Full Mueller Imaging

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Outline

- A brief overview of Imaging
- The role of antenna primary beams in Imaging
- Full Mueller Imaging
  - Who needs it
  - Its implementation in CASA as a general Imager.
- Modelling the antenna Aperture Illumination Pattern (AIP)
  - Zernike basis modelling (VLA example)

Measurement Equation

$$V_{ij}^{Obs}(\nu,t) = G_{ij}W_{ij}\int P_{ij}M_{ij}(\nu,t)I(s,\nu)e^{is.b_{ij}}ds$$

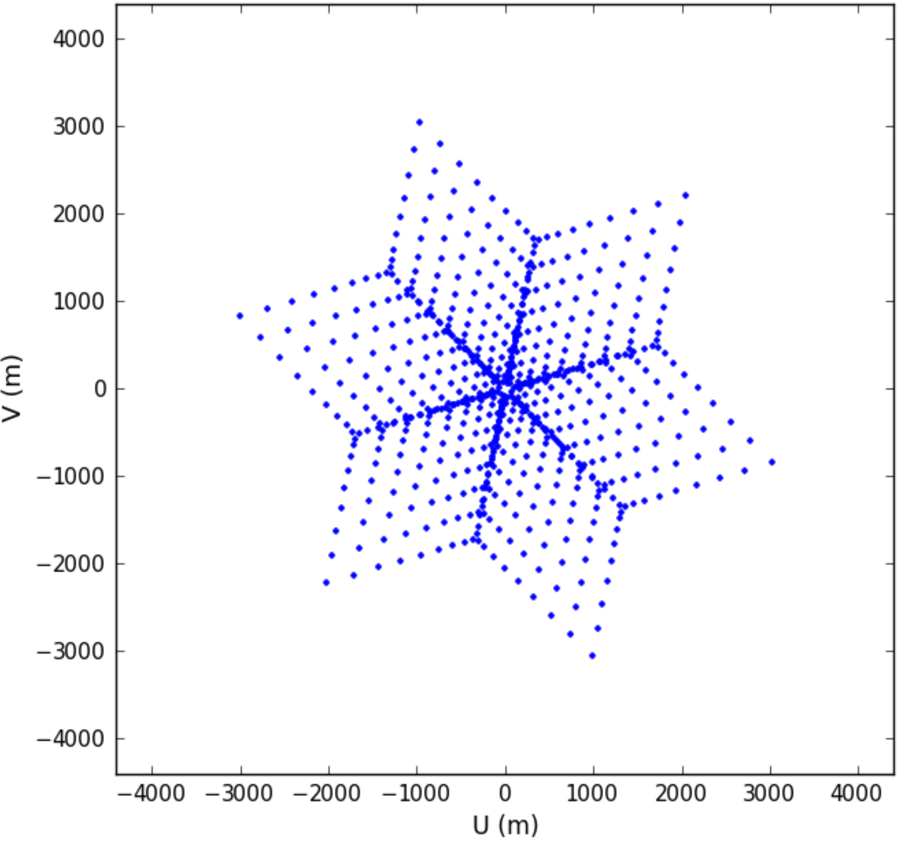
iFT

FT

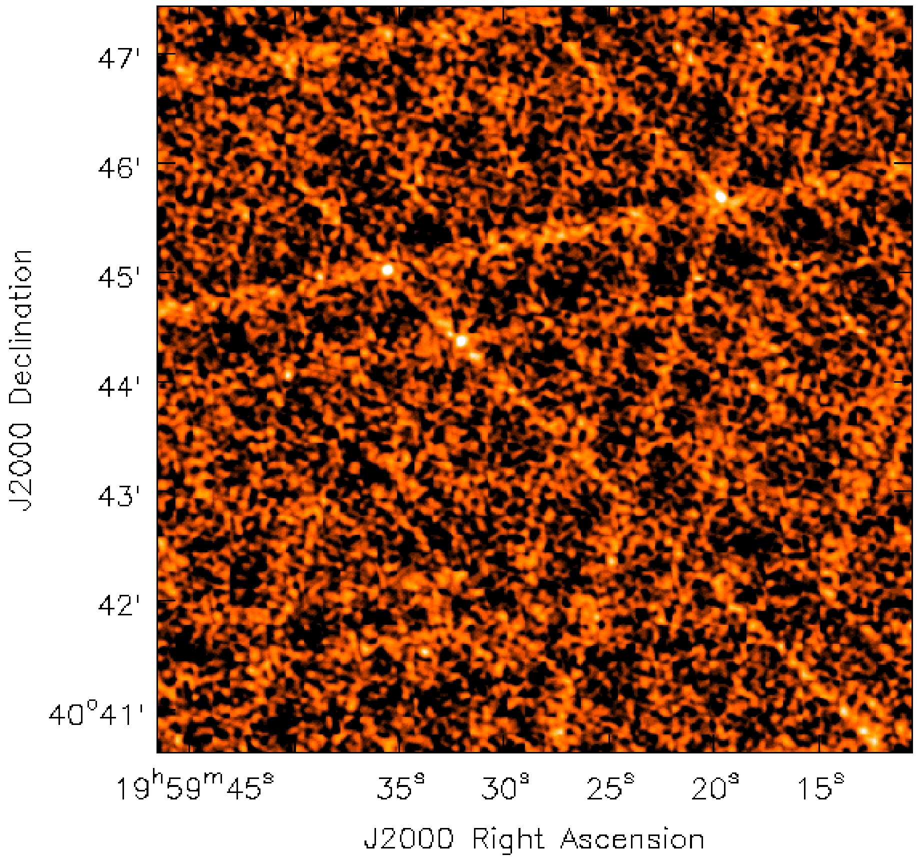
Observed Visibility

Sky Brightness Distribution

Snapshot



