Wide-field Imaging

Preshanth Jagannathan, NRAO

Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int I(l, m) e^{2\pi i(ul+vm)} dl dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

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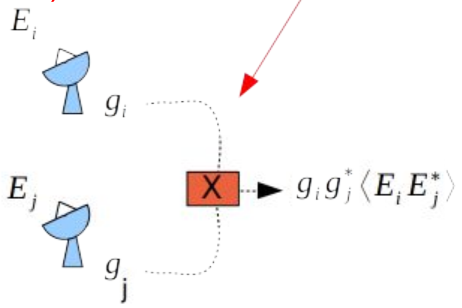

N antennas
N(N-1)/2 antenna-pairs (baselines)

Calibration

Solve for g_i and divide out $g_i g_j^*$

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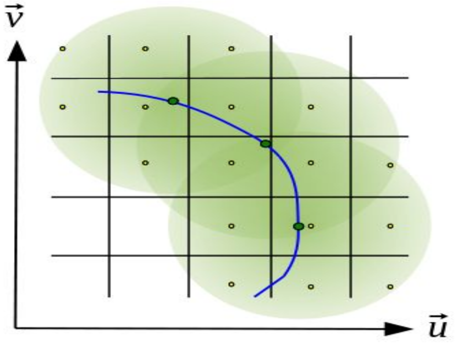
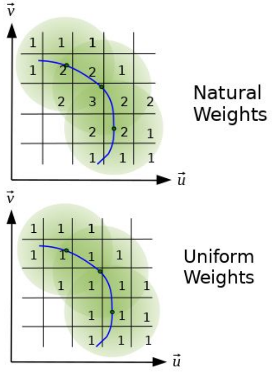
Imaging

Gridding
+
IFFT
+
Normalization





Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int I(l, m) e^{2\pi i(ul+vm)} dldm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

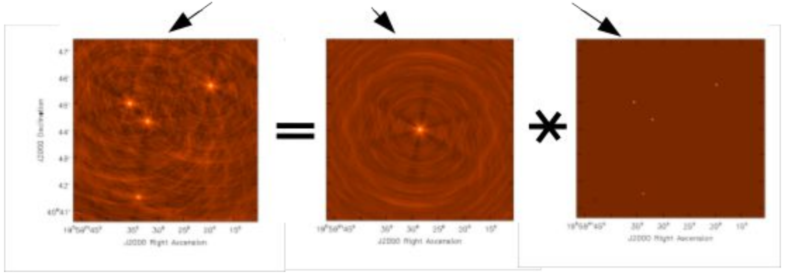
Fourier transform kernel

Gridding
+
IFFT
+
Normalization

Deconvolution

Algorithms :
Clean, MS-Clean, Asp, etc...

$$I^{obs}(l, m) = I^{PSF}(l, m) * I^{sky}(l, m)$$



Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int I(l, m) e^{2\pi i(ul+vm)} dl dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

This is an approximation

Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int I(l, m) e^{2\pi i(ul+vm)} dldm$$

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int\int M_{ij}^{S}(l, m, \nu, t) I(l, m, \nu, t) e^{2\pi i(ul+vm+w(n-1))} dldmdn$$

