References:
1. Synthesis imaging in radio astronomy II (Chp. 2, 7)
2. Interferometry and synthesis in radio astronomy (Chp. 10)
3. Low Frequency Radio Astronomy and the LOFAR observatory (Chp. 8)
4. My lecture 10 and 11 in Astro. Tech II course:
http://www.ncra.tifr.res.in/~ruta/files/2022/2022-lecture-11-ruta.pdf
http://www.ncra.tifr.res.in/~ruta/files/2022/2022-lecture-10-ruta.pdf

Deconvolution

Ruta Kale

Radio Astronomy School 2024, 19 November 2024

Outline

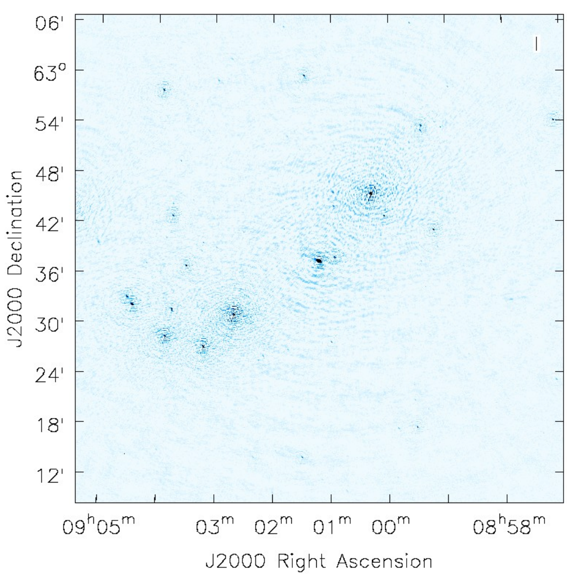
What is deconvolution ?

How is it achieved ?

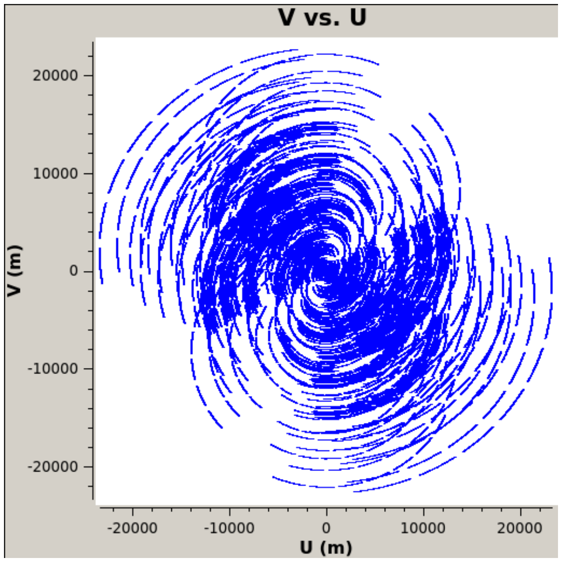
Practical considerations

Imaging

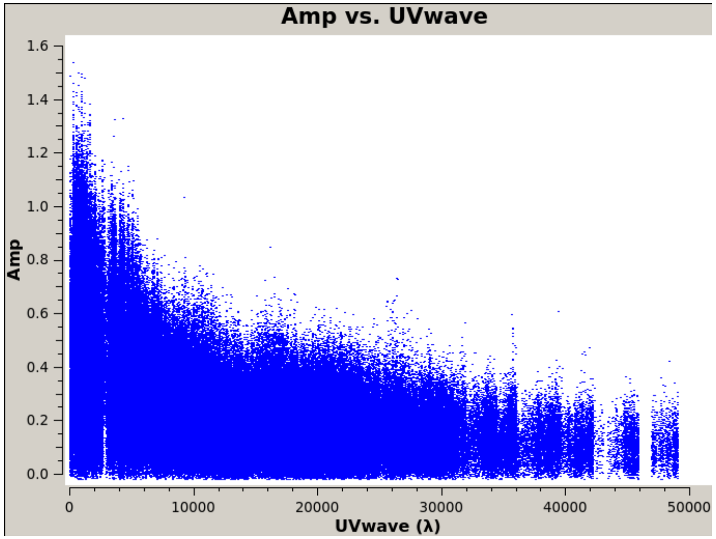
$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(u,v)V'(u,v)e^{2\pi i(ul+vm)}\,du\,dv$$