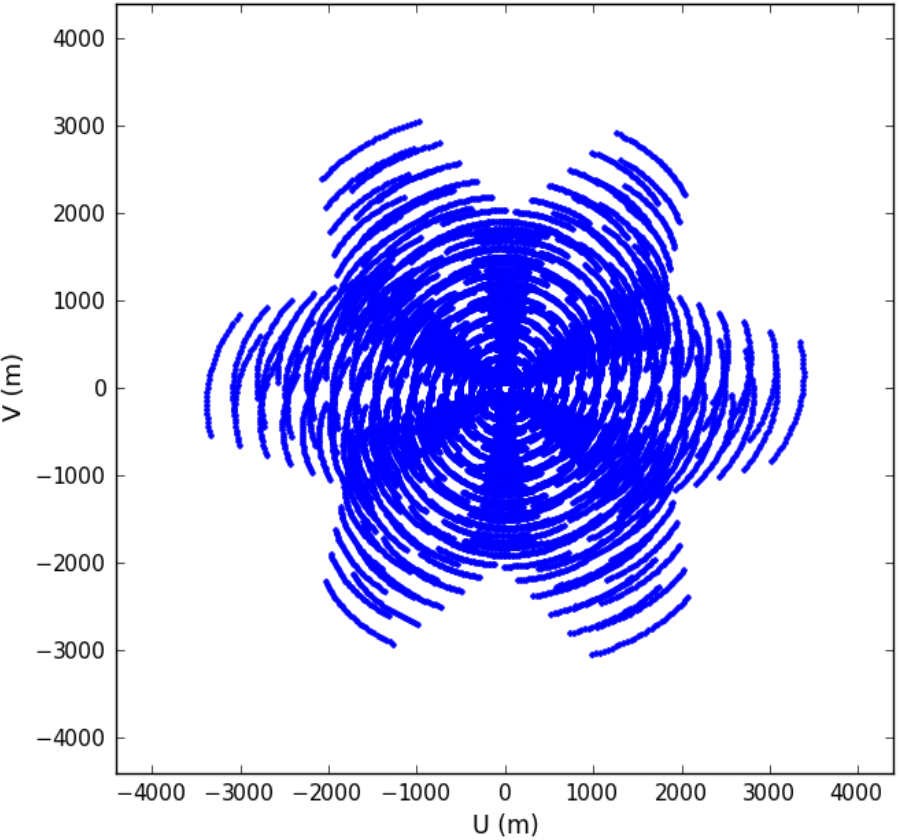
$S(u, v)$



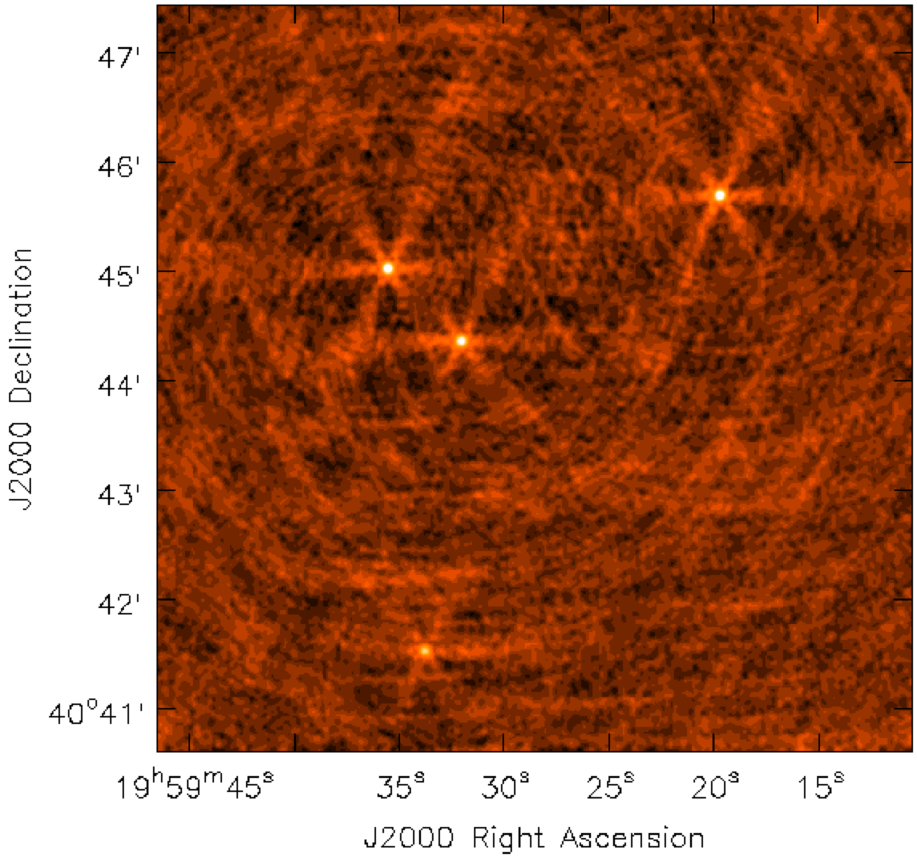
$I^{obs}(l, m)$

Courtesy : U.Rau

2 Hr synthesis



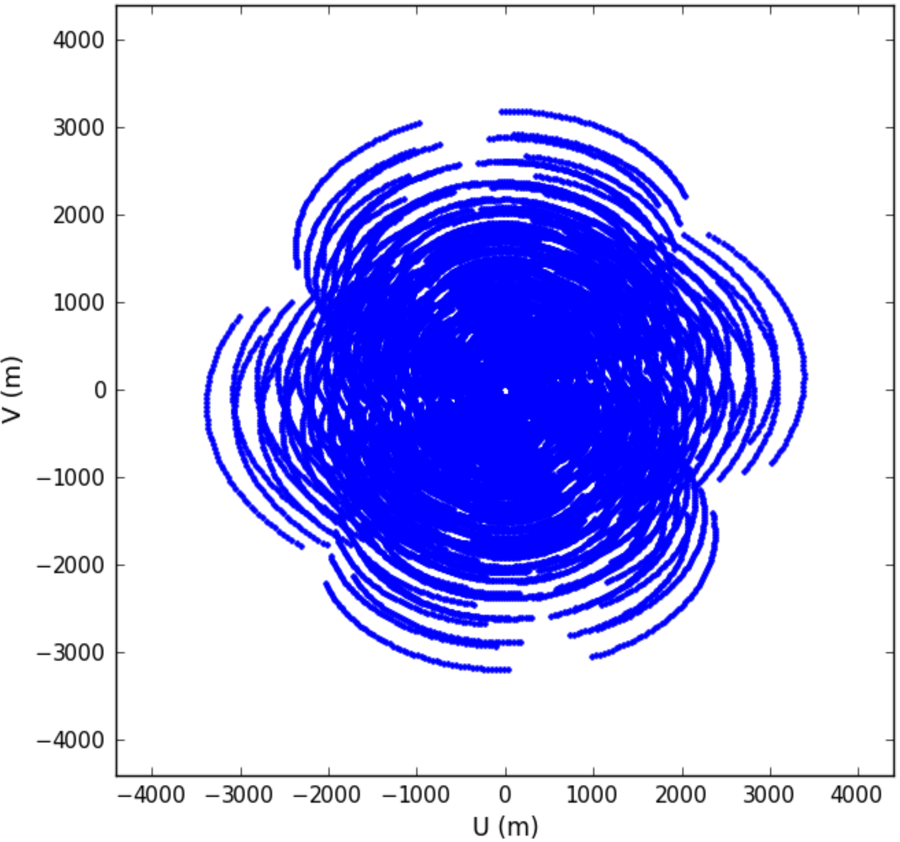
$S(u, v)$



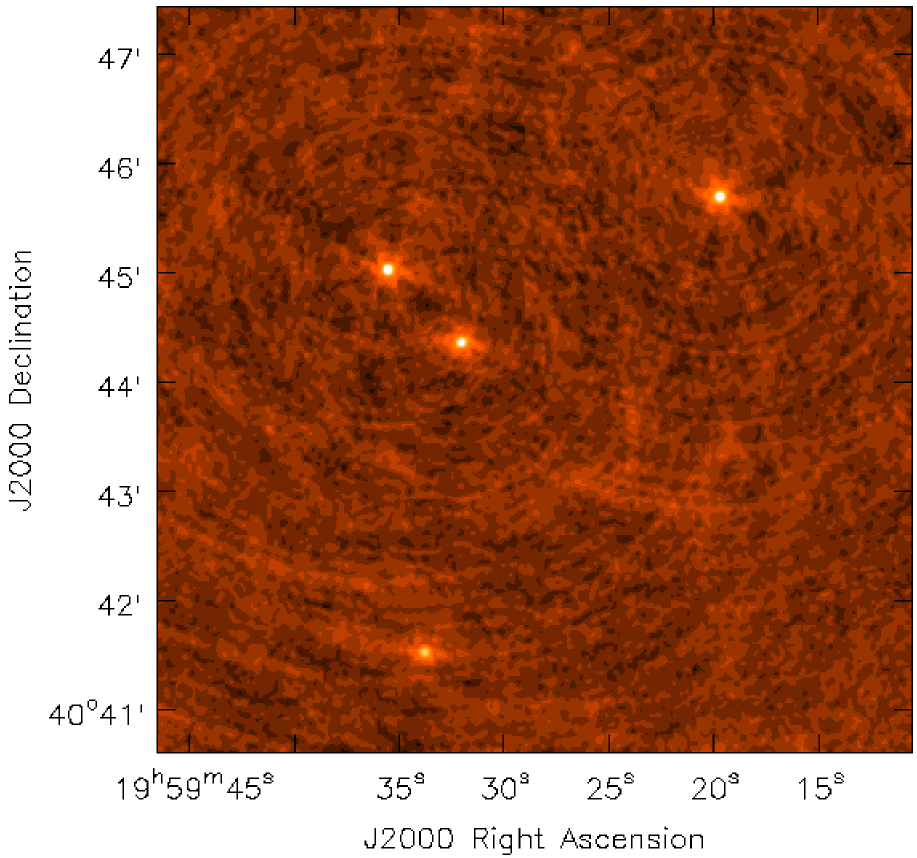
$I^{obs}(l, m)$

Courtesy : U.Rau

4 Hr synthesis



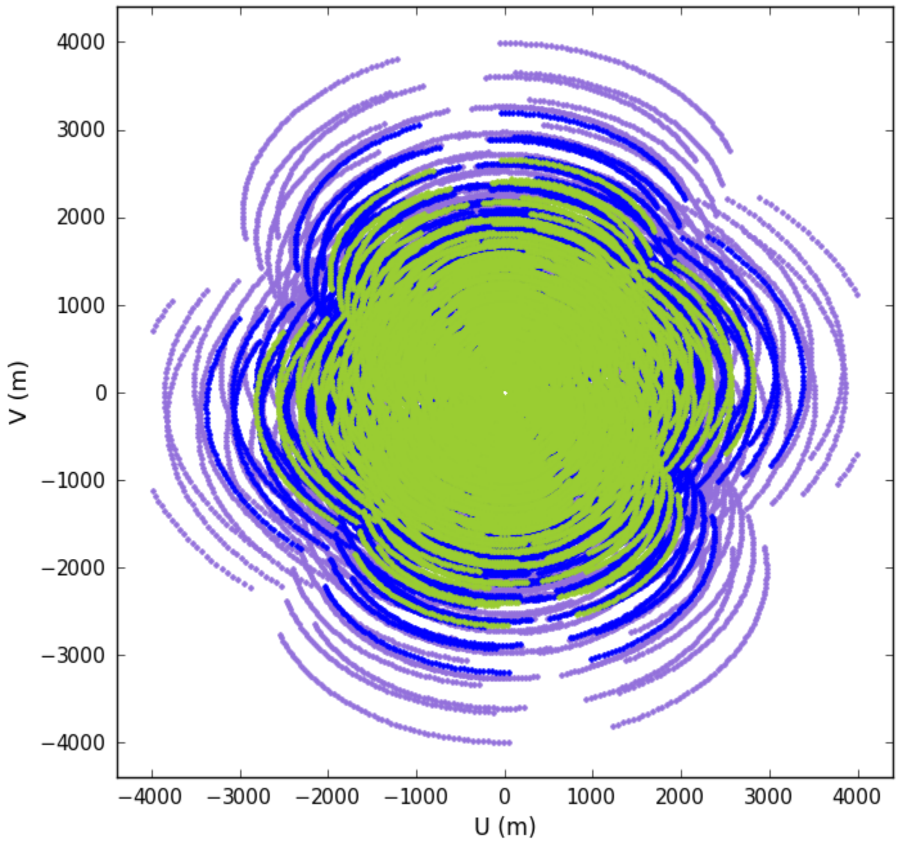
$S(u,v)$



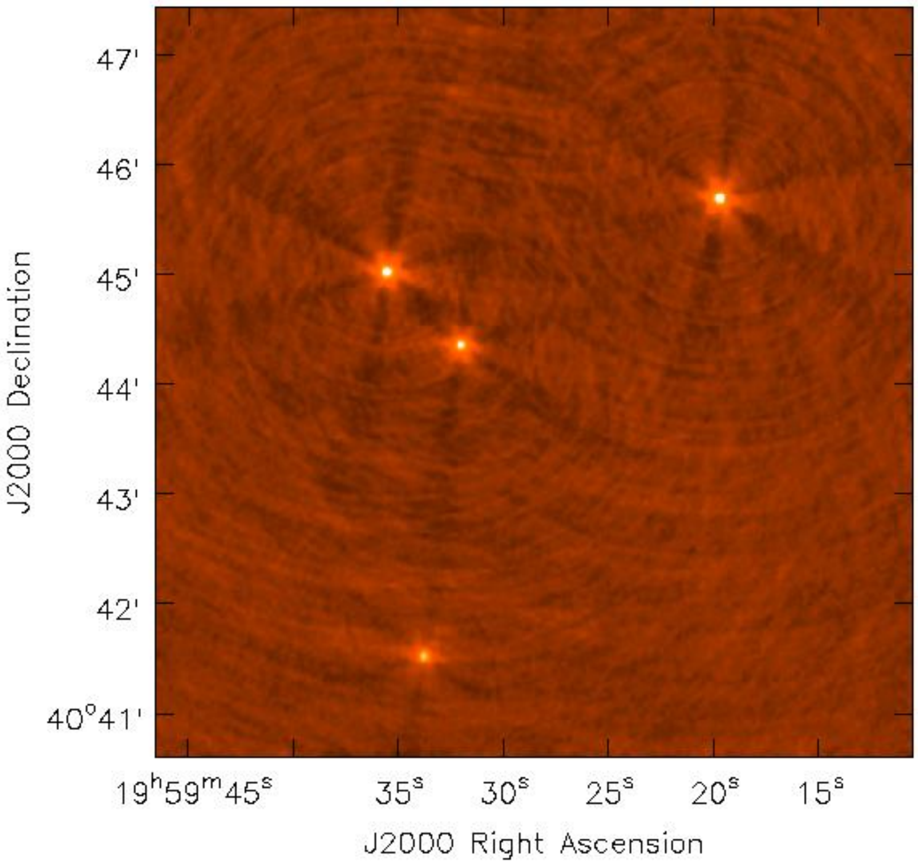
$I^{obs}(l,m)$

Courtesy : U.Rau

4 Hr Multi Frequency Synthesis



$S(u, v)$



$I^{obs}(l, m)$

Courtesy : U.Rau

Measurement Equation

$$V_{ij}^{Obs}(\nu, t) = G_{ij} W_{ij} \int P_{ij} M_{ij}(\nu, t) I(s, \nu) e^{is.b_{ij}} ds$$

