Single dish antennas and phased arrays

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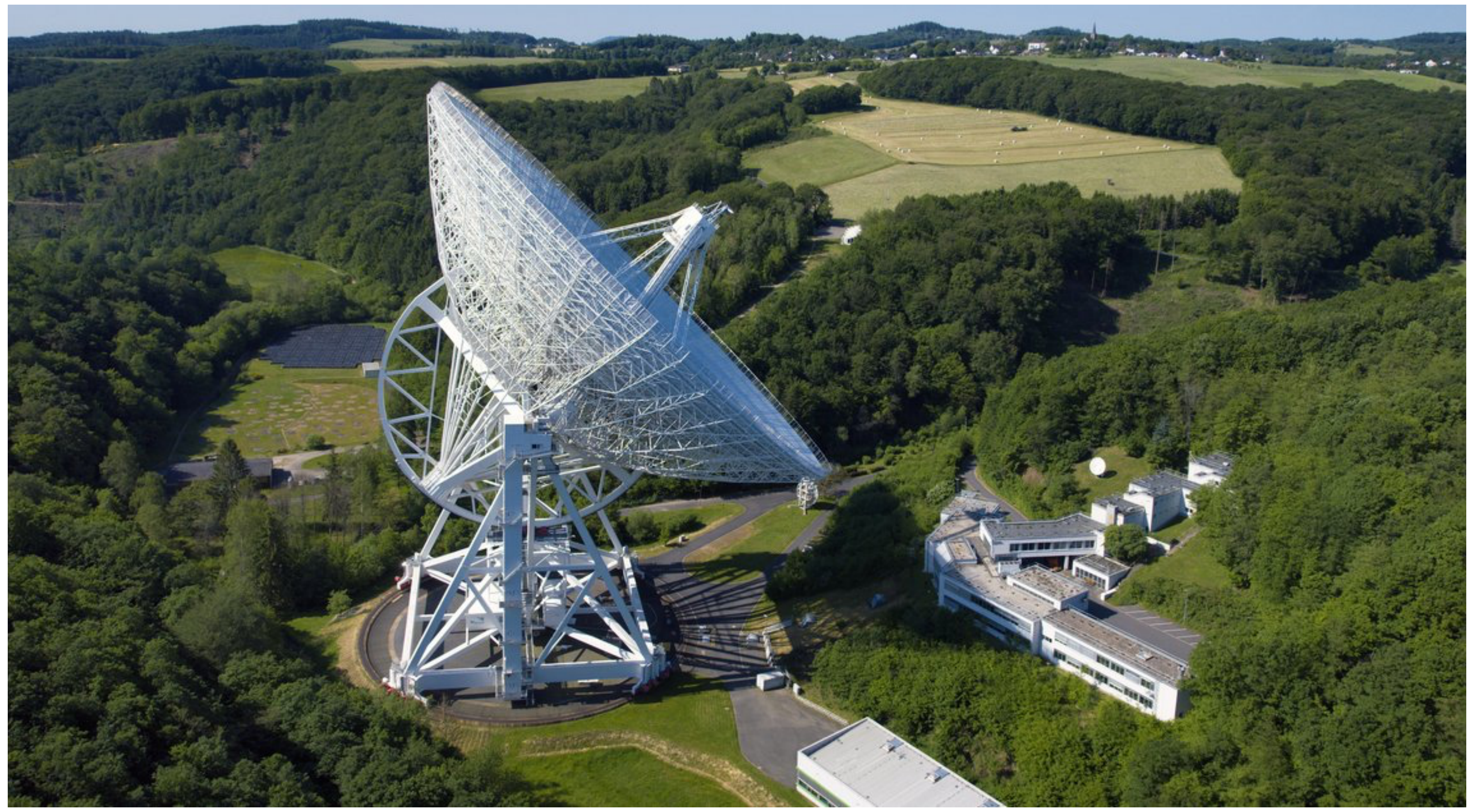
Single dish radio telescopes

