## Probing gravity in the strong field regime

# Universality of free fall from the orbital motion of a pulsar in a stellar triple system 

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Einstein's theory of gravity-the general theory of relativity ${ }^{1}$-is based on the universality of free fall, which specifies that all objects accelerate identically in an external gravitational field. In contrast to almost all alternative theories of gravity ${ }^{2}$, the strong equivalence principle of general relativity requires universality of free fall to apply even to bodies with strong self-gravity. Direct tests of this principle using Solar System bodies ${ }^{3,4}$ are limited by the weak selfgravity of the bodies, and tests using pulsar-white-dwarf binaries ${ }^{5,6}$ have been limited by the weak gravitational pull of the Milky Way. PSR J0337+1715 is a hierarchical system of three stars (a stellar triple system) in which a binary consisting of a millisecond radio pulsar and a white dwarf in a 1.6-day orbit is itself in a 327-day orbit
nonlinearity of gravity, and $\gamma$, which measures the degree to which gravity curves space-time. Both of these parameters take the value 1 in general relativity. We chose a point-particle Lagrangian that permits arbitrarily strong gravity internal to the bodies and parameterized post-Newtonian interactions between them ${ }^{10}$. We then used computer algebra ${ }^{11}$ to construct equations of motion. Each orbit was specified by an initial system configuration at modified Julian date (MJD) 55,920.0 (2011 December 25 00:00:00 UTC). The evolution of this configuration was governed by $\beta, \gamma$ and the strong equivalence principle (SEP)violation parameter $\Delta$. Because the self-gravity of the pulsar (which is a neutron star) exceeds that of the white dwarfs by a factor $10^{4}$ and the SEP violation that we seek arises from self-gravity, we neglect possi-

## "Pulsar Timing"

## Using pulsars as precision clocks



## Pulsar Timing Model

Input to PTAs
Basic Method
Actual Pulse TOA

- Thooretical Model
$=$ Timing Residual



$$
T_{\mathrm{th}}=\nu t+\frac{1}{2} \dot{\nu} t^{2}+D \frac{\int_{0}^{d} n_{e} d l}{f^{2}}-\frac{1}{c}(\vec{r} \cdot \hat{s})+\frac{V_{\mathrm{T}}^{2} t^{2}}{2 c d}-\frac{(\vec{r} \times \hat{s})^{2}}{2 c d}+\ldots
$$

## What does this teach us?



Count each pulse... for years.

## PSR JIOI 2+5307:

$P=0.005255749014115410$ +/- 0.000000000000000015s

## > 100 billion pulses in the last I5 years, and not a single one missed



## Pulsar Recycling

Alpar, Cheng, Ruderman \& Shaham 1982
Rhadakrishnan \& Srinivasan 1982


LMXB (some IMXB)

## Millisecond pulsars are the products of astrophysical accretion

## Keplerian Timing Effects



Need mass function + two other equations for m1, m2, and i

## Post-Keplerian Effects

Periastron adv. $\dot{\omega}=3\left(\frac{P_{b}}{2 \pi}\right)^{-5 / 3}\left(T_{\odot} M\right)^{2 / 3}\left(1-e^{2}\right)^{-1}$
Grav. redshift $\quad \gamma=e\left(\frac{P_{b}}{2 \pi}\right)^{1 / 3} T_{\odot}^{2 / 3} M^{-4 / 3} m_{2}\left(m_{1}+2 m_{2}\right)$
Orbital decay
Orbital decay
$\dot{P_{b}}=-\frac{192 \pi}{5}\left(\frac{P_{b}}{2 \pi}\right)^{-5 / 3}\left(1+\frac{73}{24} e^{2}+\frac{37}{96} e^{4}\right)\left(1-e^{2}\right)^{-7 / 2} T_{\odot}^{5 / 3} m_{1} m_{2} M^{-1 / 3}$
shapiro delay $\left\{\begin{array}{c}r=T_{\odot} m_{2} \quad \text { "Range" } \\ s=x\left(\frac{P_{b}}{2 \pi}\right)^{-2 / 3} T_{\odot}^{-1 / 3} M^{2 / 3} m_{2}^{-1} \text {. } \quad \operatorname{sin~i~"Shape"~}\end{array}\right.$
Depend on m 1 , m 2 , and the Keplerian parameters Measure any 2 PK params and get m1, m2

## Nobel Prize Physics 1993



Russell Hulse \& Joseph Taylor
"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"


## Pulsar riddle

## PSR J0337+1715



Time

## Pulsar riddle

## PSR J0337+1715



WHITE DWARF

## PULSAR J0337+1715

WHITE DWARF

$$
\begin{gathered}
\text { Outer Orbit } \\
P_{\text {orb }}=327 \text { days } \\
\mathrm{MwD}_{\mathrm{wd}}=0.41 \mathrm{M}_{\text {sun }}
\end{gathered}
$$