Observed Visibility

Direction Independent Gains

Weight

Direction Dependent Ionospheric Corrections

Antenna Primary Beam

MTMFS
Rau et al. 2011

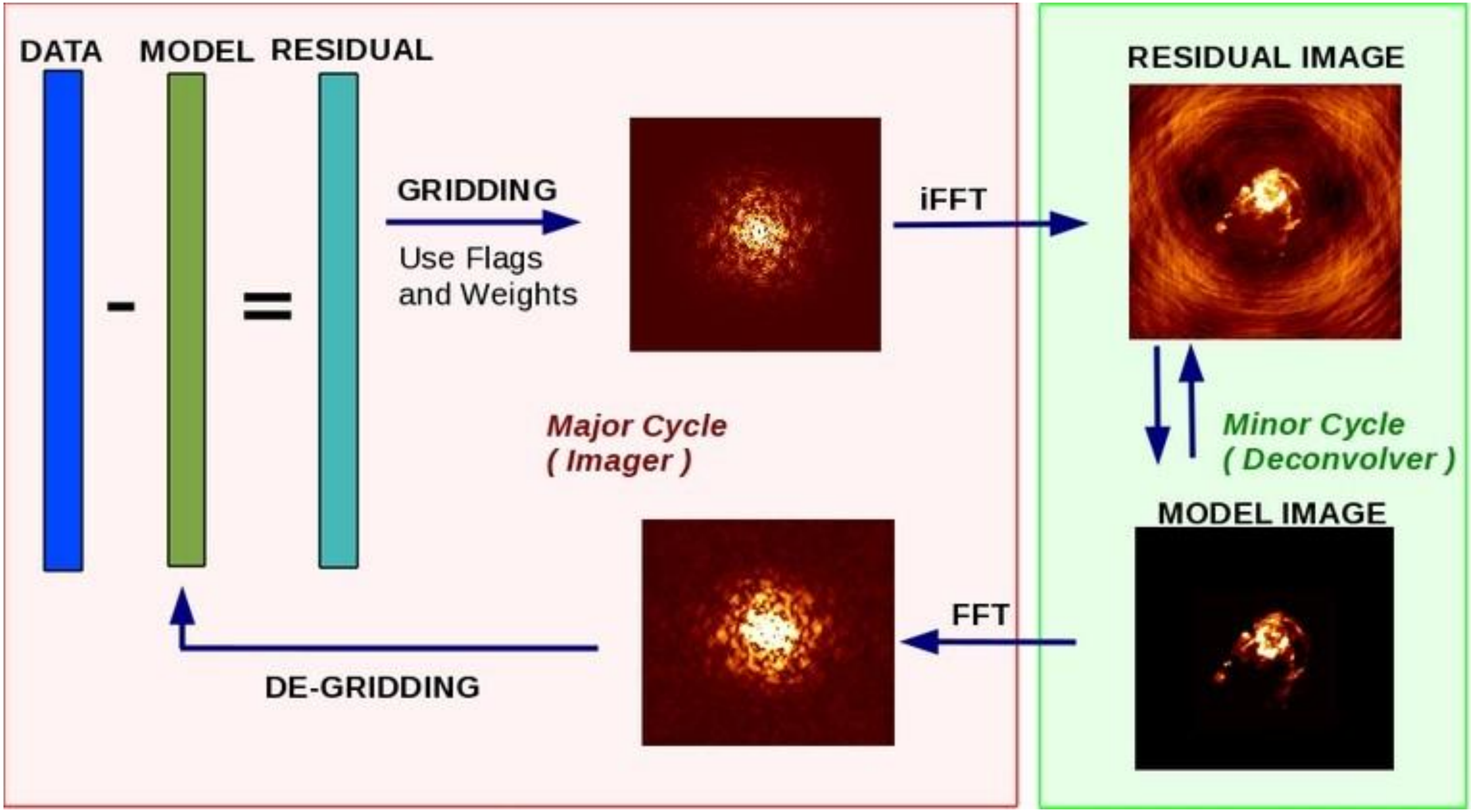
Non Co - Planar baselines

A- Projection
Bhatnagar et al 2008, 2013
Jagannathan et al 2019 (In prep)

W- Projection

Cornwell et al 2008

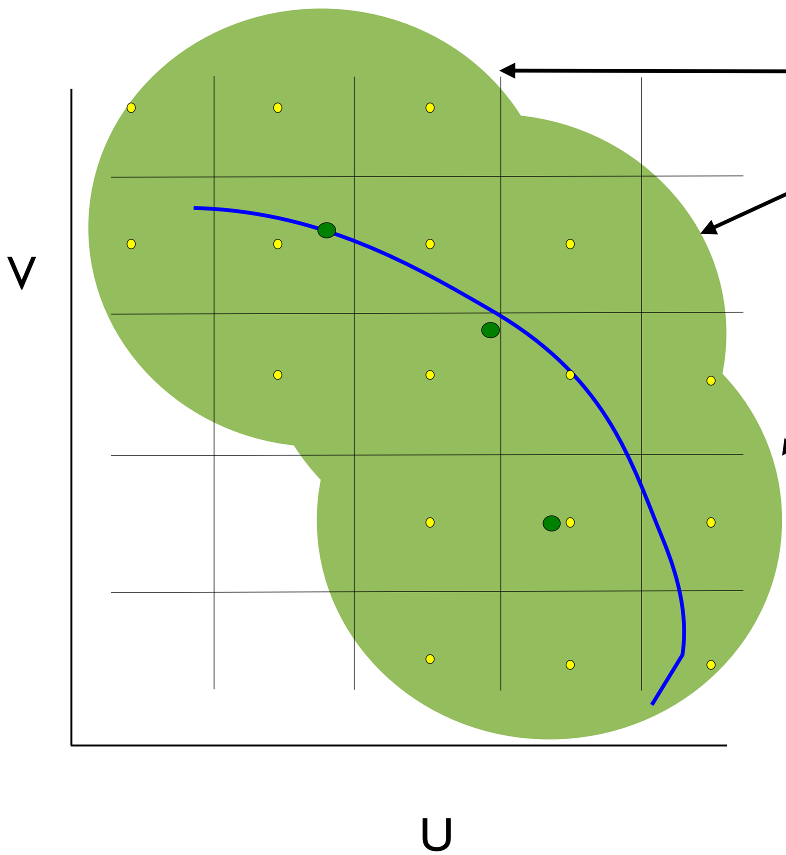
Imaging in Practice



Standard, Mosaic, W-projection, A-Projection
AW-projection

CS-Clean, Hogbom, MT MFS,
Multiscale

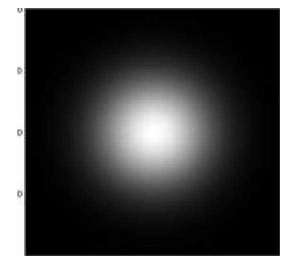
Convolutional gridding



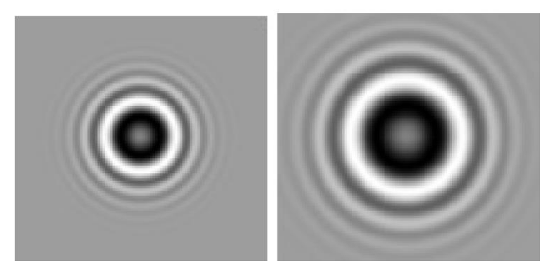
Gridding Convolution Function

- Prolate Spheroidal – CSClean
- Fresnel Kernel – W-Projection
- Aperture Illumination Function – A-Projection

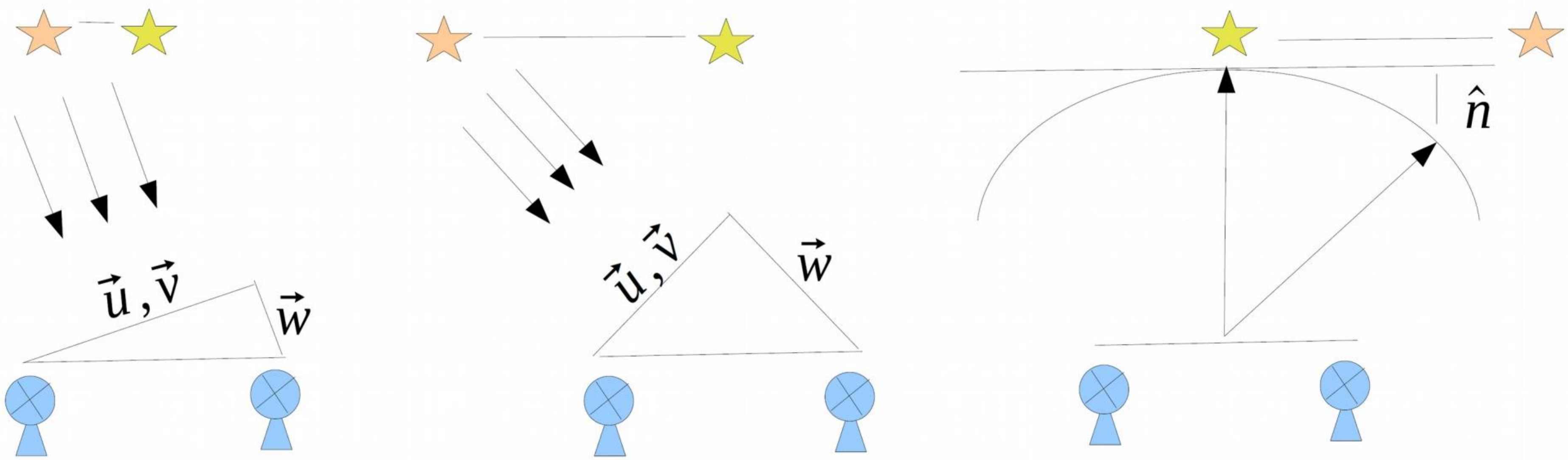
Prolate Spheroidal



Fresnel Kernel



Wide-field Imaging



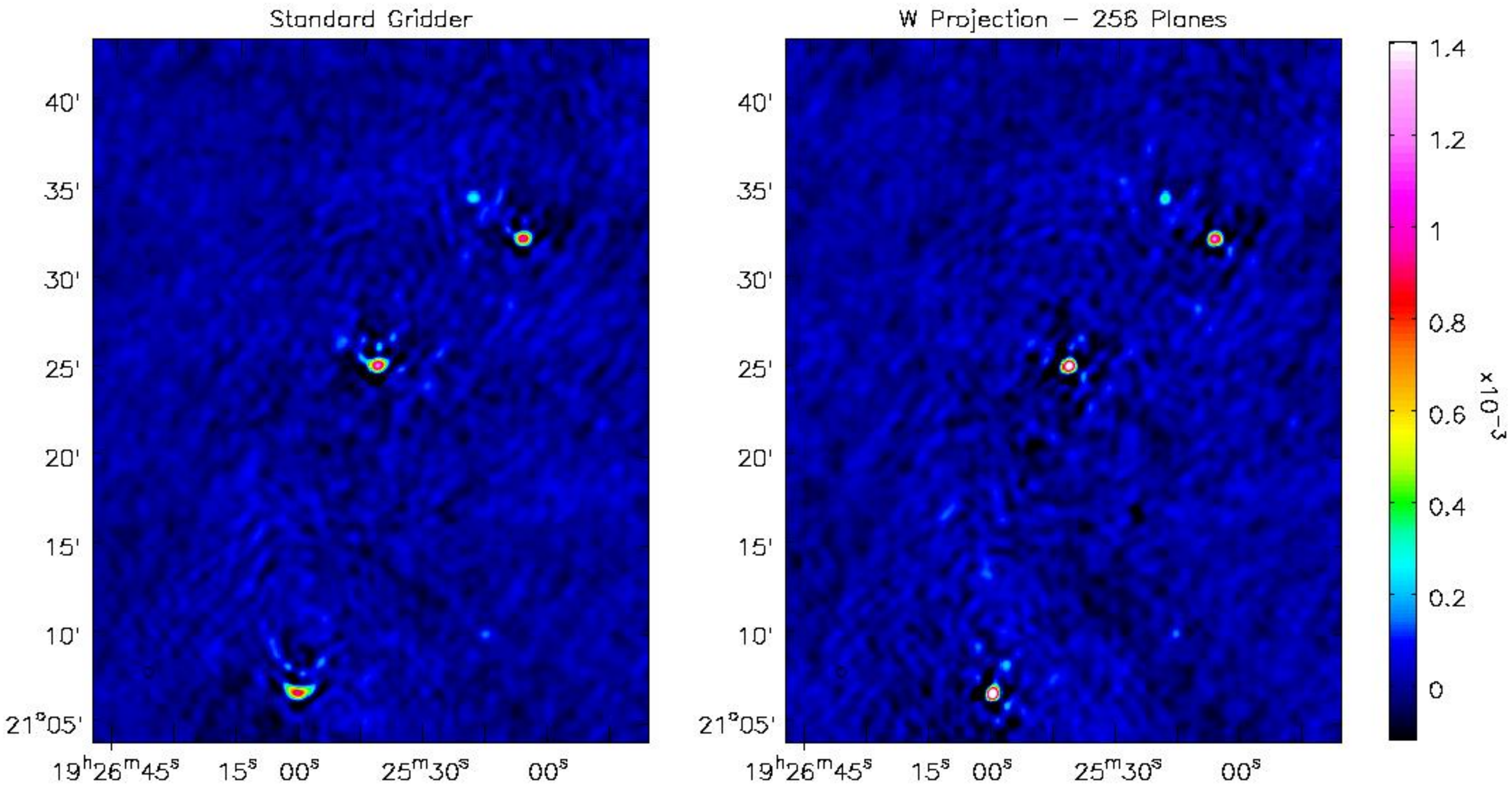
Small FoV

w-value per baseline is small.
2D vCZ holds.