Green Bank

Lovell, Jodrell Bank



Effelsberg

GMRT single dish

- Large collecting area (sensitivity)
- Fully steerable (sky coverage)
- Multiple observing bands (cost optimisation)
- Special applications (e.g. VLBI)

Parkes

Signals and noise

- In radio astronomy, signals are also noise
- Noise of interest: signal vs unwanted signal: noise
- Increase signal strength and reduce noise: wait for sensitivity lecture
- Radio astronomical signals are very weak: $1\ \mathrm{Jy} = 10^{-26}\ \mathrm{Wm^{-2}Hz^{-1}}$

Brightness : $B(\nu) = \dfrac{2h\nu^3}{c^2}\dfrac{1}{e^{h\nu/k_BT} - 1}\quad \mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$ (Planck's law)

For radio frequencies, $h\nu \ll k_BT$, so $B(\nu) \approx \dfrac{2h\nu^3}{c^2}\dfrac{k_BT}{h\nu} = \dfrac{2k_BT}{\lambda^2}$

Brightness temperature, $T_b(\nu) = B(\nu)\dfrac{\lambda^2}{2k_B}$

Signals to Noise ratio

Let a radio telescope have a gain G. Let the instantaneous RMS fluctuations in the output be $T_{\rm sys}$

RMS Fluctuations can be reduced: $\dfrac{T_{\rm sys}}{\sqrt{\Delta\nu\tau}}$

For a source with brightness S (Jy), $T_A = GS$

Signal-to-noise, $S/N = \dfrac{GS}{T_{\rm sys}/\sqrt{\Delta\nu\tau}} = \dfrac{GS\sqrt{\Delta\nu\tau}}{T_{\rm sys}}$

S/N is usually a number. If S/N > threshold, the source is detected. Increasing integration time and/or bandwidth improves S/N

$$T_A = \frac{P}{k_B}$$

$$T_{\rm sys} = \frac{\text{Total power referred to receiver inputs}}{k_B}$$

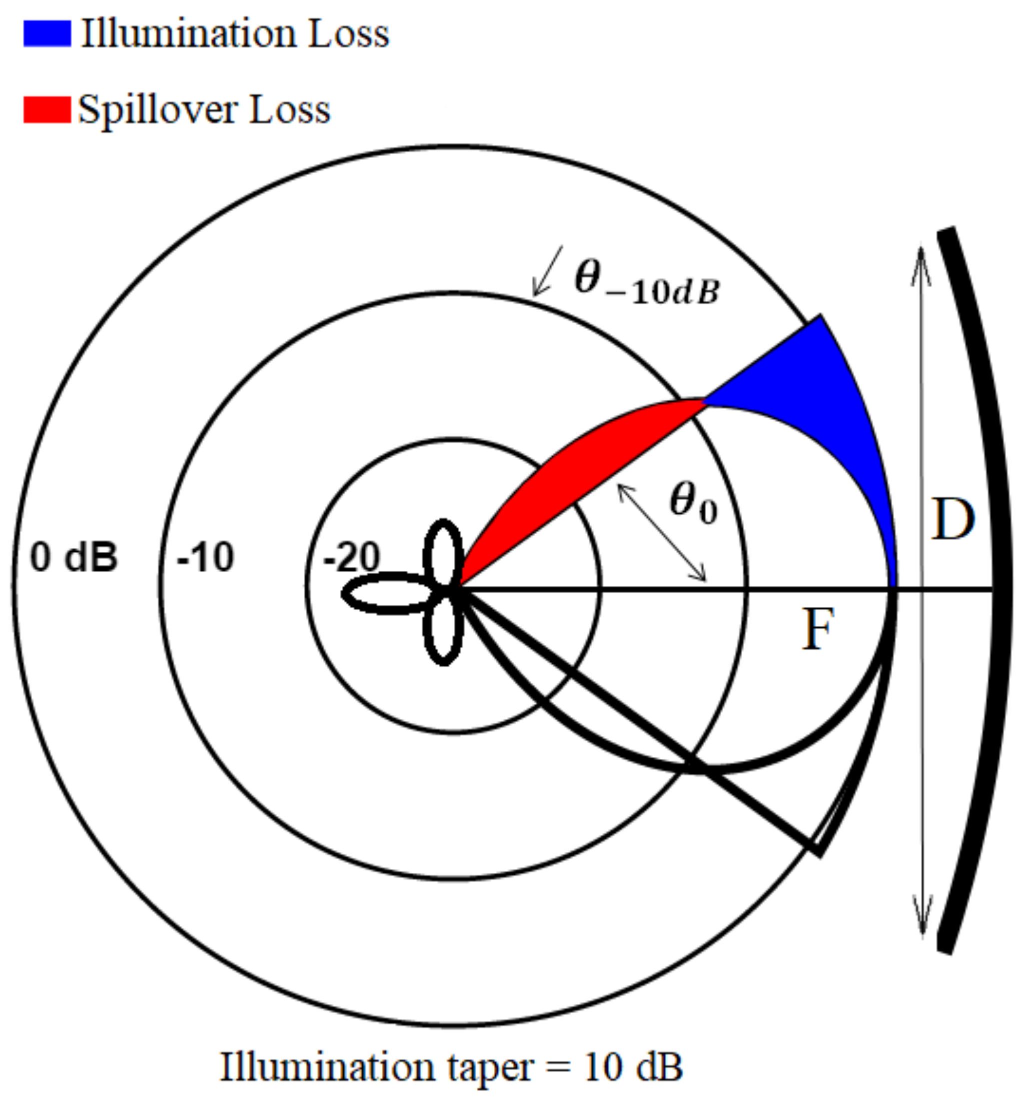
where $T_{\rm sys} = T_{\rm sys} + T_{\rm rec} + T_{\rm spill} + T_{\rm loss}$