"Dirty" image



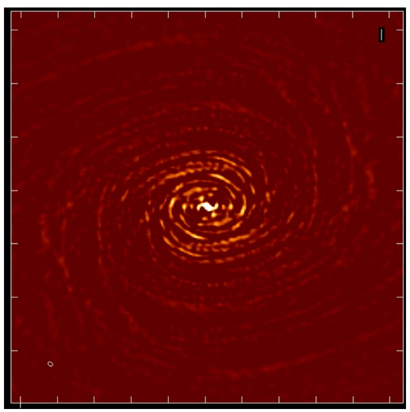
Sampling



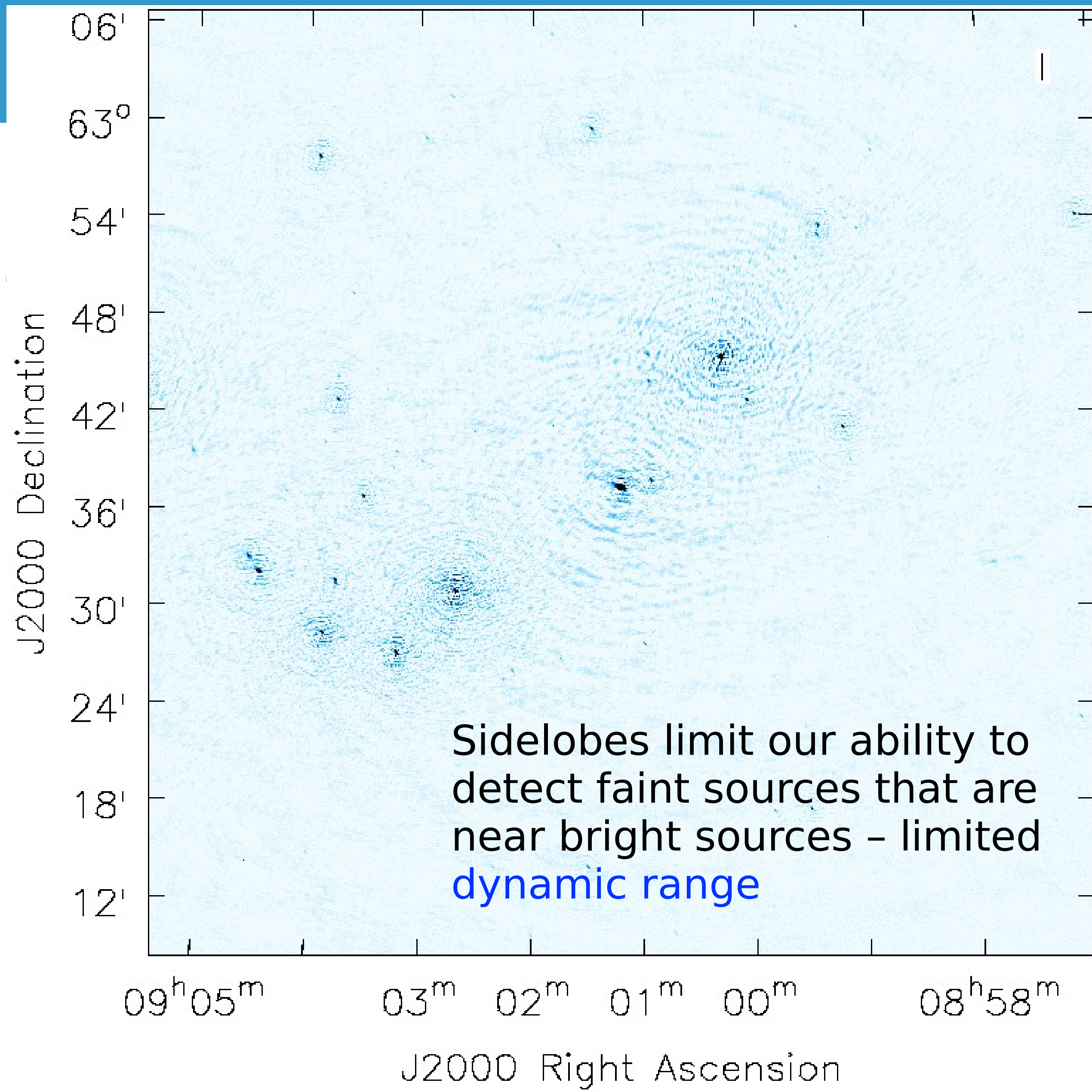
Observed visibilities (complex numbers) Only amp. Shown.

Imaging

$$I^D(l,m) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$



Point source response or synthesized beam



Sidelobes limit our ability to detect faint sources that are near bright sources – limited dynamic range

“Dirty” image

Imaging: deconvolution

$$V(u,v) = \int\int_S I(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm$$

Only a finite number of measurements (noisy too) of the visibilities are available; thus recovering *I(l,m)* has limitations.

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A model with a finite number of parameters is needed.
A general purpose model of the sky is that of a 2-D grid of delta functions with strengths $\widehat{I}(p\Delta l, q\Delta m)$ can be considered.

The delta_l and delta_m are the separations of grid elements in two orthogonal sky coordinates.

Imaging: deconvolution

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The visibility predicted by this model is given by:

$$\widehat{V}(u,v) = \sum_{p=1}^{N_l} \sum_{q=1}^{N_m} \widehat{I}(p\Delta l, q\Delta m) e^{-2\pi i(pu\Delta l + qu\Delta m)}$$

Imaging: deconvolution

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N_l and N_m are pixels on each side. And the range of the uv points sampled are required to be:

$$\Delta l \leq \frac{1}{2u_{\max}},\ \Delta m \leq \frac{1}{2v_{\max}}$$

~Pixel size in the image

$$N_l \Delta l \geq \frac{1}{u_{\min}},\ \text{and } N_m \Delta m \geq \frac{1}{v_{\min}}$$

~Size of the image

One can estimate source features with widths in the range:

Minimum= $\mathcal{O}(1/\max(u,v))$ Maximum= $\mathcal{O}(1/\min(u,v))$

$N_l N_m$ free parameters that are the cell flux densities.