Large FoV or Non-Coplanar Array

Large w-values.
Need to account for w in imaging
Algorithms – facetting, w-projection

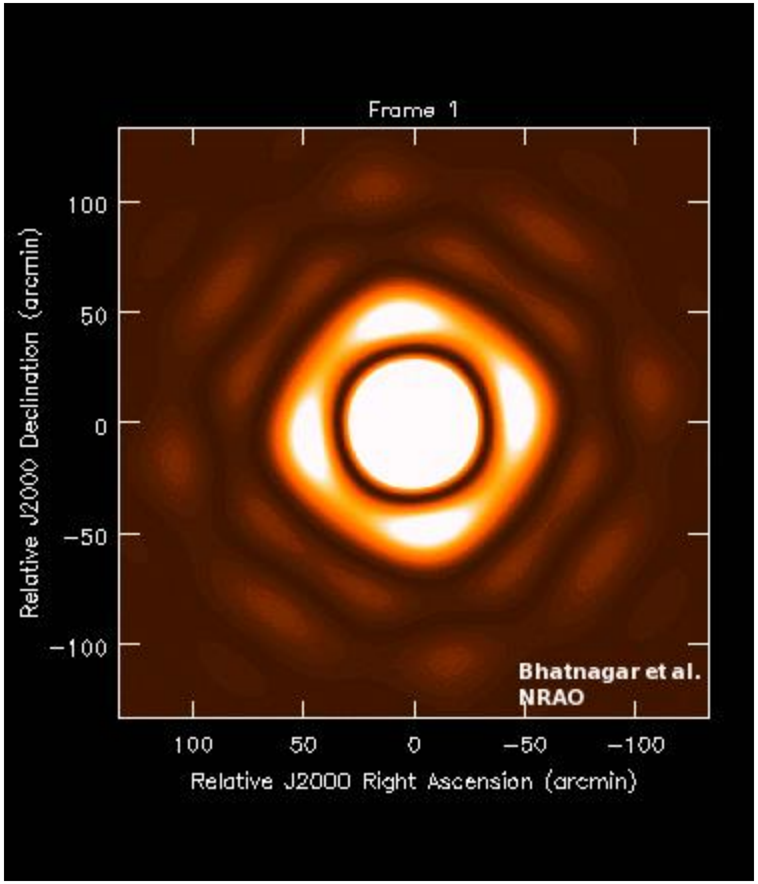
W-Projection



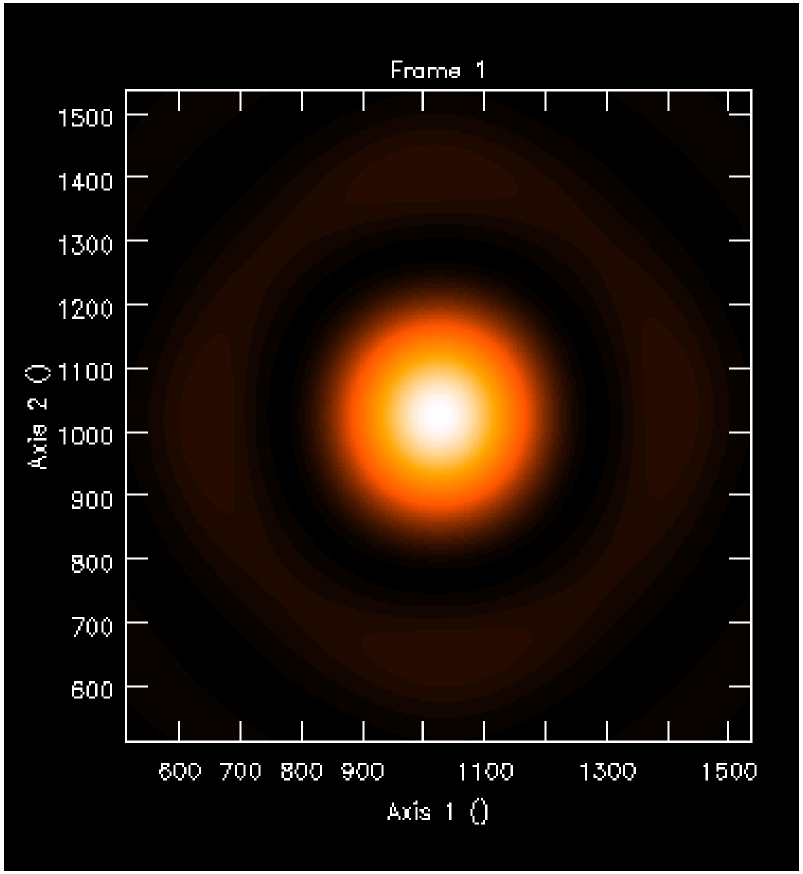
Antenna Primary Beam

Varies with time, frequency and direction on the sky

Time



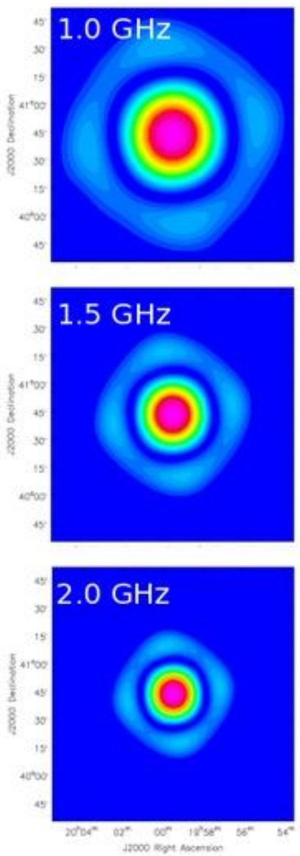
Frequency



Courtesy: S. Bhatnagar

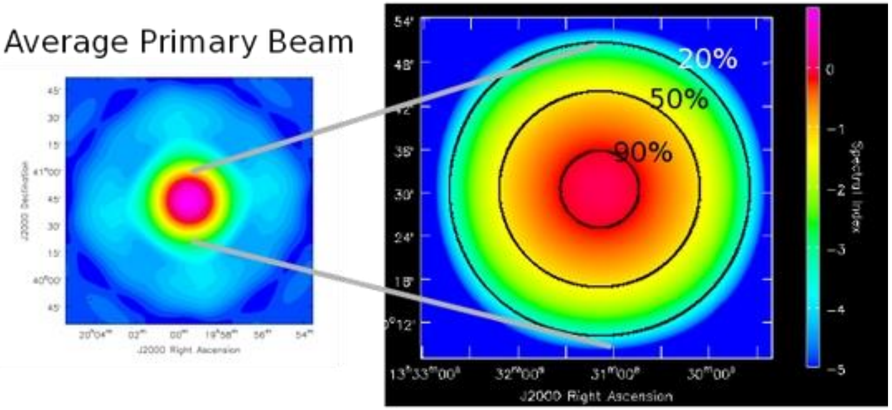
Antenna Primary Beam

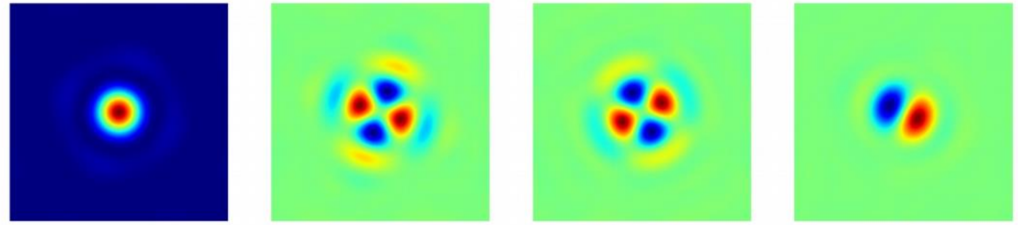
VLA PBs



Spectral Index of PB

Average Primary Beam





Polarized Beam

FM AW-Projection -I

$$\vec{V}_k^{Obs} = (A_k^\circ \star \vec{V}^\circ)\delta_k \qquad k = ij, \nu, t$$

Observed Visbilities

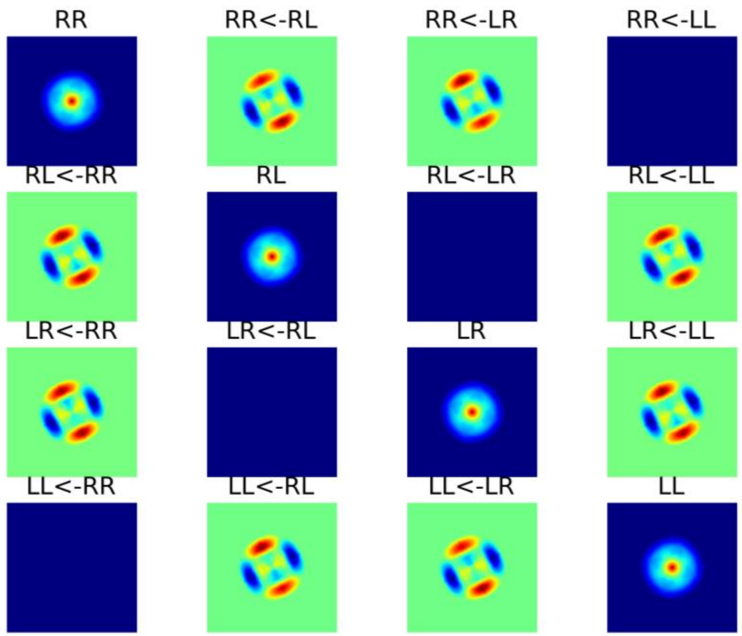
Continuous Coherence Function

Sampling Function

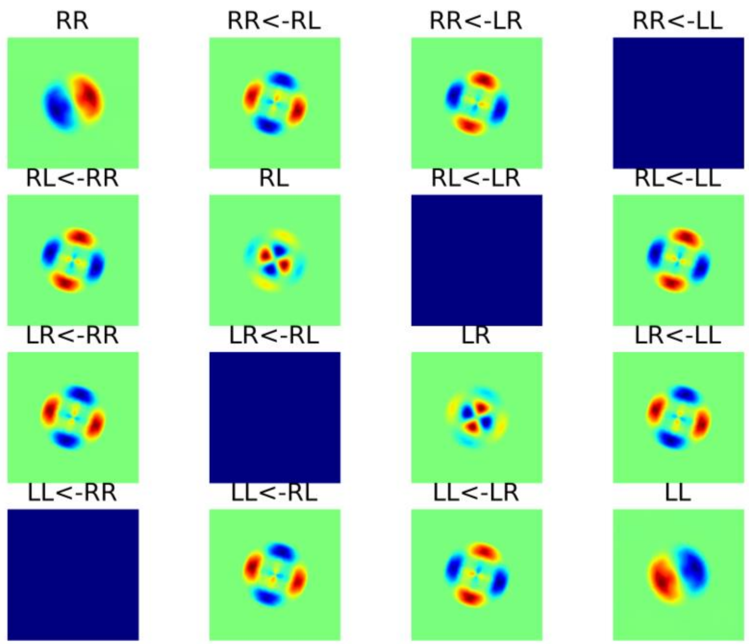
$$A_k^{\circ^{-1}} = \frac{adj(A_k^\circ)}{det(A_k^\circ)}$$

Correction Matrix is applied in two steps.