Antenna Gain

Effective area, $A_e = \frac{\text{Power density available at antennas termainls}}{\text{Flux density of incident radio wave}} = \frac{W/Hz}{W/m^2/Hz} = m^2$

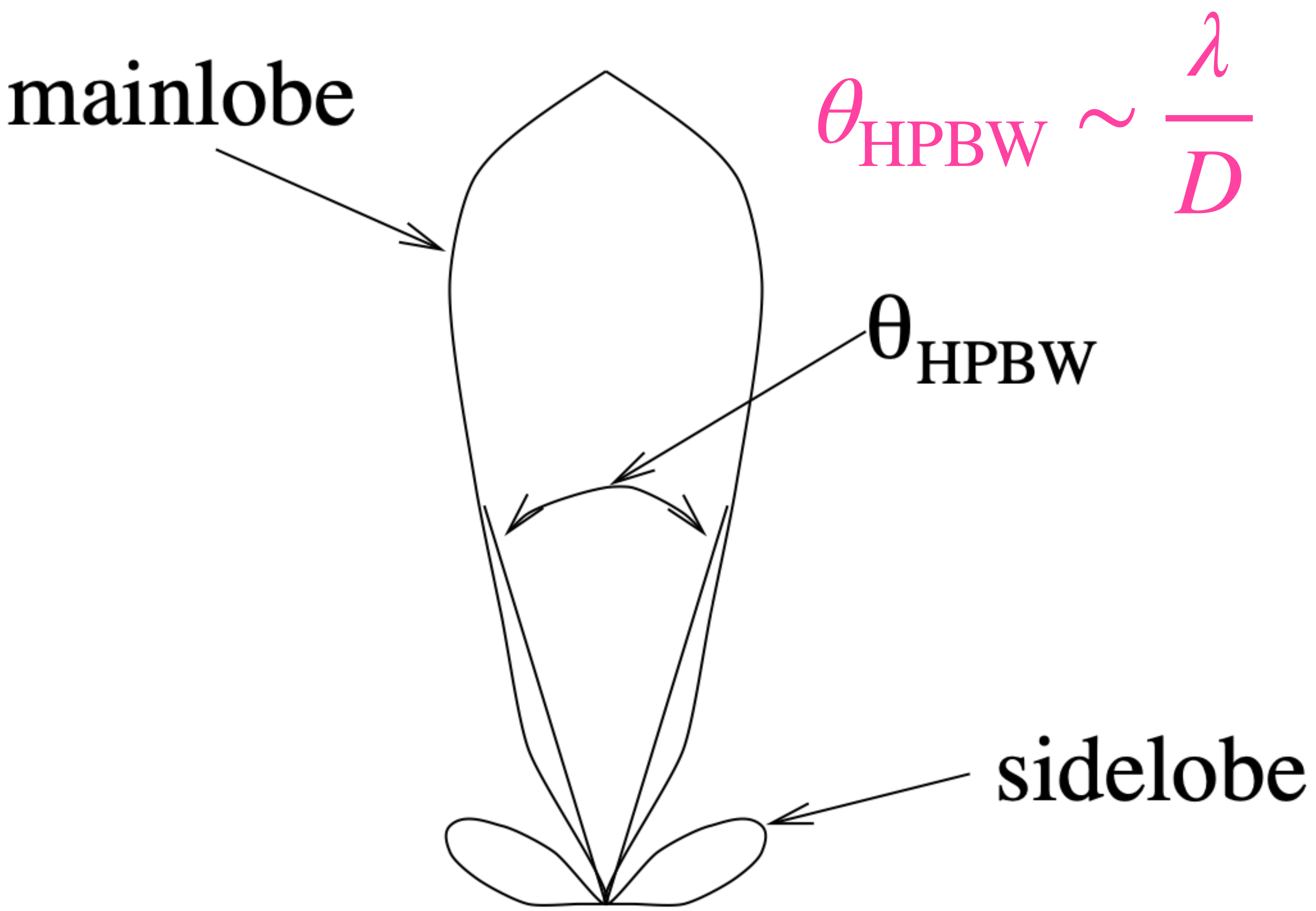
Antenna works better in some directions, so A_e is a function of angle: $A_e(\theta, \phi)$