Direction Independent Gains

UV sampling function

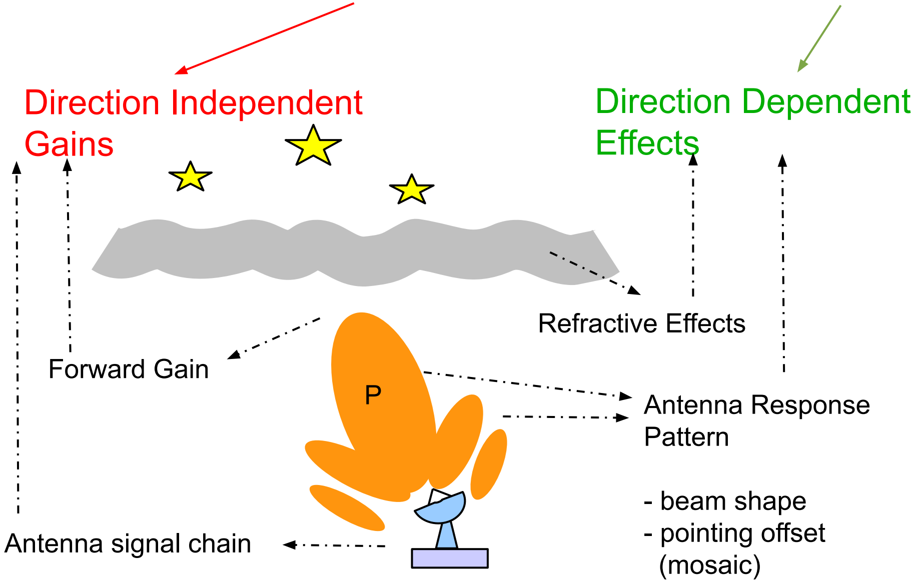
Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

Widefield Imaging - Calibration + Imaging

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$



On-axis gains : **Calibration**

Off-axis effects : Use primary-beam or aperture models in **Imaging**

What models ?

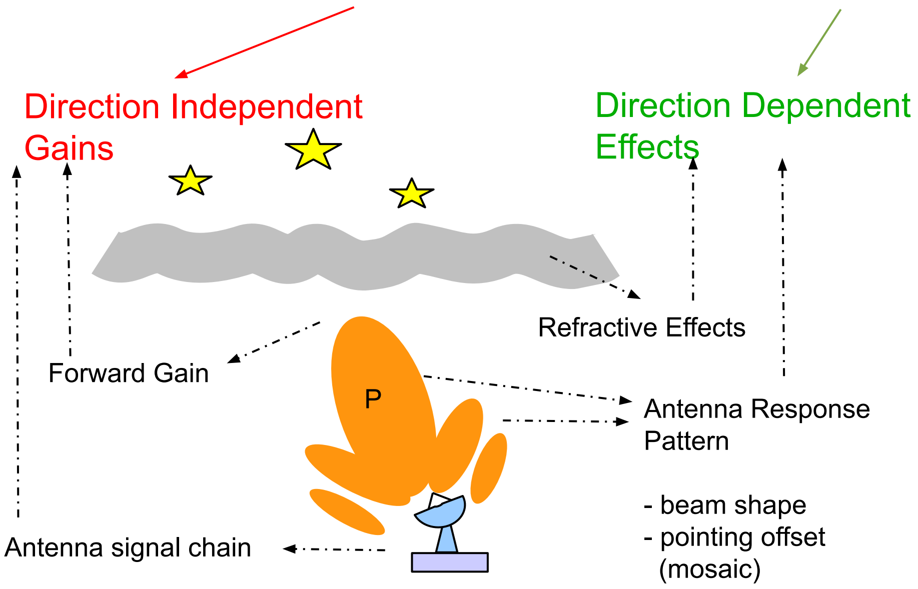
– Theoretical, or measured+fitted (ray-traced, holography, ionosphere maps)

(or)

– Solve for DD gains or PB model params from the data themselves (direction-dependent calibration)

Widefield Imaging - Calibration + Imaging

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \int\int\int M_{ij}^{S}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$



$$I^{obs}(l,m) = \sum_{ij,t,\nu} I_{ij}^{PSF}(l,m,t,\nu) * \left[P_{ij}(l,m,t,\nu) \cdot I^{sky}(l,m) \right]$$

The Sky is multiplied by a Primary Beam, **before** being sampled by each baseline

Not a simple convolution equation anymore

One can to first order assume a uniform azimuthally symmetric PB and divide it out.