Imaging: deconvolution

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Estimate these for the GMRT

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Imaging: deconvolution

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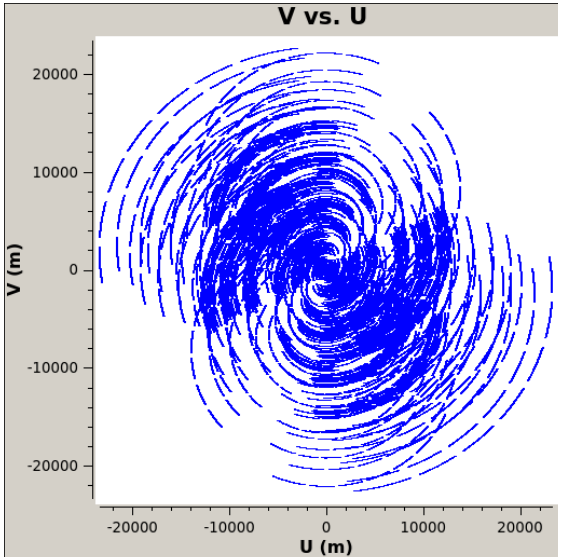
The measurements constrain the model such that at the sampled u,v points

$$V(u_k, v_k) = \widehat{V(u_k, v_k)} + \epsilon(u_k, v_k)$$

$\epsilon(u_k, v_k)$ is a complex, normally distributed random error due to receiver noise.

At the points in the plane where no sample was taken the model is free to take on any value.



Imaging: deconvolution

$V(u_k, v_k) = \widehat{V(u_k, v_k)} + \epsilon(u_k, v_k)$ can be expressed as a

multiplicative relation

$$V(u,v) = W(u,v)(\widehat{V}(u,v) + \epsilon(u,v))$$

W is the weighted sampling function;

$$W(u,v) = \sum_k W_k \delta(u - u_k, v - v_k)$$

Non-zero only for the sampled points.

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In the image plane this translates to a convolution relation:

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Noise image

Dirty image

$$I^D_{p,q} = \sum_k W(u_k, v_k)\ \mathrm{Re}\left(V(u_k, v_k) e^{2\pi i (p u_k \Delta l + q v_k \Delta m)}\right)$$

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Dirty image

Dirty beam

Imaging : deconvolution

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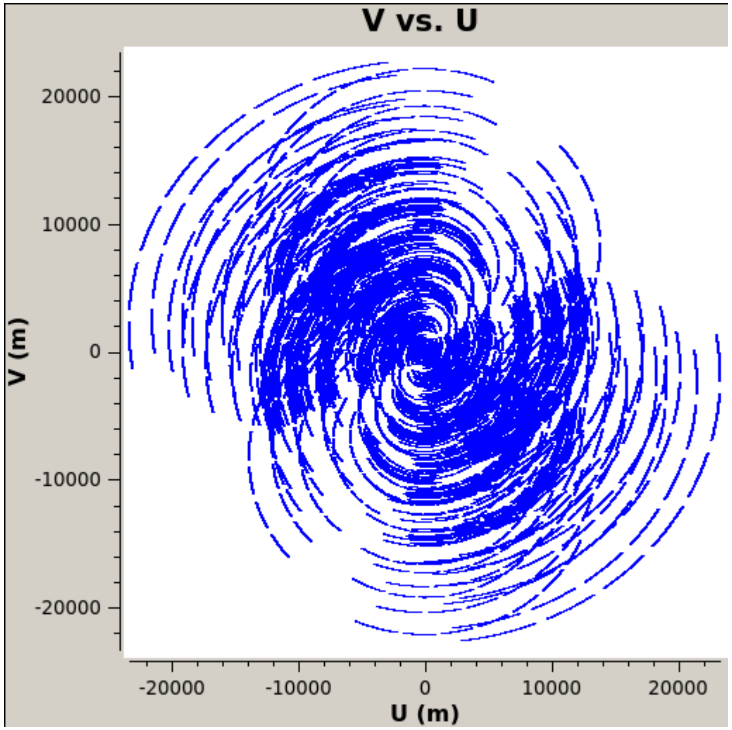
$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

The model when convolved with the point spread function (dirty beam) corresponding to the sampled weighted (u,v) coverage should yield the dirty image.



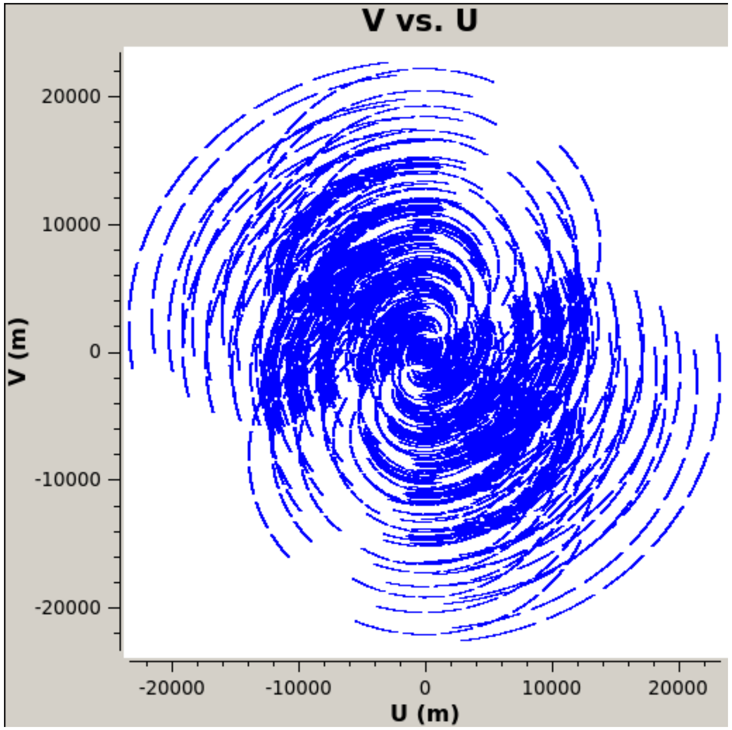
Principal solution and invisible distributions