$$P(\theta, \phi) = 10\ \log_{10}\left(\frac{A_e(\theta, \phi)}{A_e^{max}}\right)\ \text{dB}$$



Aperture efficiency

$$\eta = \frac{A_e^{max}}{A_g}$$



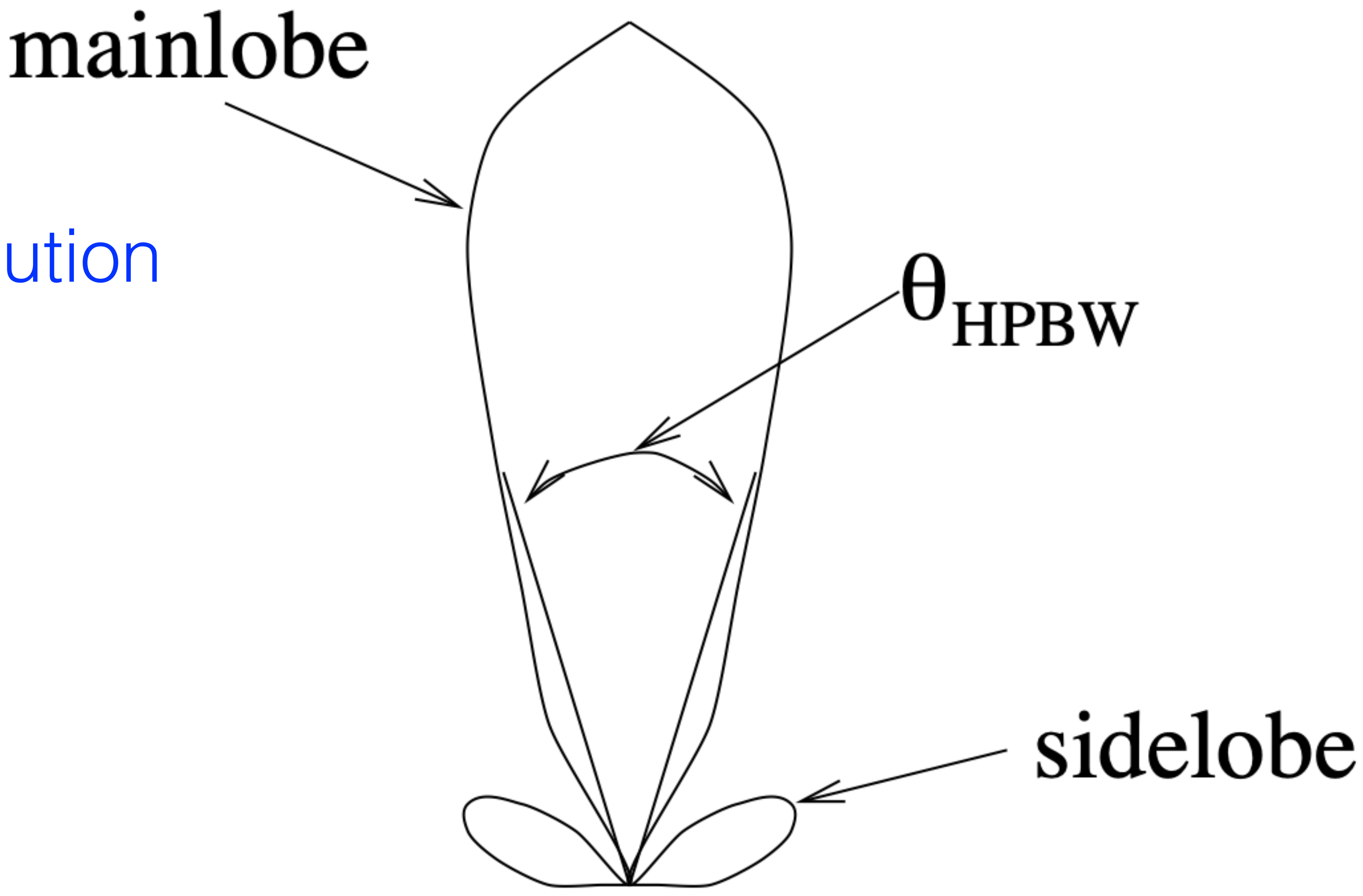
Measured brightness and temperature

Brightness measured at antenna terminals

$$W(\theta', \phi') = \frac{1}{2} \int B(\theta, \phi)\ A_e(\theta - \theta', \phi - \phi')\ \sin\theta\ d\theta\ d\phi$$

Antenna temperature corresponding to the brightness distribution

$$T_A(\theta', \phi') = \frac{A_e^{\max}}{\lambda^2} \int T_B(\theta, \phi)\ P(\theta - \theta', \phi - \phi')\ \sin\theta\ d\theta\ d\phi$$



Antenna temperature is a smoothed version of the sky brightness temperature

$T_B(\theta, \phi)$ could be a single, infinitely peaked source or a collection of faint sources, producing the same $T_A(\theta, \phi)$. The lowest detectable $T_A(\theta, \phi)$ sets the confusion limit.