Limits achievable dynamic range

Widefield Effects - Antenna PB

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \int\int\int M_{ij}^{S}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

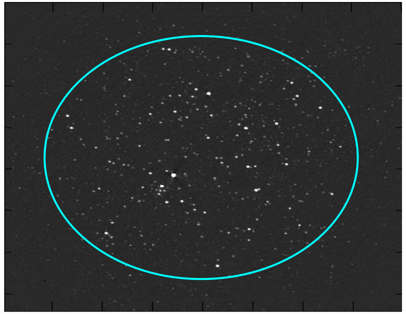
$$I^{obs}(l,m) = \sum_{ij,t,\nu} I_{ij}^{PSF}(l,m,t,\nu) * [P_{ij}(l,m,t,\nu) I_{ij}^{sky}(l,m,t,\nu)]$$

The Sky is multiplied by a Primary Beam, **before** being sampled by each baseline

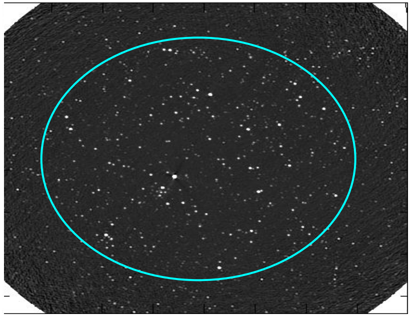
Not a simple convolution anymore

Normally we assume a singular non time varying model for the PB and divide it out of the equation