$$I^D_{p,q} = \sum_{p',q'} B_{p-p',q-q'} \widehat{I}_{p',q'} + E_{p,q}$$

Is the solution unique ?
Solution not unique in the absence of boundary conditions.
Existence of “homogenous” solutions: called invisible distributions in radio astronomy.

Invisible distribution is that which has non-zero amplitude in only the unsampled spatial frequencies.

Also called as “ghosts”!



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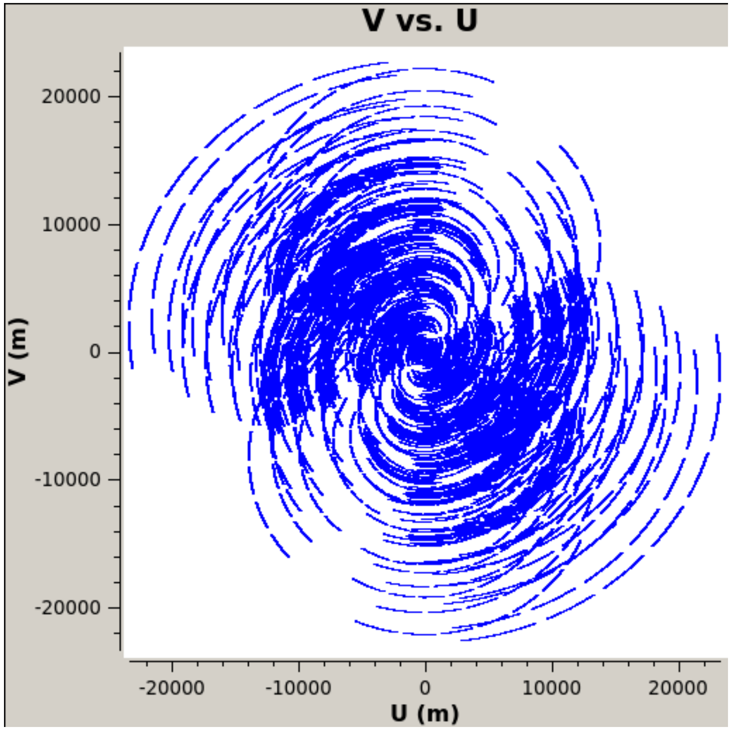

Invisible distributions arise due to :
- Limit on the extent of u,v coverage.
- Holes in the u,v coverage

Principal solution and invisible distributions

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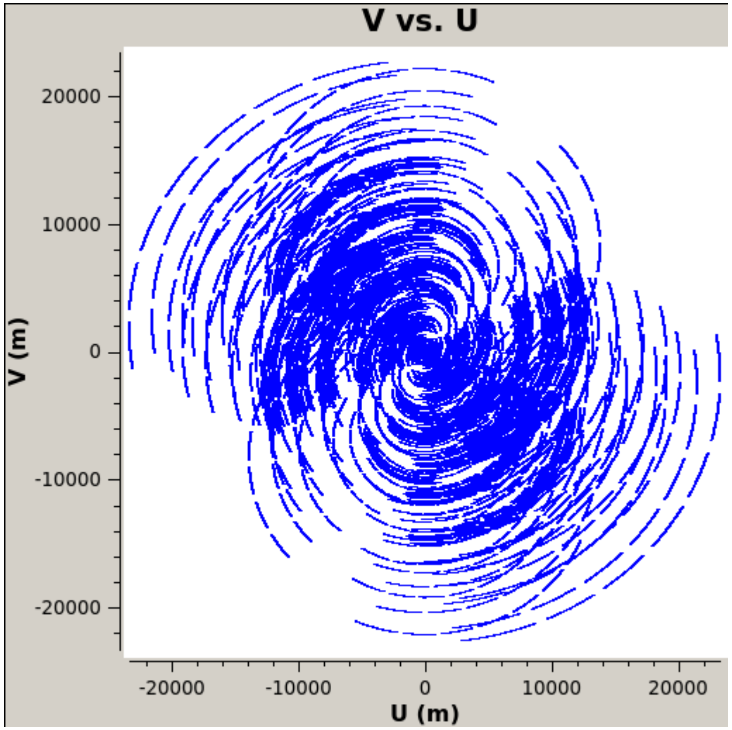


Invisible distribution is that which has non-zero amplitude in only the unsampled spatial frequencies.

- The solution having zero amplitude in all the unsampled spatial frequencies is called the *principal solution*.