Beam pattern

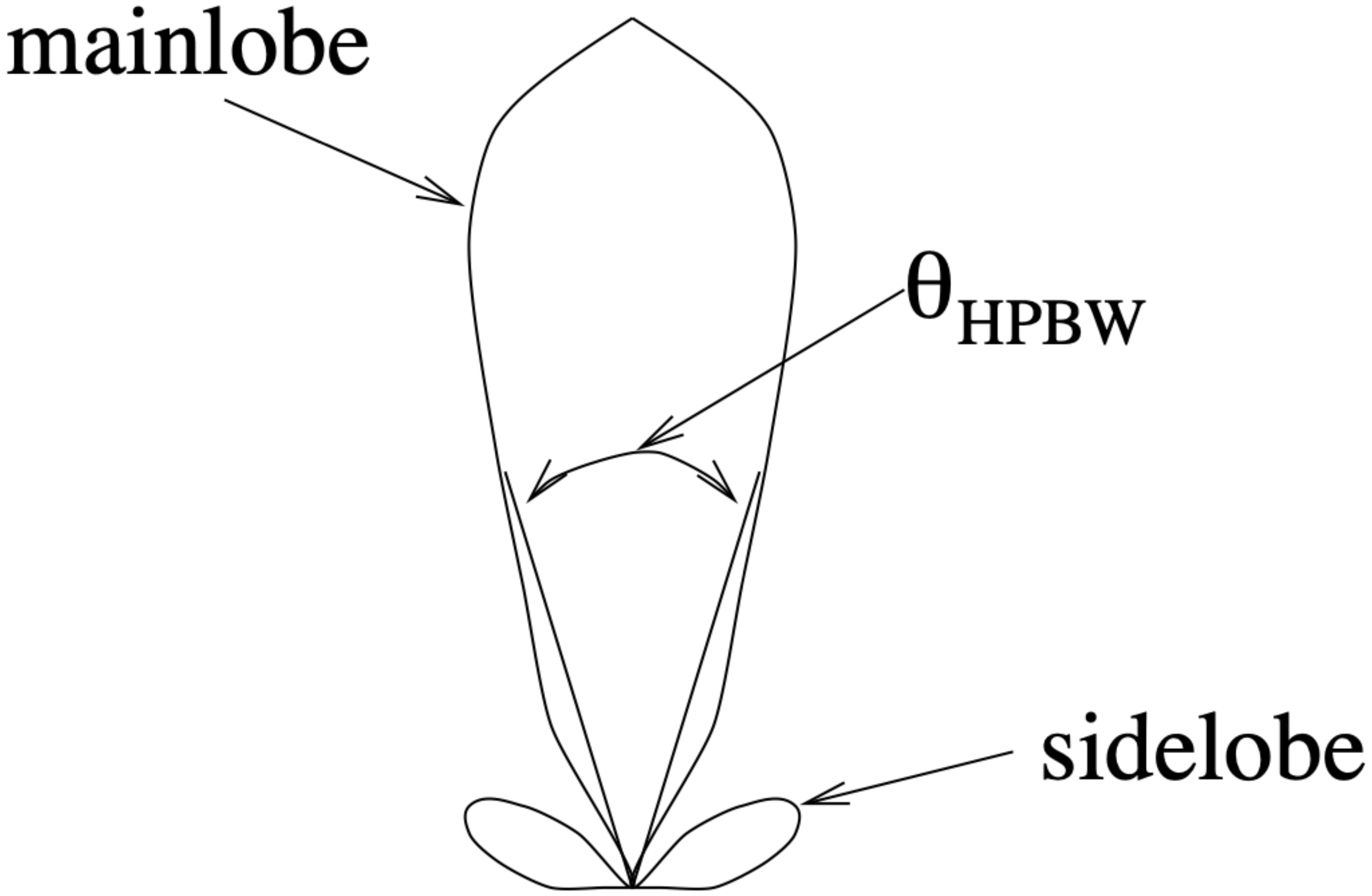
Beam pattern is related to the aperture size and shape

$$U(\theta) = \int u(l)\, e^{-2\pi\theta l}\, dl$$

Circular aperture < — > circularly symmetric beam pattern

The relationship is a Fourier transform pair

- 1D rectangular <—> 1D sinc function
- 2D rectangular <—> 2D sinc function
- Circular <—> Bessel function: $\theta_{\mathrm{HPBW}} \sim \dfrac{\lambda}{D}$
- Gaussian <— > Gaussian



Phased arrays

Resolution: ability to distinguish directions

Single dish resolution: HPBW

$$\theta_{\mathrm{HPBW}} \sim \frac{\lambda}{D}$$

Large HPBW increases confusion, reduces detectability of fainter signals

Increasing resolution: decrease λ or increase D

Increasing is very expensive, scales as $D^{2.7}$

Increase the number of antennas: N antennas of aperture D, instead of one antenna of area $\sim (\sqrt{N}D)^2$

These antennas can be made to work like a single antenna: phased array

Phased arrays: *Linear arrays*

$$E(\theta) = E_0 + E_0 e^{i\psi} + E_0 e^{i2\psi} + E_0 e^{i3\psi} + \ldots + + E_0 e^{i(n-1)\psi}$$