## PSR 10337+1715 . Inner Orbit

Triple System
$P_{\text {orb }}=1.6$ days
$M_{\text {PSR }}=1.44 M_{\text {sun }}$ $M_{w d}=0.20 M_{\text {sun }}$



## Pulsar riddle

PSR J0337+ 1715
Dynamical interactions between the two nested binaries


Westerbork data

## Pulsar riddle

## PSR J0337+1715



Work done by Anne Archibald, directly integrating the motions of the 3 bodies

## J0337+17 I5 - Timing model



Pulsar mass: I.4378(13) MSun "Inner" WD mass: 0.1975I(I5) MSun "Outer"WD mass: 0.410I(3) MSun



## Strong Equivalence Principle



All objects should fall with same acceleration regardless of their mass or composition

## Effects of an SEP violation

Key idea: test whether two bodies fall the same way in the gravitational field of a third
Need: binary falling in an external gravitational field

- Earth and Moon falling in Sun's gravity (LLR)
- Pulsar-WD binary falling in Galactic potential (e.g. Gonzalez et al.)
- Triple system: pulsar and inner WD falling in gravity of outer WD

Fractional difference in acceleration ( $\Delta=M_{g} / M_{i}-1$ ) shifts the massive object's orbit in the direction of the external acceleration


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## Observations

| Tel. | Band | Num. | Hours | Date range |
| :--- | :---: | :---: | :---: | :---: |
| AO | 1400 | 92 | 58.9 | 2012 Mar-2017 Mar |
| GBT | 1400 | 172 | 236.0 | 2012 Feb-2017 May |
| WSRT | 1400 | 439 | 836.7 | 2012 Jan - 2013 Jul |
| AO | 430 | 36 | 12.9 | 2012 May-2017 Mar |
| WSRT | 350 | 20 | 17.3 | 2012 Feb-2013 Jul |



## Timing model

No adequate formula is known for directly describing the three-body orbit, so we use direct integration of equations of motion:

$$
\begin{equation*}
F_{j}=M_{j, l} a_{j} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
F_{j}=-\sum_{k} \frac{G M_{j, G} M_{k, G}}{r_{j k}^{2}} \hat{r}_{j k} \tag{2}
\end{equation*}
$$

A standard ODE solver allows us to calculate an orbit given initial conditions.
This scheme is easily adapted to allow gravitational mass different from inertial mass.

## Relativistic timing model

- Nordtvedt (1985) derives a "point particle" Lagrangian
- Taylor expansion around the Newtonian Lagrangian
- Lorentz invariance and symmetry used to eliminate terms
- Bodies may contain strong fields but internal structure is frozen
- Fields away from bodies approximated to first post-Newtonian order
- Computer algebra straightforwardly yields equations of motion
- Direct integration simulates orbits

$$
\begin{aligned}
L_{P P N}= & -\sum_{i} M_{i, l}\left(1-\frac{v_{i}^{2}}{2}-\frac{v_{i}^{4}}{8}\right) \\
& +\frac{1}{2} \sum_{i, j} \frac{M_{i, G} M_{j, G}}{r_{i j}}\left(1+\frac{v_{i}^{2}+v_{j}^{2}}{2}-\frac{3 v_{i} \cdot v_{j}}{2}-\frac{\left(v_{i} \cdot \hat{r}_{i j}\right)\left(v_{j} \cdot \hat{r}_{i j}\right)}{2}\right) \\
& +\frac{\gamma}{2} \sum_{i, j} \frac{M_{i, G} M_{j, G}}{r_{i j}}\left(v_{i}-v_{j}\right)^{2}+\left(\frac{1}{2}-\beta\right) \sum_{i, j, k} \frac{M_{i, G} M_{j, G} M_{k, G}}{r_{i j} r_{i k}}
\end{aligned}
$$

## Testing the SEP



In principle we simply:

- include $\Delta$ in the timing model,
- fit timing model to TOAs, and
- determine best-fit values and uncertainties.

Ideally, the value of $\Delta$ and its uncertainty would determine how well we constrain SEP violation and whether GR is violated.

But: only correct once we've accounted for all systematics, and formally the effects of $\Delta$ are constrained at the 7 ns level.

## Known systematics

| Cause | Remedy |
| :--- | :--- |
| Profile variation with frequency | TOAs no more than 20 MHz |
| Telescope polarization variations | Matrix template matching |
| Intrinsic profile variations | $?$ |
| Interstellar DM variations | Variable DM fitting |
| Interplanetary medium effects | IPM fitting |
| Tidal effects in inner WD | Too small |
| GW losses | Too small |
| Red noise | Too small at freq. of interest |
| Uncertainty in DE435 ephemeris | Position fitting |
| Kopeikin and inverse parallax | Too small |
| Kabouters | $?$ |

We need to estimate the impact of unknown or poorly modeled systematics.

## The signature of an SEP violation

Key idea: look for structure in the residuals that looks like SEP violations.
SEP violation produces a shift in the pulsar's orbit toward the the outer companion: approximately a sinusoid with frequency $2 f_{\text {inner }}-f_{\text {outer }}$.