Problems with the principal solution

- The solution having zero amplitude in all the unsampled spatial frequencies is called the *principal solution*.

Principal solution is not enough as we cannot make out the if the source is a point source or is shaped like the beam !

Also it will change as we change the visibilities. *We need a method to estimate the visibilities in the unsampled range.*

We can use the information at the total intensity of the source must be positive.

Use of a priori information is the key to making an image of the sky.

Deconvolution algorithms use this to obtain better estimates of the sky than given by the principal solution.

Deconvolution: non-linear, iterative image re-construction

CLEAN algorithm : Hogbom 1974

Deconvolution methods originate in a “deceptively” simple idea proposed by J. Hogbom (A&AS, 15, 417, 1974) – turned out to be a breakthrough for radio astronomy!

The CLEAN algorithm (Högbom 1974)

- Provides a solution to the convolution equation by representing any source as a collection of point sources. An iterative approach is used to find the positions and strengths of the point sources.

- It makes use of the fact that the dirty beam is known and thus we can remove features of it and tell them apart from a real source.

- The final “deconvolved” image is called CLEAN image – it is the sum of the point source components convolved with the CLEAN beam – chosen usually to be a Gaussian.

Högbom’s CLEAN algorithm

1. Find the position and strength of the brightest point in the dirty image, $I^{D}_{p,q}$.

2. Multiply the peak with the dirty beam B and a “damping factor” (loop gain) and subtract from the dirty image.

3. Save the position and strength of the peak in a “model image”.

4. Go to (1) and repeat for the next peak until there is no peak above a user specified level.

Finally one will have “residual” image.

5. Convolve the model image with an idealized CLEAN beam (Gaussian fitted to the central peak of the dirty beam) to form a CLEAN image.

6. Add the residuals and the CLEAN image.

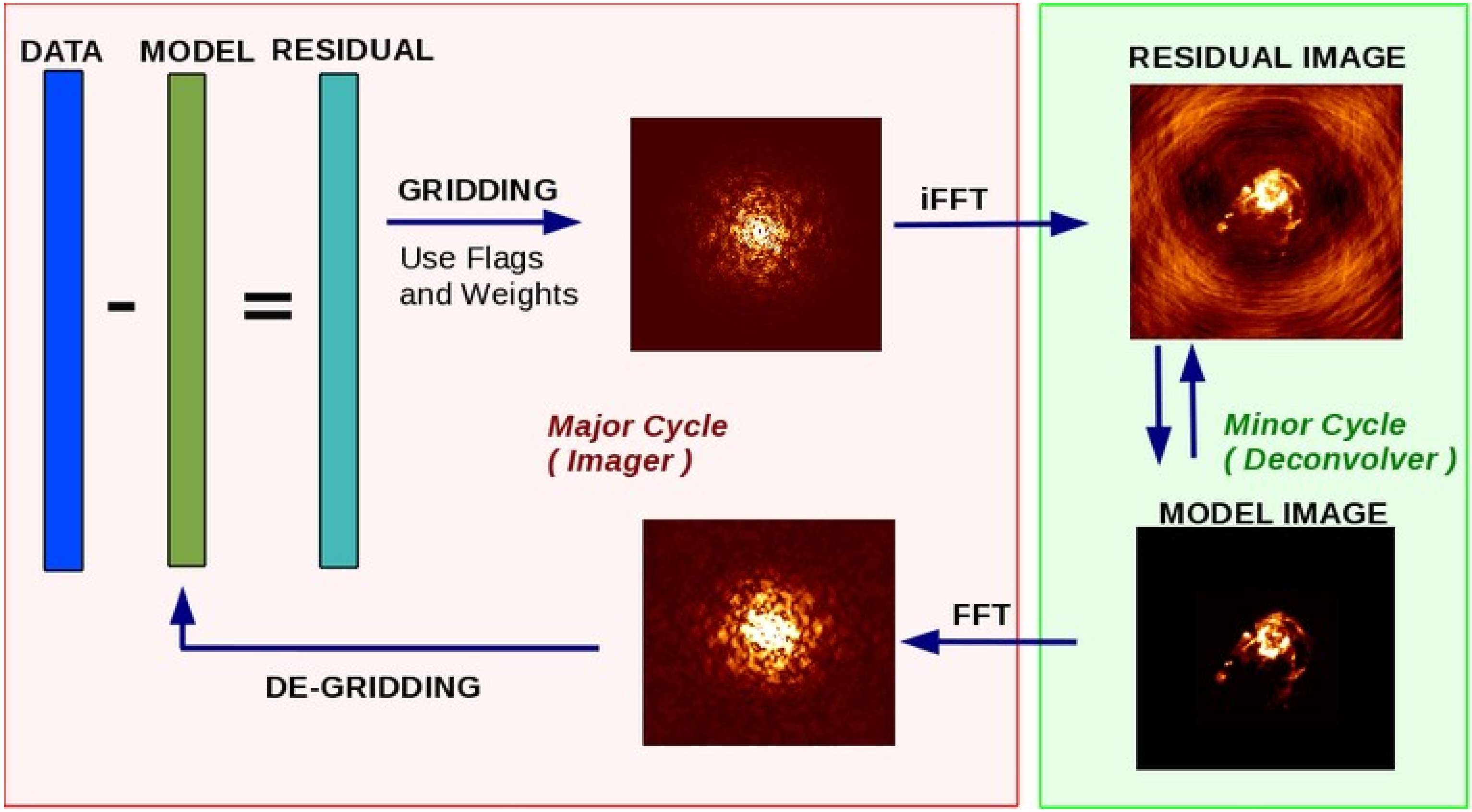
CLEAN algorithm

Clark (1980) CLEAN: use of psf patches

Minor cycle:beam patch to select for components; proceeds like Hogbom CLEAN
Major cycle: Point source model is transformed via FFT, transformed back and subtracted from dirty image.