Full Mueller A-Projection - Convolution Functions



Aperture Real



Aperture Imaginary

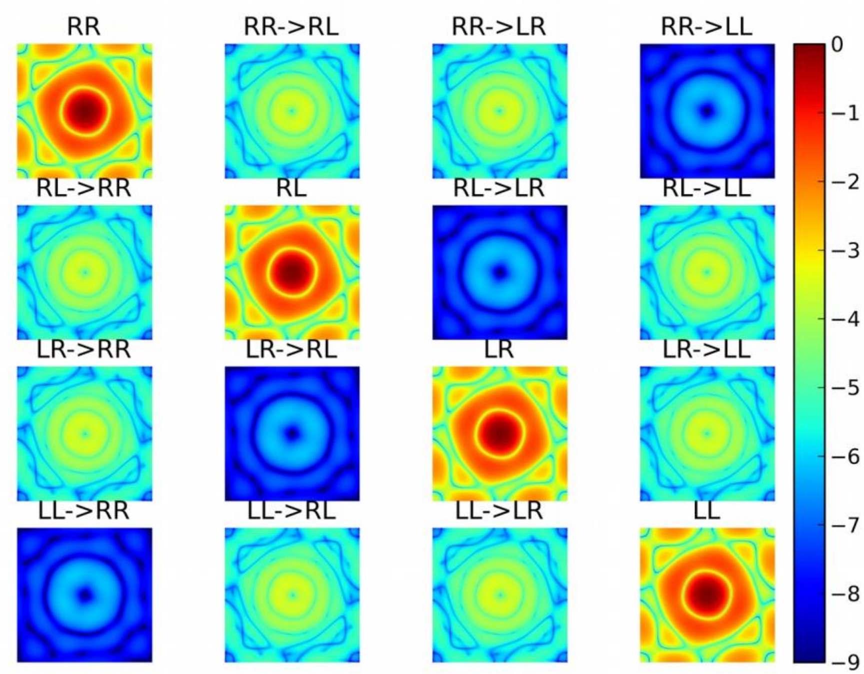
16

Full Mueller A-Projection – Mueller Matrix

The cross correlation of two antenna jones matrices is the mueller matrix

In data domain the mueller matrix tranforms into the ~unitary A matrix.

$$\begin{pmatrix} J_i^R J_j^{R*} & -J_i^R J_j^{RL*} & -J_i^{RL} J_j^{R*} & J_i^{RL} J_j^{RL*} \\ J_i^R J_j^{LR*} & J_i^R J_j^{L*} & -J_i^{RL} J_j^{LR*} & -J_i^{RL} J_j^{L*} \\ J_i^{LR} J_j^{R*} & -J_i^{LR} J_j^{RL*} & J_i^L J_j^{R*} & -J_i^L J_j^{RL*} \\ J_i^{LR} J_j^{LR*} & J_i^{LR} J_j^{L*} & J_i^L J_j^{LR*} & J_i^L J_j^{L*} \end{pmatrix}$$



FM AW-Projection - II

Gridding with the hermitian conjugate transpose of A

$$\vec{V}_k^M = \sum_k A^{M^\dagger} \star (A_k^\circ \star \vec{V}^\circ)\delta_k$$

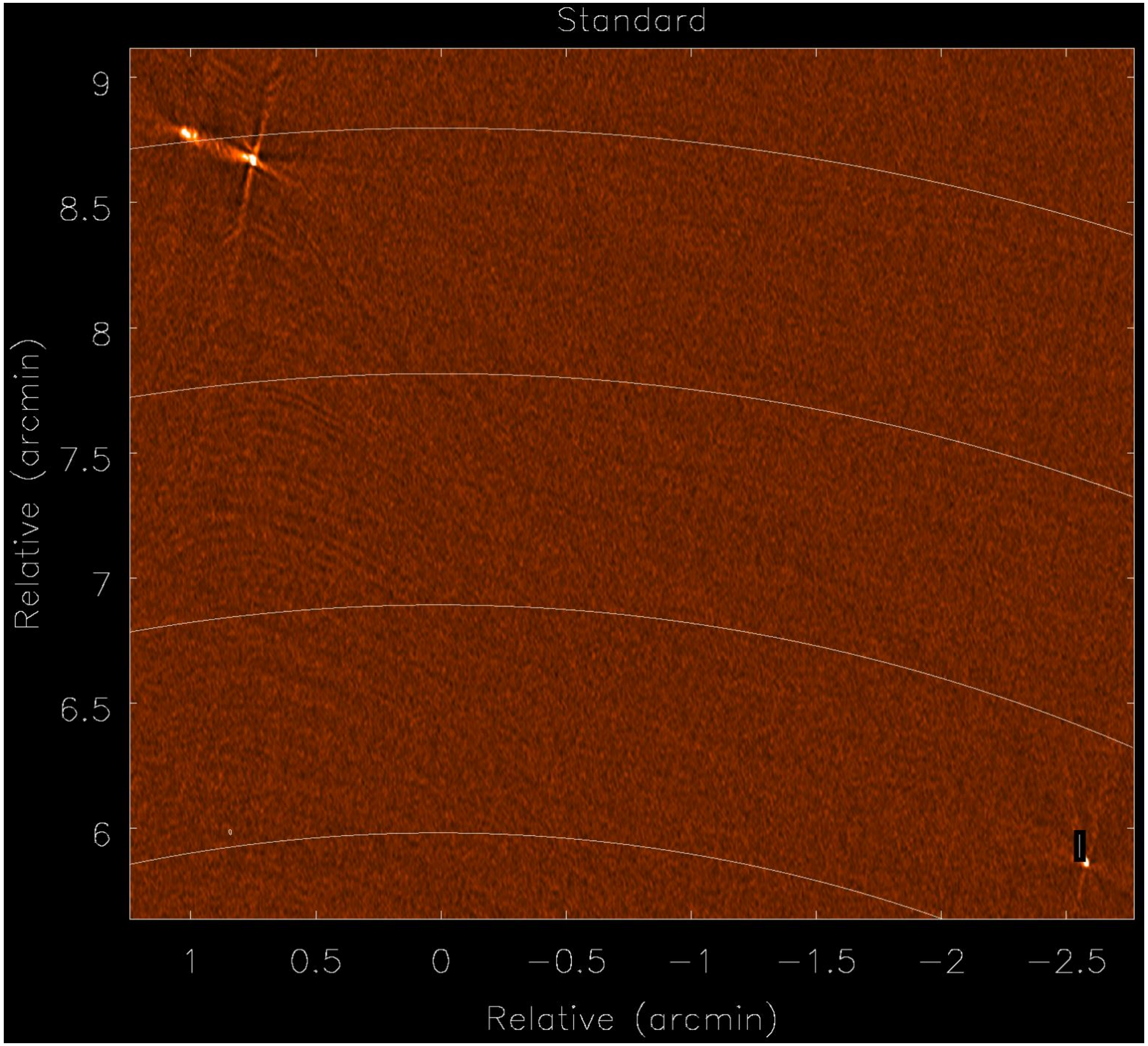
Accumulating the averaged convolution function for normalization

$$\overline{A^M} = \sum_k A_k^M$$

Normalized Residual Image - End of Major cycle

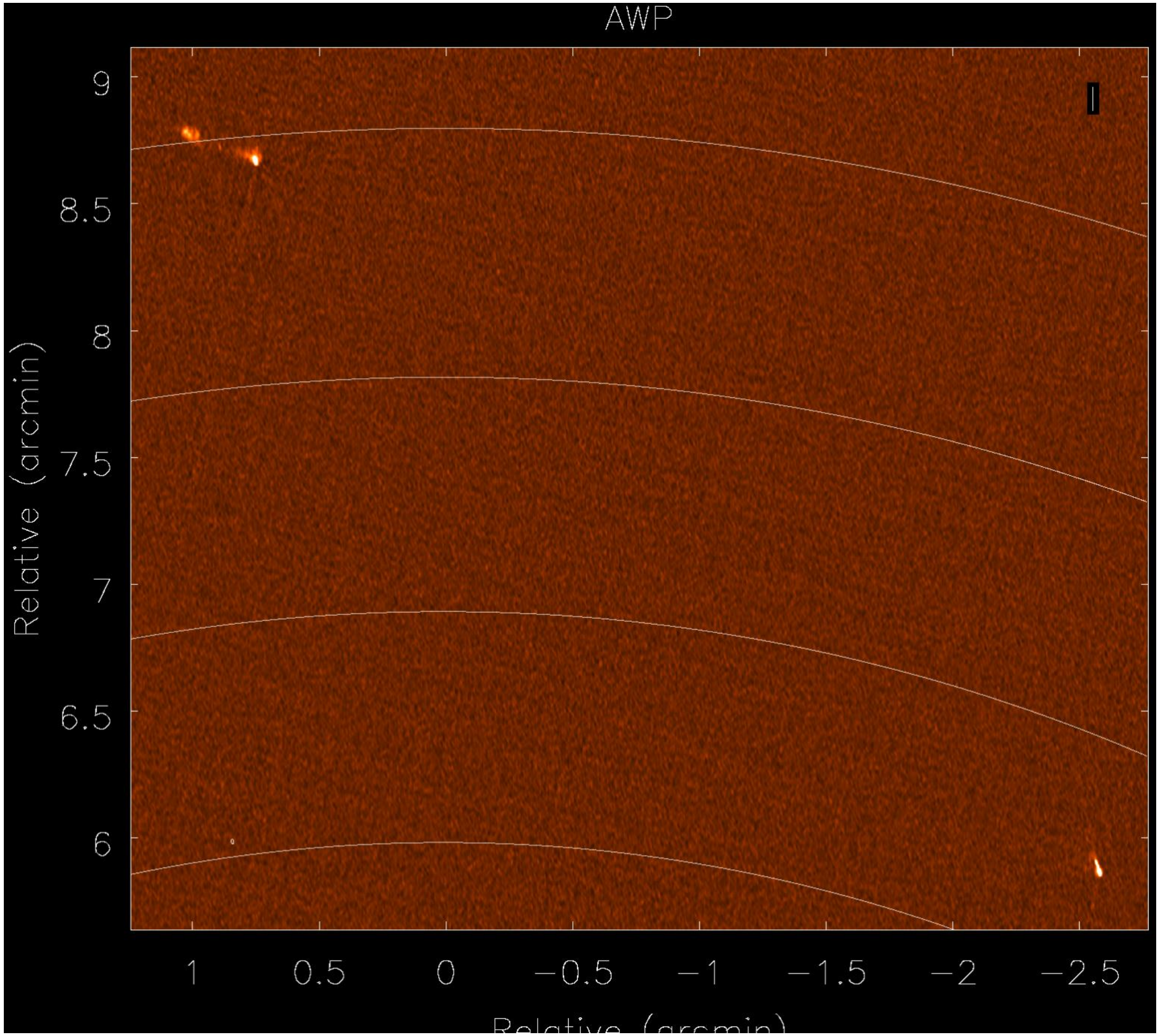
$$\vec{I}^R = \frac{F \sum_k \vec{V}_k^{Obs} - \vec{V}_k^M}{det(F\overline{A^M})}$$

