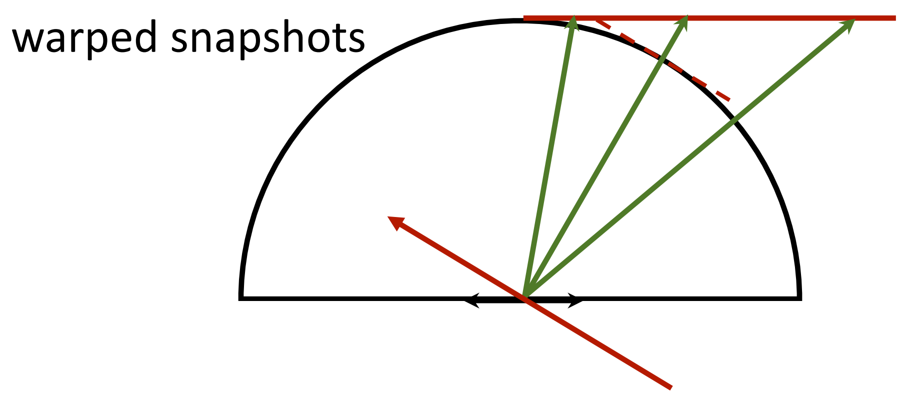
van der Tol, et al. (2018) A&A 616, A27

- transform small regions of the uv plane back to the image plane and apply the convolutions as multiplications
- e.g. $\mathfrak{F}(\ A^*_{jk} \times e^{i2\pi w_{jk}\left(\sqrt{1-l^2-m^2}-1\right)} \times e^{i2\pi(ul+vm)} \times V_{jk}))$

Image Corrections: Visibility Segmentation

- Grid visibilities to their nearest plane
- FFT each plane separately
- Apply wide-field corrections
- Stack