Cotton-Schwab CLEAN: Periodically predict model-visibilities, calculate residual visibilities and re-grid – major and minor cycles: works on ungridded visibilities
Minor cycle: each field cleaned independently but in major cycle components from all the fields are removed – relevant for the non-coplanar baselines case.

Major and minor cycles



Major and minor cycles implement an iterative weighted χ2 minimization process that solves the measurement equation.

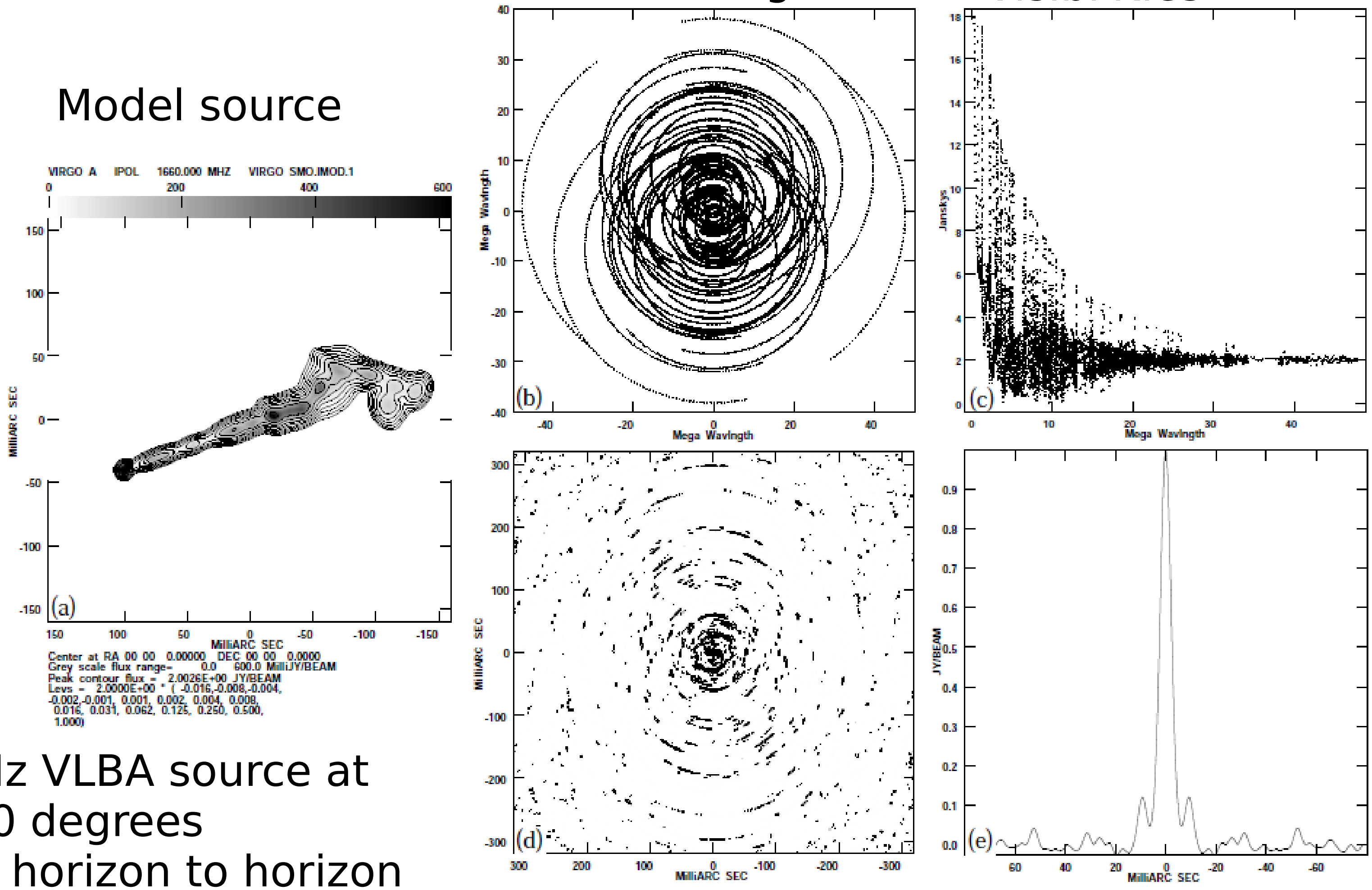
Schematic from U. Rau

Example

uv-coverage

Visibilities

Model source



1.6 GHz VLBA source at dec. 50 degrees
Nearly horizon to horizon coverage.

Dirty beam

Dirty beam

Softwares implementing CLEAN

NRAO AIPS: Astronomical Image Processing System

NRAO CASA: Common Astronomy Software Applications

WS-clean (A. Offringa)

Practical considerations

Given visibilities you need to make some estimates of the expected outcomes of the imaging.