HUDF – Standard gridder 3GHz



0.4 microJy

HUDF – AWP gridder 3GHz

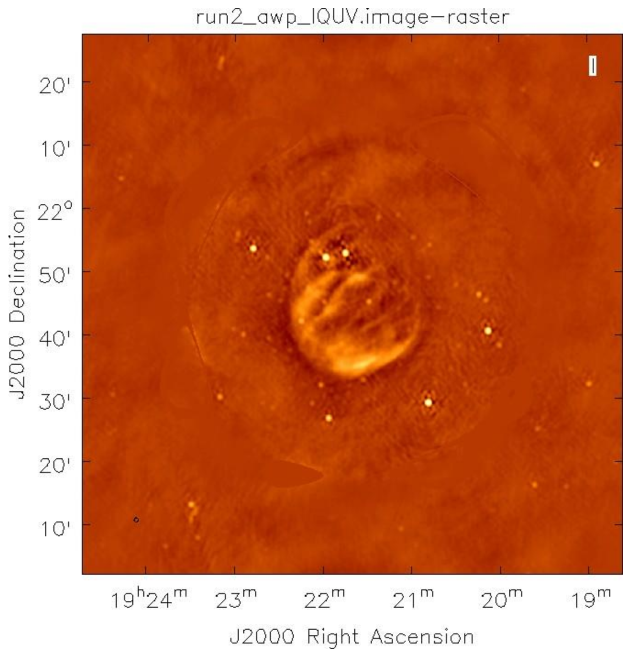


0.4 microJy

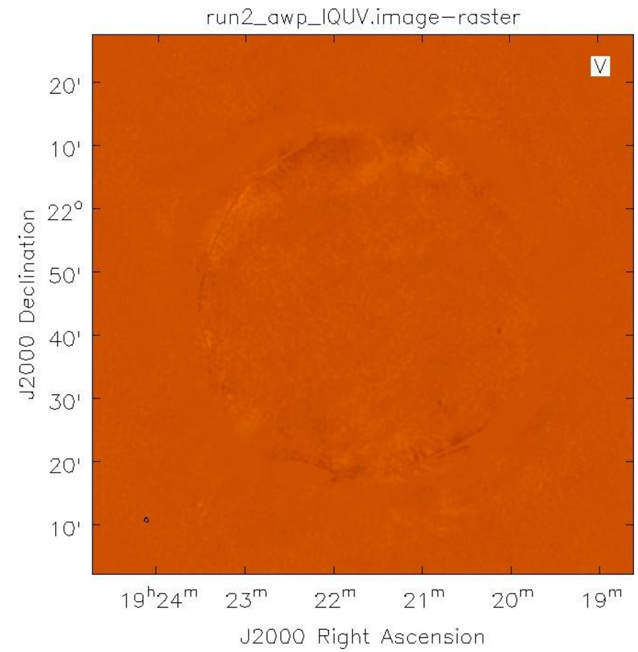
Full Mueller A-Projection - In practice

- As shown in the algorithm above it is a major cycle algorithm applied in two steps.
- A gridding kernel that is the conjugate transpose of the A matrix.
  - We call these convolution functions
  - Ray traced models based on Grasp 8 simulations
  - Oversampled in the data domain
  - In the Full-Mueller Case
- Anti-diagonal is negligible so 12 instead on 16.
- Rotate cache for the corresponding parallactic angle on the fly, else one unique set of convolution functions for each parallactic angle.
- Gridding in serial takes very long even for small data sets. Only feasible when done in parallel.

FM AW-Projection – III

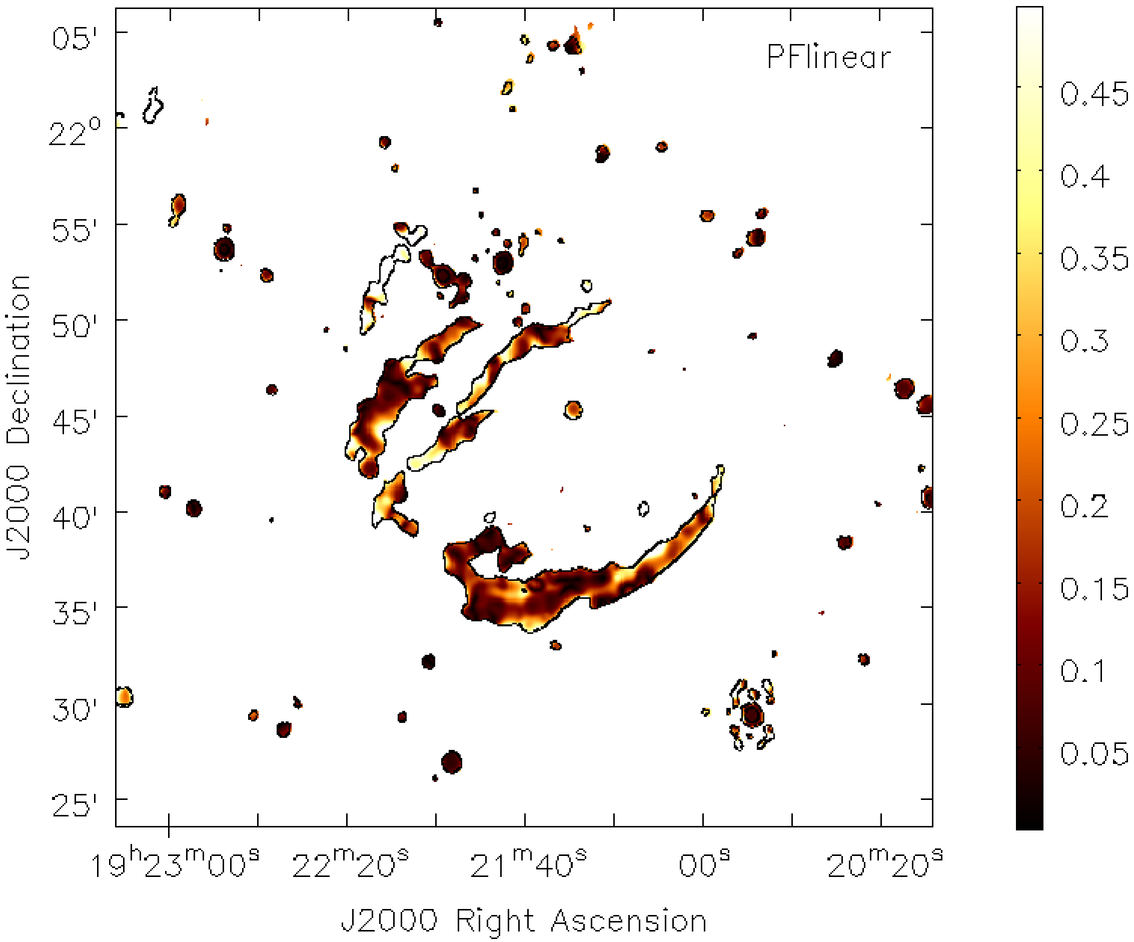


4.4 microJy



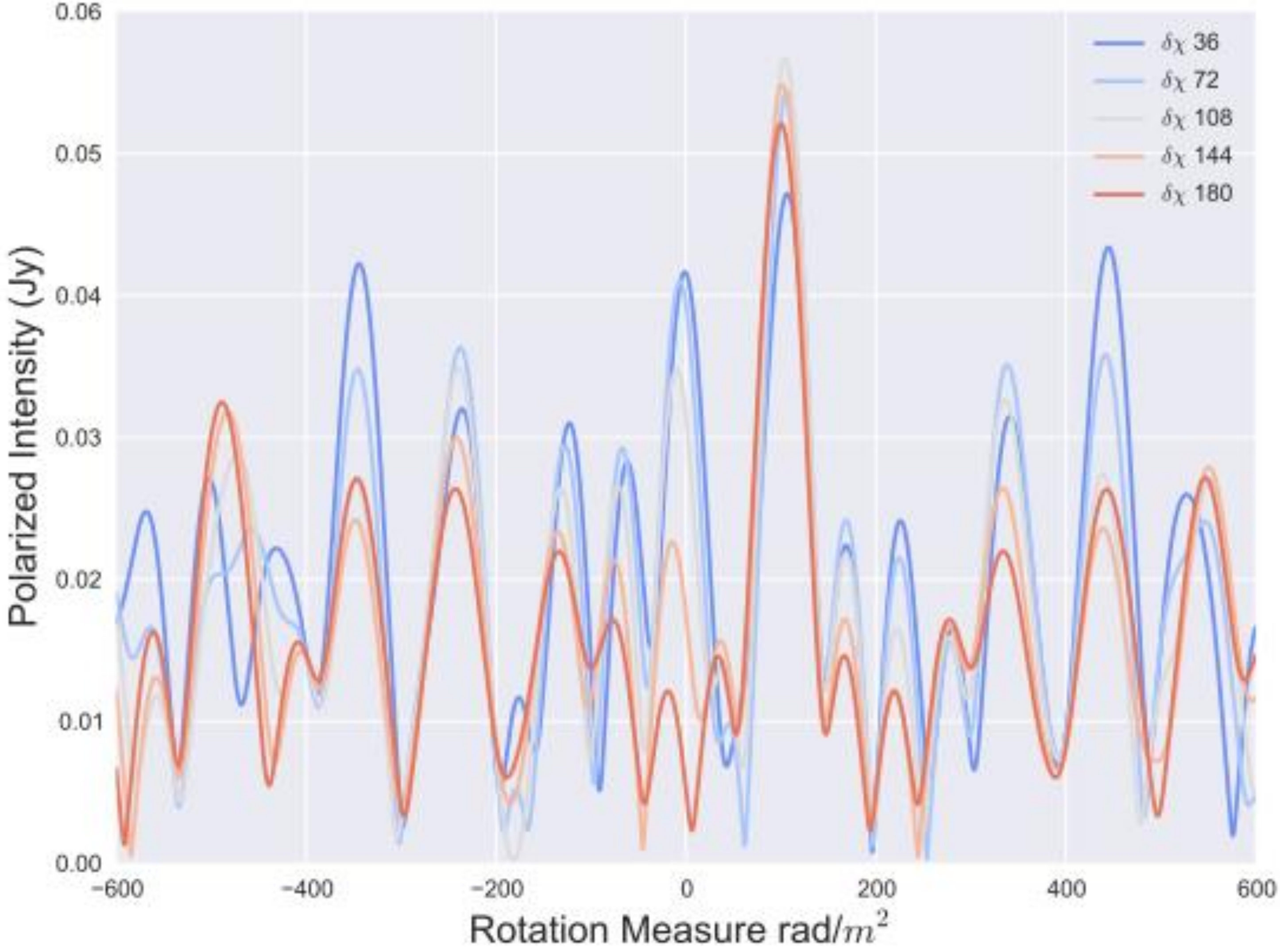
4 microJy theoretical sensitivity

FM AW-Projection – IV – Polarized Intensity



RM~120 rad/m/m

RMSF Polarized Source RM=100



Who needs FM AW-Projection

1. What fraction of the field of view contains your source of interest
   - 50% in power and beyond at reference frequency

2. What is your bandwidth ratio ?
   - Bandwidth/(Reference Frequency) > 0.2

3. Did you make a pointed mosaic for sensitvity ?
   - Narrow band mosaic ?
   - Broad band mosaic ?

4. Did you integrate for a very long time for increased senstivity ?

5. What fidelity do you require in your source flux density and spectral index ?

6. Do you need wide-field polarization leakage correction ?

PART II
Modelling Aperture Illumination Pattern

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Aperture Illumination Pattern