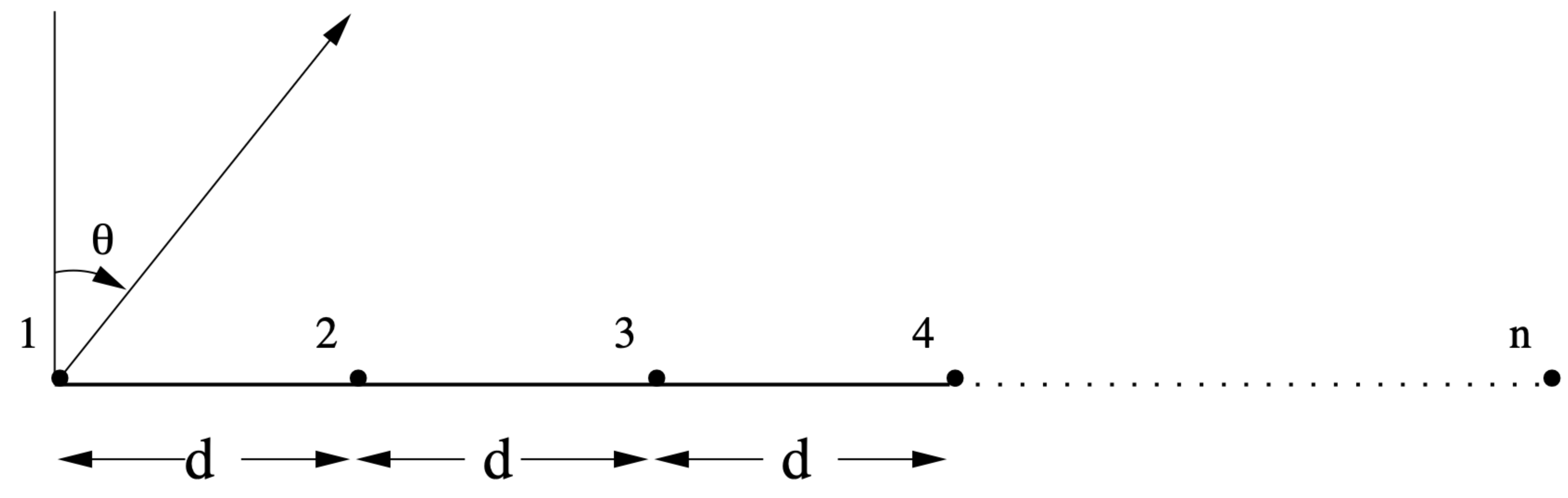
$$E(\theta) = E_0 \left(1 + e^{-i\psi} + e^{-i2\psi} + e^{-i3\psi} + \ldots + e^{-i(n-1)\psi}\right) = E_0 \frac{sin(n\psi/2)}{sin(\psi/2)} e^{-i(n-1)\psi/2}$$

Notice that this is a Fourier transform

When the array elements have the same phase, the array is said to be 'phased'.

The direction of the beam can be changed by introducing a phase gradient across the array.

Effectively, the beam can be 'steered' to a desired direction.



- Resolution: $\theta_{res} \sim \frac{\lambda}{D_{max}}$
- Increase sensitivity and resolution
- Reduces confusion

Phased arrays vs Incoherent arrays

Phased array	Incoherent array
$V_{\mathrm{PA}} = \left\langle \left(\sum_{i=1}^{n} V_i \right)^2 \right\rangle$	$V_{\mathrm{IA}} = \left\langle \left(\sum_{i=1}^{n} V_i^2 \right) \right\rangle$
Voltages are added in phase (before detection)	Powers are added (after detection)
Beam size N times smaller	Beam size same as single antenna
Multiple beams can be obtained (offline)	One incoherent beam can be obtained
High resolution beam	Low resolution beam
Useful when source position is well-known	Useful when source position is not known precisely
N times sensitive than a single antenna	$\sqrt{N}$ times sensitive than single antenna

Questions