Find the expected synthesized beam: needed to set the pixel size (cellsize)

Find the size of the primary beam: needed to set the image size

Given the on-source time find the theoretical noise – revise this estimate based on the state of the visibilities at the start of imaging.

Based on the kind of source you set out to image and the telescope specifications, choose deconvolver and gridders accordingly.

Practical considerations

Deconvolvers (minor cycle): CASA
Minor cycle algorithms (**Hogbom, Clark, Multi-Scale, Multi-Term**)

Since Hogbom CLEAN uses only delta functions, it is most appropriate for fields of isolated point sources. It will incur errors when imaging extended emission and this is typically seen as a mottled appearance of smooth structure and the presence of correlated residuals.

Clark: residual image updates only a part of the image; iterations stop when the peak is below the first sidelobe level of brightest source

Practical considerations

"Multi-scale": Cornwell-Holdaway Multi-Scale CLEAN: deconvolution algorithm designed for images with complicated spatial structure

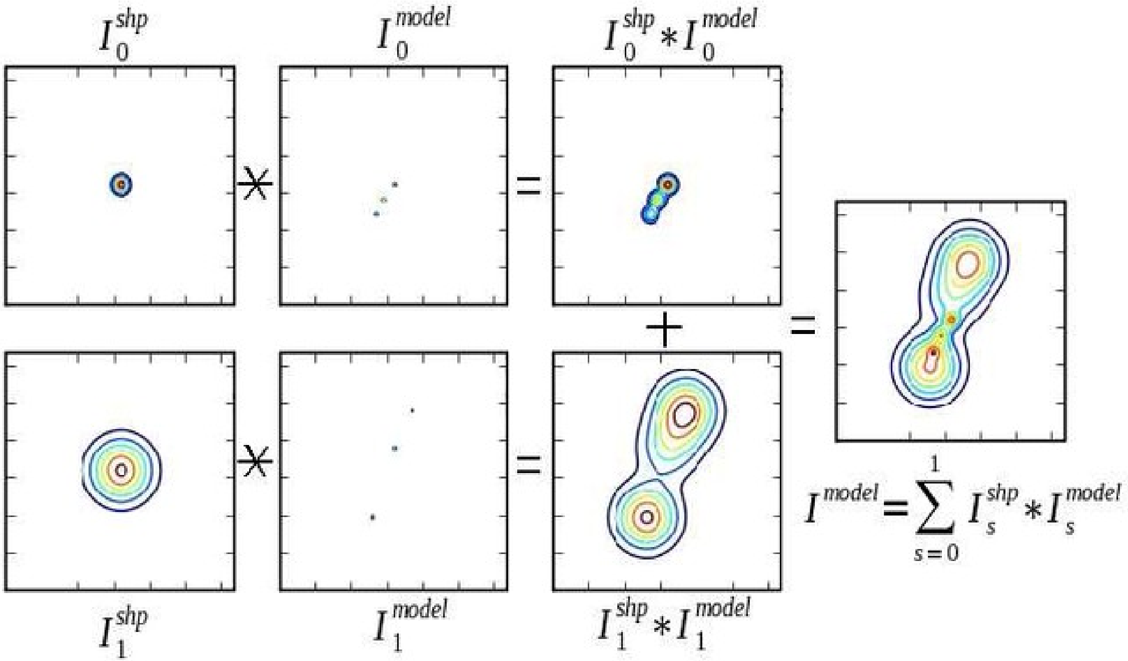


Image credit: U. Rau

Practical considerations

"Multi-scale": Cornwell-Holdaway Multi-Scale CLEAN: deconvolution algorithm designed for images with complicated spatial structure

Need to choose scale sizes:

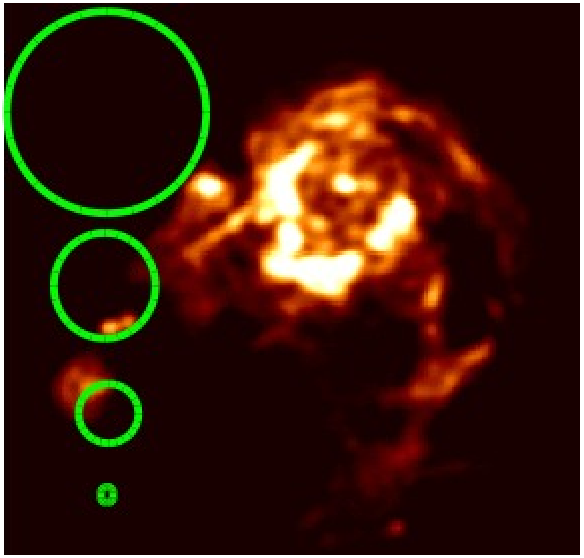


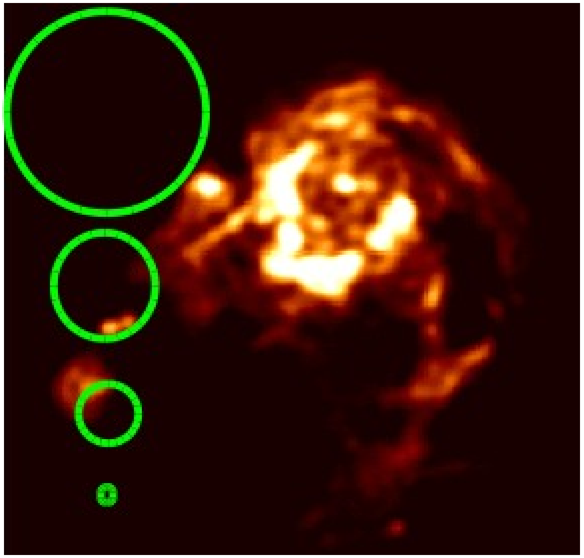
Image credit: U. Rau

Upgraded GMRT

Multi-Scale Multi-Frequency synthesis (MSMFS)