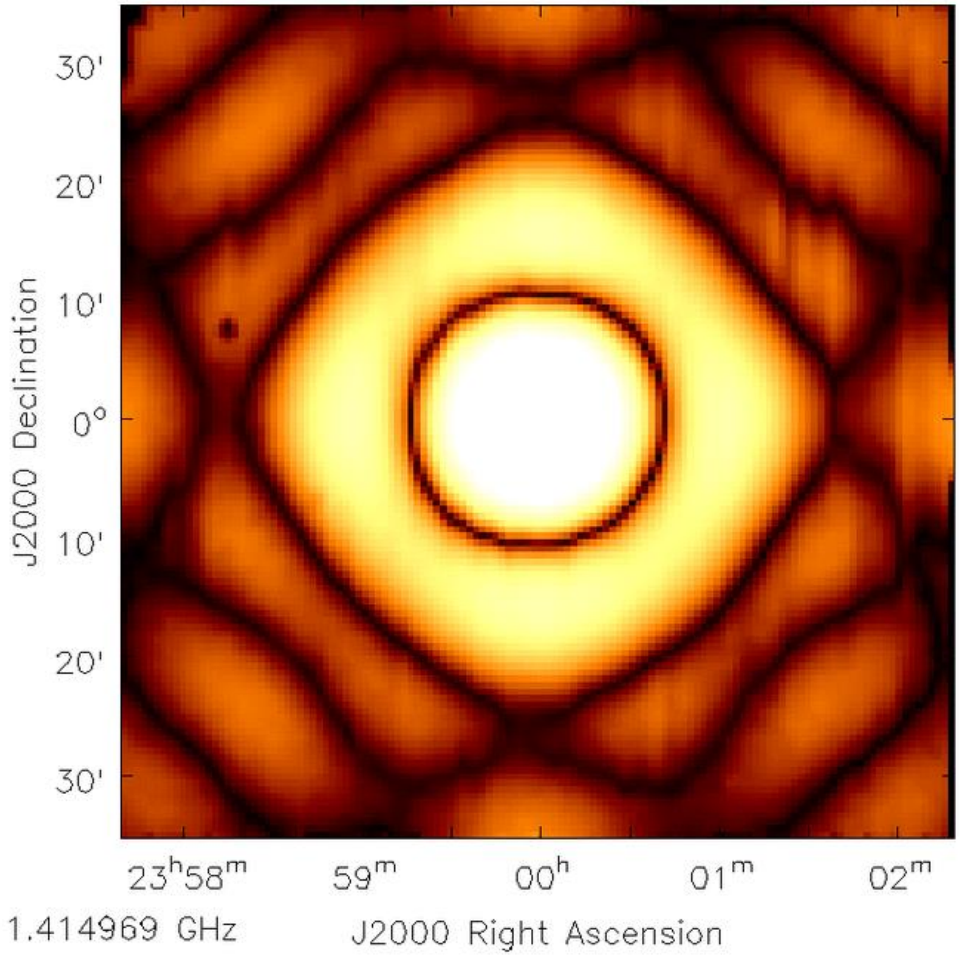
- High fidelity and high dynamic range imaging requires
  - Accurate convolution functions to project out WB A term.
  - Reach true thermal noise across the wide FOV.
  - A good model of off-axis polarization leakage.
- Convolution Functions
  - In feed basis
  - Aperture Illumination Pattern (AIP) of the antenna
  - In CASA they are produced through Ray tracing.

Reality Check

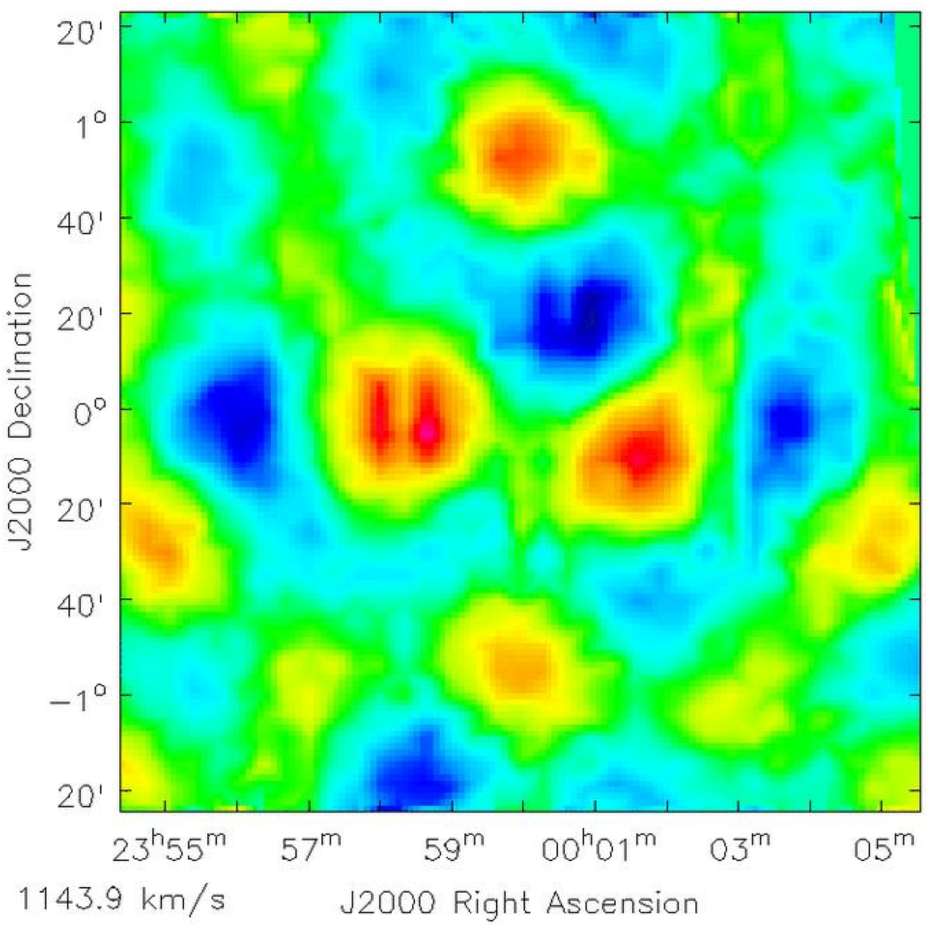
- Does ray tracing reproduce the aperture illumination pattern ?
  - Mostly Correct in Stokes I as a function of Frequency.
  - Stokes Q and U only approximately

- How do we compare the efficacy of ray tracing ?
  - We compare the ray traced AIP, and look at its FT, the antenna far field voltage pattern, also called the primary beam.
  - Primary beam of an interferometric array is measured through holography.
  - The VLA carried out holography measurements. (Perley, VLA Memo 195). Data were re-reduced in CASA.

Holographic Measurements

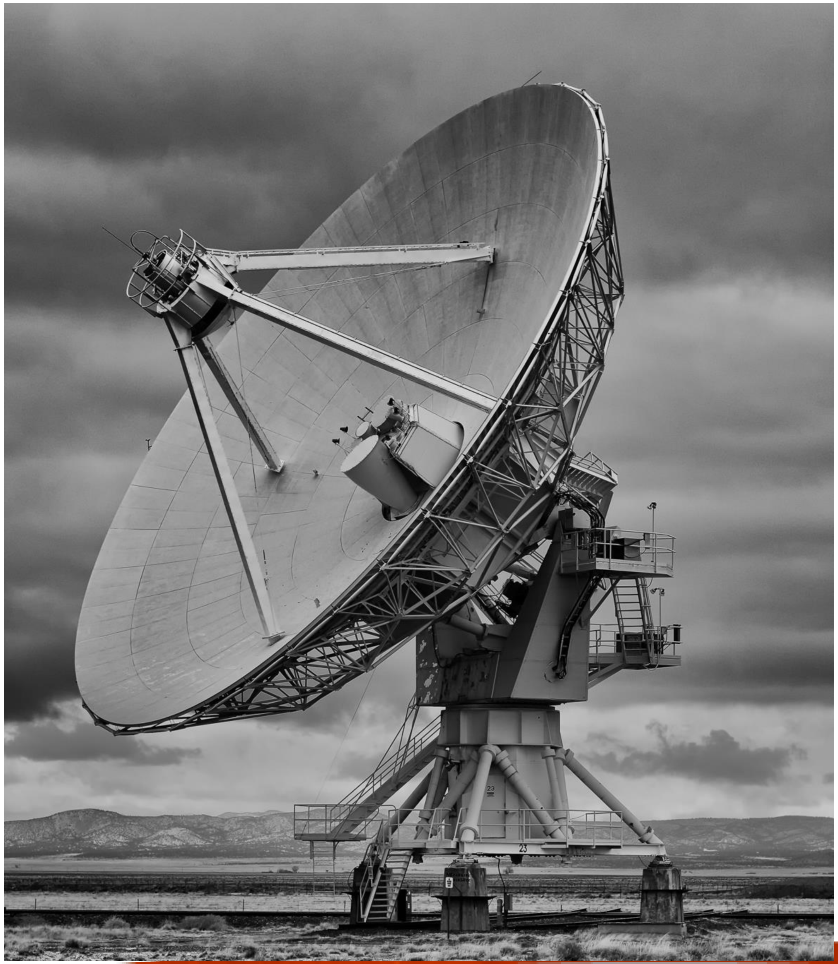


Holographic Measurements



Making the model mimic reality

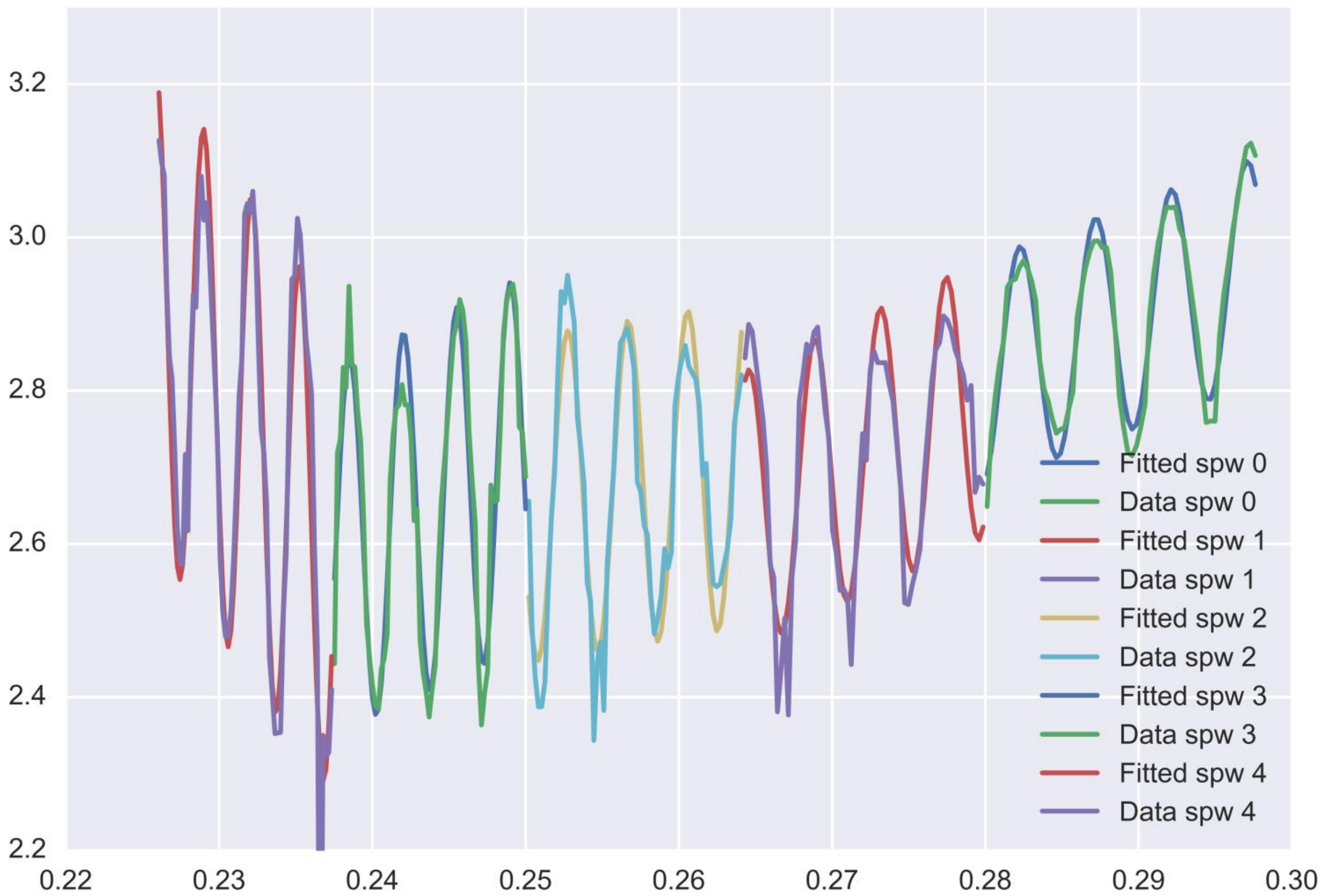
- Fix stokes I side-lobe levels by tweaking the model.
- Tweaks must be physically motivated.
  - Apparent blockage due to the antenna secondary.(Moves power from main-lobe into the side-lobes)
  - Feed Illumination taper.(alters side-lobe power)



Minimization - (Model - Measurement)

- Multi parameter minimization
  - Apparent illumination blocked by secondary
  - Feed Illumination taper polynomial
  - Pointing offset
- A stand alone C++ code to tweak the ray traced beam was created.
- Python wrappers to access the convolution functions and run the minimization in parallel using multi-processes.
- Serial run time of 4-6 hours for one channel in an antenna.
- Parallel implementation moved to two AWS clusters and 1024 channels reduced in 8 hours time, accounting for spot pricing. (Erik Bryer and James Robnett).