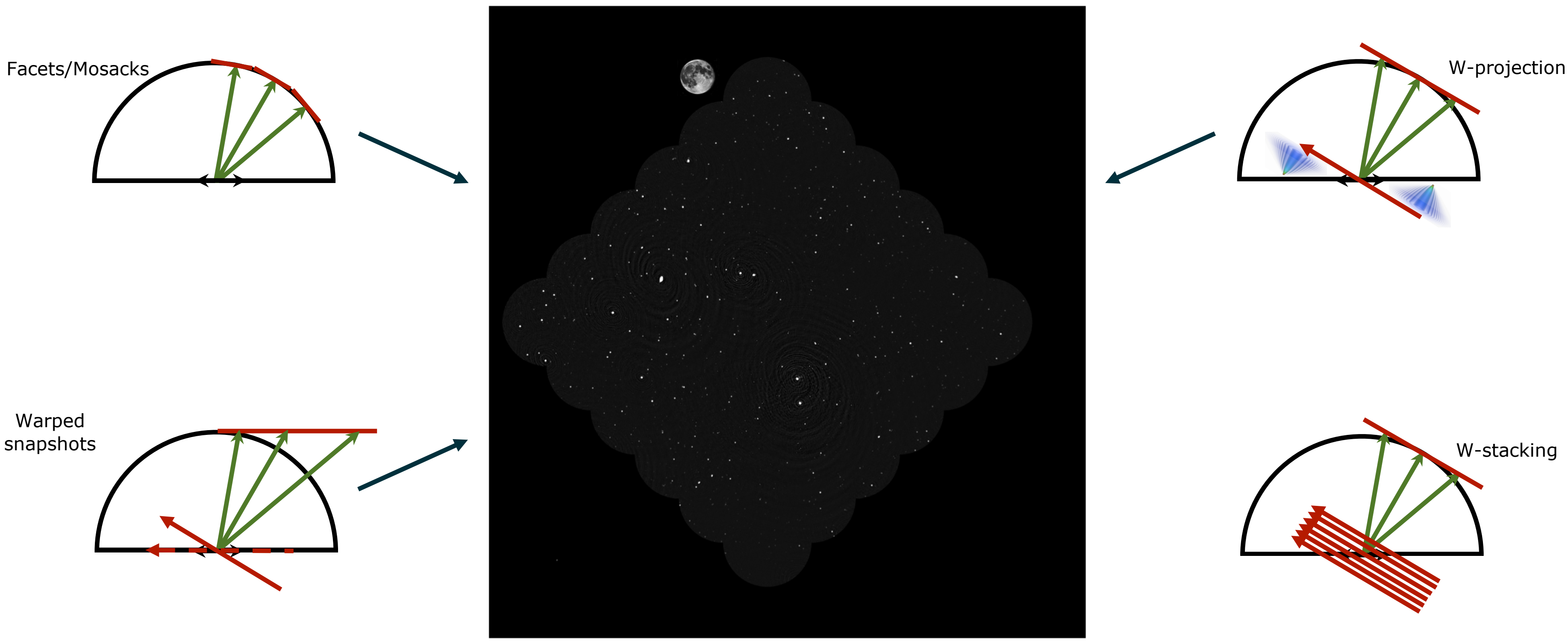


- Grid vis to the nearest w plane,
- FFT each plane
- Correct planes via multiplications

- Grid vis to the nearest time plane,
- FFT each plane
- Correct planes via regridding

Combined Approaches

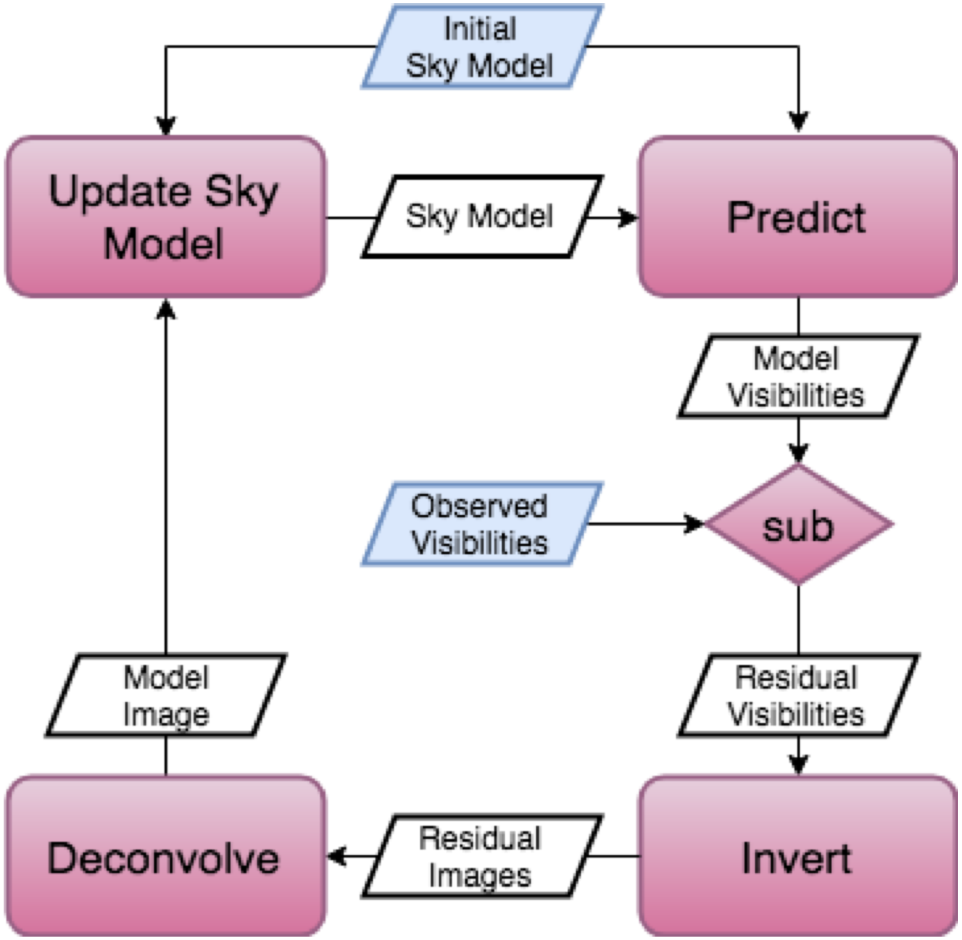
Most packages support multiple approaches that can be / have been blended together to meet computing and/or scientific needs.



ASKAPSoft example

Major Cycle

- Accumulate CLEAN components?
- Run source finder on restored images?



High-accuracy generation of model visibilities

Subtract model visibilities (from observed or previous residual set)

Image residual visibilities

Minor Cycle: CLEAN down to the sidelobe level of brightest pixel in the residual image

Thank you

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