- applicable for uGMRT (wideband data)

For uGMRT typical continuum dataset using the full bandwidth:
Wide-field, wide-band, multi-scale



Clean bias

Systematic lowering of source brightness.

CLEAN algorithm is more error-prone in the low SNR regime. A systematic lowering of source brightness can be explained by the algorithm constructing many artificial source components from the sidelobes of real sources.

Crowded fields

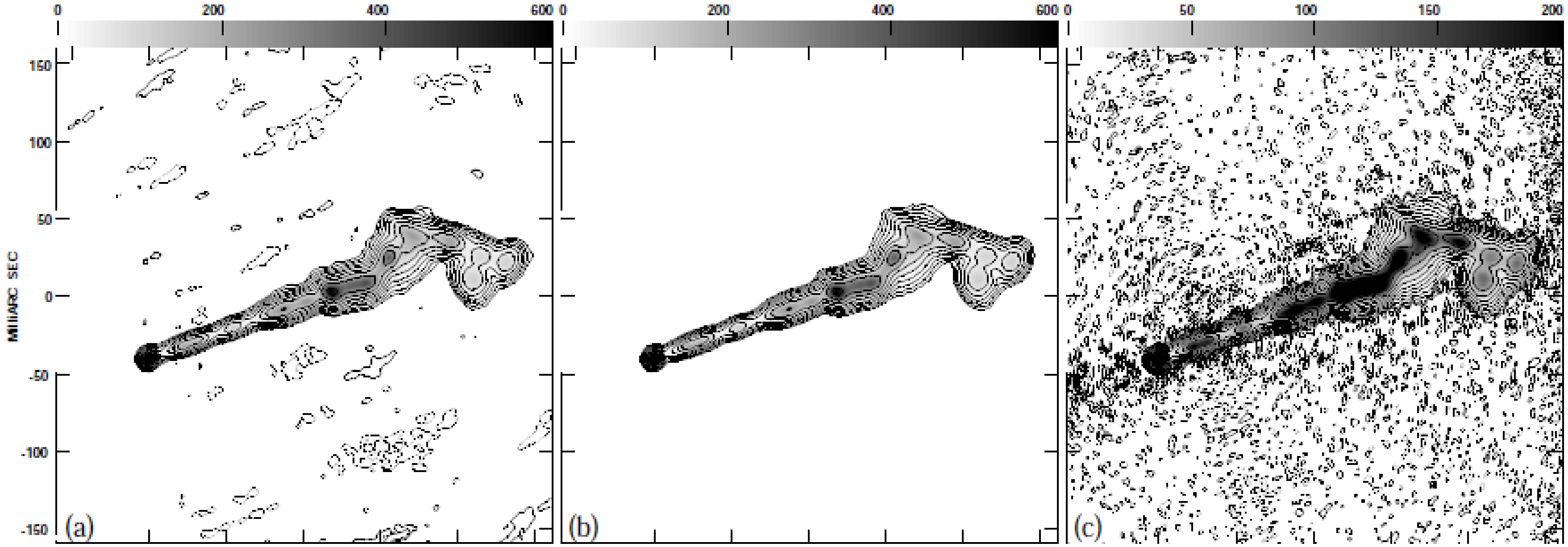
Masking

Instead of a blind search for peaks in the image, provide a mask that restricts the search to regions where sources are expected to be present.

Sky is not completely unknown anymore (to a large extent): Initial mask based on images from all sky surveys.

Mask can be further refined.

In CASA "tclean" task to make the image (choice of deconvolver, gridder): auto-multithresh – an automated way of improving the mask



Restored CLEAN image;
CLEANing without constraint.

Restored CLEAN image;
CLEANing with a constraint to be within the region of the source.

Same as panel b but with contours drawn starting at 10 times lower level to show the pattern in the rest of the image.

Practical considerations

Tutorial sessions !

Alternatives to CLEAN

Maximum Entropy Method (MEM)

In the problem of deconvolution we are trying to select one answer from many possible answers – basically one image from the many possible that can fit the visibilities.

MEM uses a statistical approach to find the most likely image.

We will discuss MEM and other more recent approaches in the last lecture in this course.

Summary

Obtaining the image: A problem that need an iterative solution

Deconvolution: Process by which the image is reconstructed starting from the dirty (true sky convolved with the synthesized beam) image.

CLEAN algorithms: Hogbom, Clarke, Cotton-Schwab Multi-scale

Clean bias

CASA tclean: choice of deconvolvers, gridders and masking

Applications of these to be seen during tutorial sessions.