Output Image = PB x I

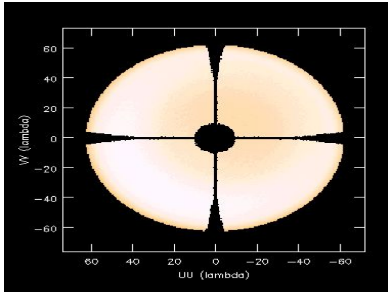


PB-corrected Image

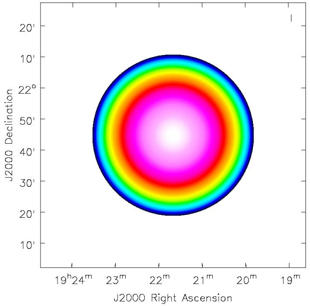


Useful Fourier Pairs

Aperture

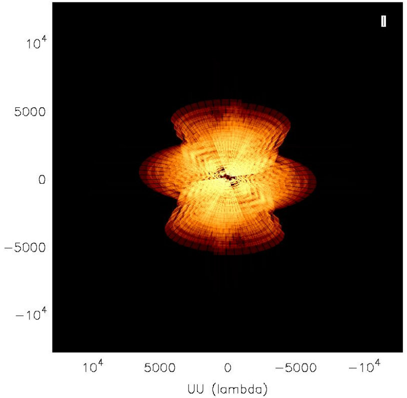


iFFT

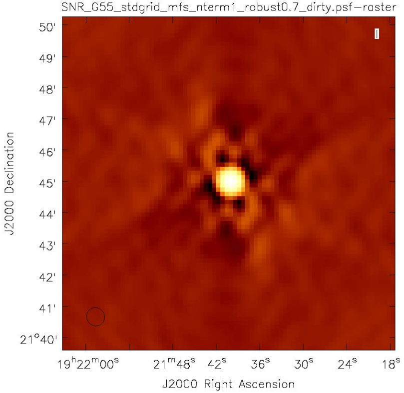


Primary Beam

UV Coverage



FFT



Synthesized Beam - PSF

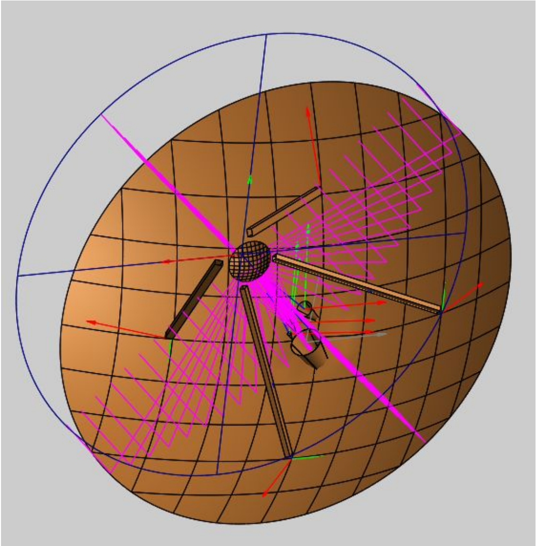
Measurement Equation - I

$$\vec{e_a} = J_i.\vec{\epsilon} \qquad \vec{e_b} = J_j.\vec{\epsilon}$$

$$\vec{I_{ab}} = \vec{e_a} \otimes \vec{e_b}$$

$$\vec{V}_{pq}^{obs} = \mathcal{F}_{pq} S^{ab} \vec{I}_{ab}$$

The measured power at the correlator is a cross correlation of the voltages received by two antennas.
Note that the antennas can be of different types i,j
The basis a,b are the sky stokes basis
The matrix s performs the transfer of basis from stokes to an orthogonal feed basis p,q
vCZ theorem relates the sky brightness distribution to the measured interferometric voltages.



L-Band VLA Grasp 10 simulation .
Bruce Veidt DRAO.

Measurement Equation - II

$$\vec{I}_{ab} = (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}_b^{\,*} \vec{J}_j^{\,*})$$

$$\vec{V}_{pq}^{obs} = \mathcal{F}_{pq} S^{ab} (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}_b^{\,*} \vec{J}_j^{\,*})$$

$$\vec{V}_{pq}^{obs} =_{pq} S^{ab} (\vec{A}_i \circledast \vec{A}_j^{\,*}) \star \vec{V}_{ab}$$

Our goal then is to be able to reconstruct true sky brightness or the true sky coherence function

A-Projection

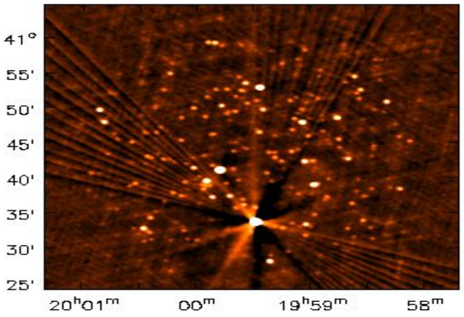
If the antenna A term were (approximately hermitian) we could consider an inversion operation of the form

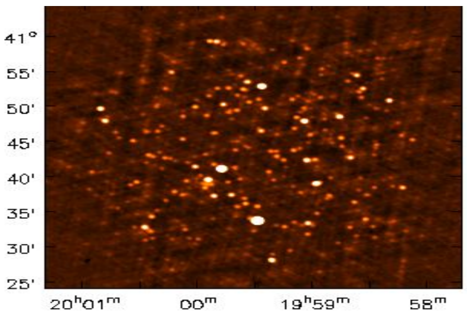
$$(\vec{A}_i \circledast \vec{A}_j)^{M^\dagger} \star \vec{V}_{ab}^{obs} = |Aij|^2 \star \vec{V}_{ab}$$

This term that is applied on the left is the kernel for A-projection at the time of gridding. The term in the modulus is the square of the antenna PB (forward gain) in the image plane so we divide it out after we take the FT to go from data to the image.

$$\frac{\mathcal{F}^\dagger(\vec{A}_i \circledast \vec{A}_j)^{M^\dagger} \star \vec{V}_{ab}^{obs}}{|Mij|} = PB.I_{ab}$$

$$\mathcal{F}^\dagger(\vec{A}_{ij}) = \vec{M}_{ij}$$



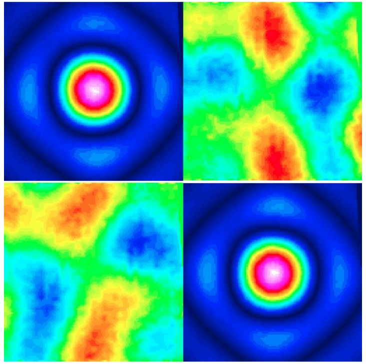


Widefield Polarimetry

$$\vec{I}_{ab} = (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}_b^{\,*} \vec{J}_j^{\,*})$$