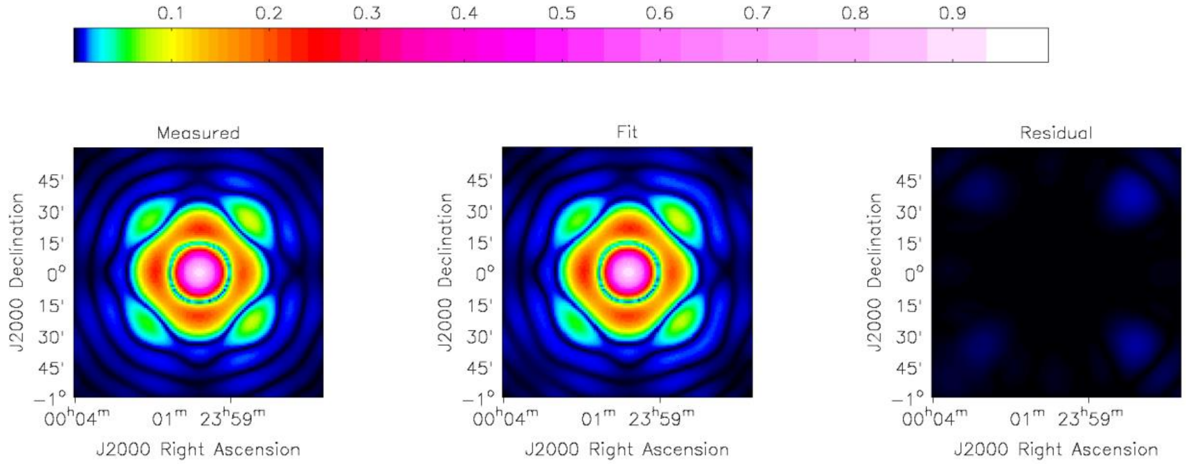
Aperture Blockage



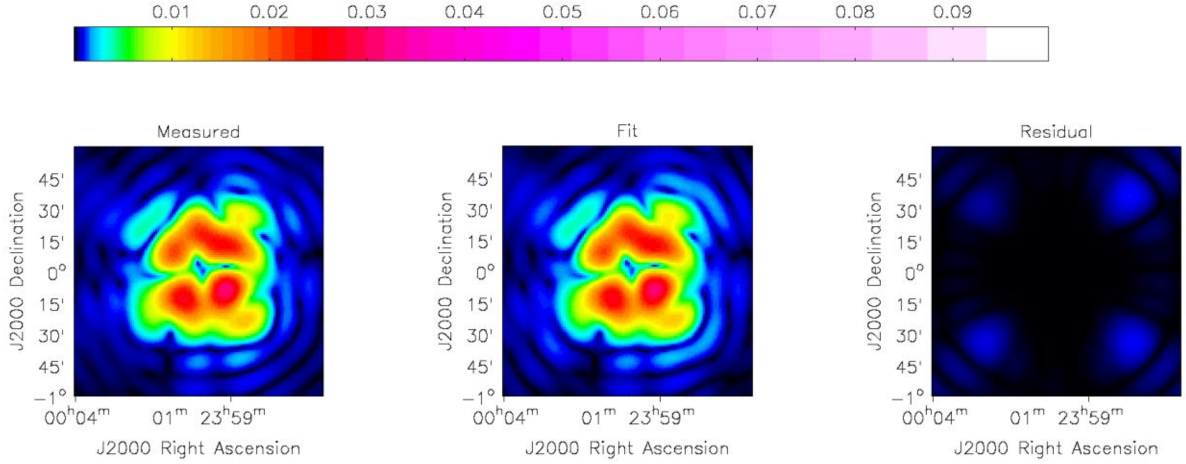
Zernike Modelling of the Aperture

- Purely python code does minimization for the coeffecients per channel in under 6seconds.
- All minimization was carried out on a single node on a single thread within CASA.
- Six orders of Zernike polynomials were used to model the AIP without having to transform into PB ever.
- Measured AIP were obtained by padding and FT of the measured PB through Holography
- 3 Different telescope AIP were modelled this way, VLA, ALMA and MeerKAT.

Zernike Modelling VLA

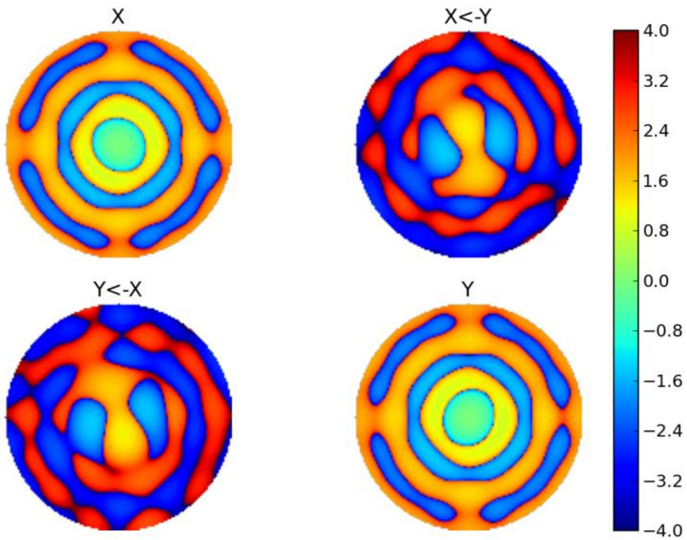


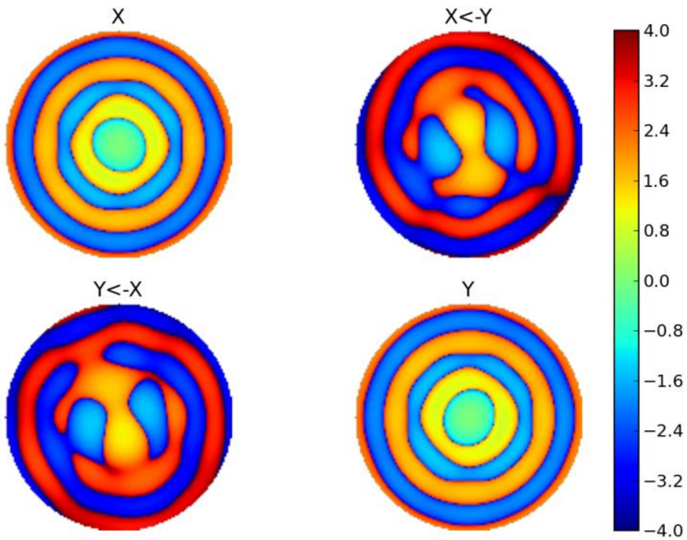
R



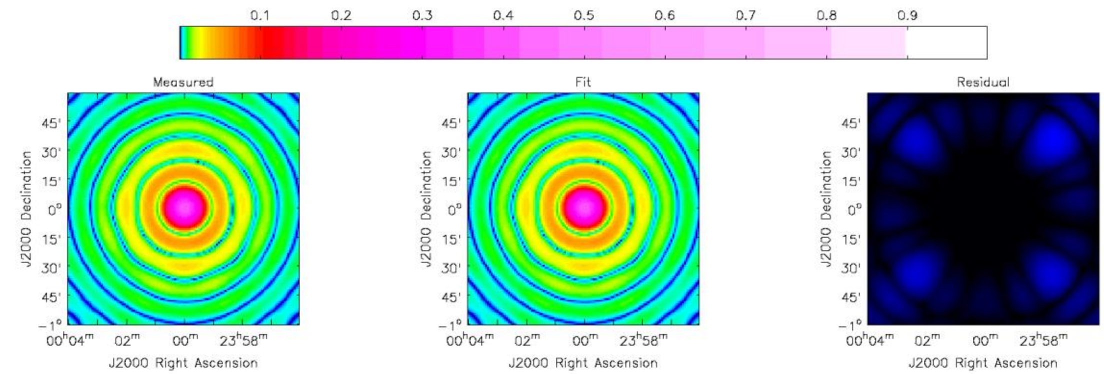
R > L

Zernike Modelling ALMA

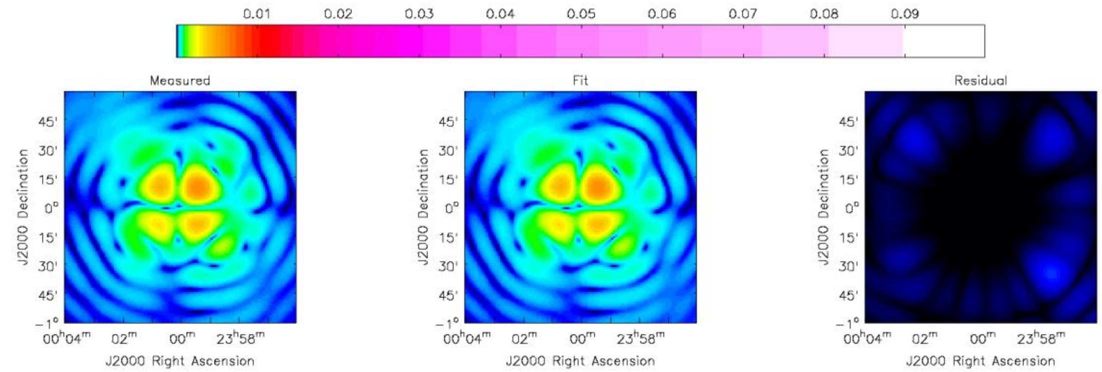




Zernike Modelling MeerKAT



X



X > Y

Conclusions

- Full Mueller AWP required for wide field Polarimetry. A requirement for most deep surveys with current and future radio interferometric arrays.
- In the absence of FM AWP the beam leaks flux from Stokes I to Stokes Q and U and alters the true polarized signal and for short integrations reduced RM signal and P due to depolarization.
- Extremely important to have the correct full polarization convolution functions for performing Full Mueller AWP.
- Ray-tracing has proved very useful in the Stokes-I case but falls short of modelling the off-diagonal leakage terms
- Zernike polynomial modelling – reproduces the AIP of quickly and accurately across all polarization and is basis agnostic.

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