## The signature of an SEP violation

Key idea: look for structure in the residuals that looks like SEP violations.
SEP violation produces a shift in the pulsar's orbit toward the the outer companion: approximately a sinusoid with frequency $2 f_{\text {inner }}-f_{\text {outer }}$.


(turns)

## Wiggles in our residuals

Look at sinusoids with frequency $k f_{\text {inner }}+l f_{\text {outer }}$ :


Estimate no more than $\sim 77 \mathrm{~ns}$ in the SEP position based on distribution of all arrows.

## Best-fit values

When we carry out the basic fitting, we obtain

$$
\Delta=(-1.1 \pm 0.2) \times 10^{-6} .
$$

But: that's a $\sigma$ corresponding to a 7 ns uncertainty. If we take into account all the wiggles we see in the data from our arrow plot we get a more realistic $\sigma$ corresponding to a 22 ns uncertainty:

$$
\Delta=(-1.1 \pm 0.7) \times 10^{-6}
$$

We conclude that our result agrees with General Relativity at the $1.6 \sigma$ level.

## An upper limit on SEP violation

With the best-fit value and uncertainty we computed, we can set a $2 \sigma$ upper limit on SEP violation. We can say that for a $1.4378 M_{\odot}$ neutron star, its acceleration differs from that of its white dwarf companion:

$$
|\Delta|<2.6 \times 10^{-6}
$$

Fundamentally, this difference in acceleration is the key quantity we limit. So we constrain any theory that predicts such an anomalous difference in acceleration, for example, Einstein-Aether or scalar-tensor theories.

But: how does our result compare to existing tests?

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$$

(Triple system)

The wide pulsar-white-dwarf binary PSR J1713+0747 falling in the Galactic potential gives:

$$
\begin{equation*}
|\Delta|<2 \times 10^{-3} \tag{WB}
\end{equation*}
$$

But: how do we compare this to lunar laser ranging or dipole gravitational wave tests?

## The Nordtvedt parameter

In PPN we measure a theory's SEP violation by using the Nordtvedt parameter:

$$
\Delta=\eta_{N} \frac{E_{g}}{M c^{2}}
$$

Lunar Laser Ranging constrains the Earth-Moon-Sun system to $|\Delta|<1.3 \times 10^{-13}$, and for the Earth $E_{g} / M c^{2} \sim-4.5 \times 10^{-10}$, so $\left|\eta_{N}\right|<2.4 \times 10^{-4}$.
In the triple system, the pulsar interior is not 1PN!

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In the triple system, the pulsar interior is not 1PN!
We can calculate the "strong-field Nordtvedt parameter" $\hat{\eta}_{N}$ the same way:

$$
\Delta=\hat{\eta}_{N} \frac{E_{g}}{M c^{2}}
$$

Since $|\Delta|<2.6 \times 10^{-6}$ and $E_{g} / M c^{2} \sim-0.1,\left|\hat{\eta}_{N}\right|<2.6 \times 10^{-5}$ - improving on LLR by a factor of about ten.

But: funny things can happen in the strong field!

## Quasi-Brans-Dicke scalar-tensor theories

These theories include a scalar field $\phi$ in addition to the metric that mediates gravity. Matter responds to a modified version of the metric:

$$
\tilde{g}_{\mu \nu}=e^{2\left(\alpha_{0} \phi+\beta_{0} \phi^{2} / 2\right)} g_{\mu \nu}^{*}
$$

The scalar field is sourced in matter:

$$
\square \phi=-\frac{4 \pi G^{*}}{c^{4}}\left(\alpha_{0}+\beta_{0} \phi\right) T_{*}
$$

If $\beta_{0} \lesssim-4$ spontaneous scalarization can occur, resulting in order-unity deviations from GR in strong fields, no matter how small the weak-field effects are.

## Our constraint on quasi-Brans-Dicke theories




Our constraint $|\Delta|<2.6 \times 10^{-6}$ rules out the light-gray area.

## Universality of free fall



Tests the foundation on which Einstein's theory of gravity, general relativity, is built:

- Used I200hr of WSRT, GBT and Arecibo data to see if pulsar and white dwarf fall differently towards a 2nd white dwarf companion.
- Find equal accelerations to within a few parts in a million. Best-ever test of the strong equivalence principle:

Pulsar triple system PSR J0337+1715

$$
|\Delta|<2.6 \times 10^{-6}
$$

# Does Extreme Gravity Change How Things Fall? The Triple System PSR J0337+1715 <br> Microsecond-accurate measurements of the 



Universality of Free Fall from the Orbital Motion of a Pulsar in a Stellar Triple Șystem
Naturé, 2018 July 5
Archibald, A. M., Gusinskaia, N. V., Hesselș, J. W. T.; Deller, A. T., Kaplan, D. L., Lorimer, D. R., Lynch, R. S., Ransom, S. M., Stairs, I. H.

Even with its extreme gravity, the pulsar falls . exactly the way Einstein predicted