The antenna jones Can be measured through holography.

$$\vec{J}_i =$$

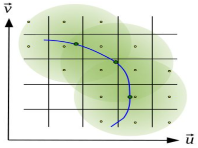


S-band ~ 3GHz, In feed basis
I.e. pq

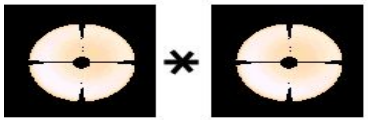
A-Projection + Mosaic

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$

A-Projection : Use aperture illumination functions to construct gridding convolution functions.

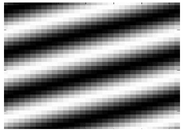


Use as the



FT => Primary Beam

Add a phase gradient



FT => Shift the Primary Beam

A-Projection + Mosaic

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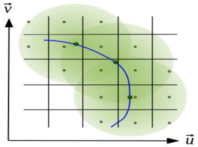
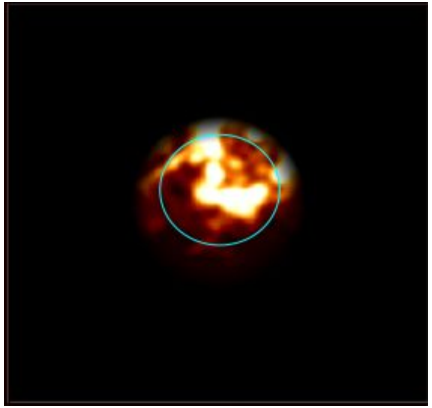


Each pointing is gridded with a *different* phase gradient => shift the PB



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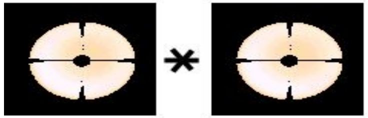

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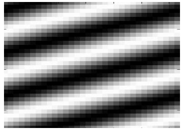

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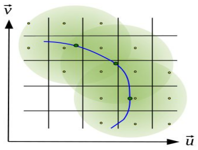

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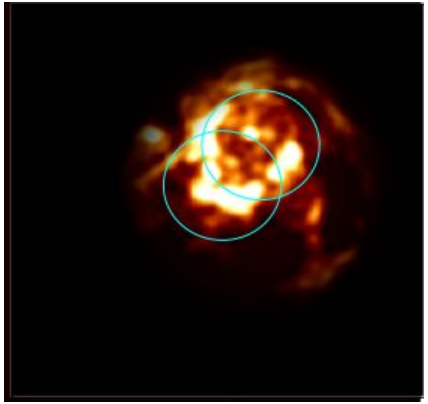

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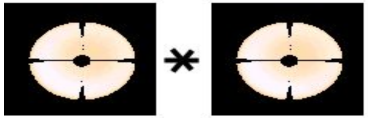

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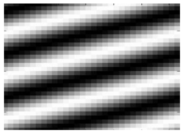

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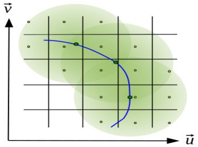

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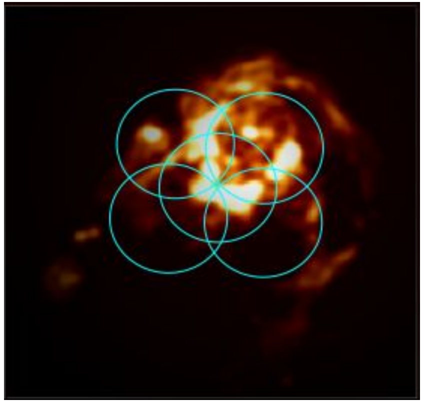

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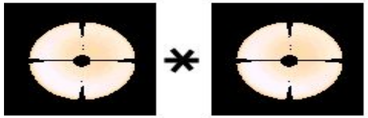

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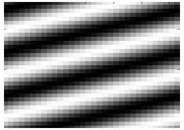
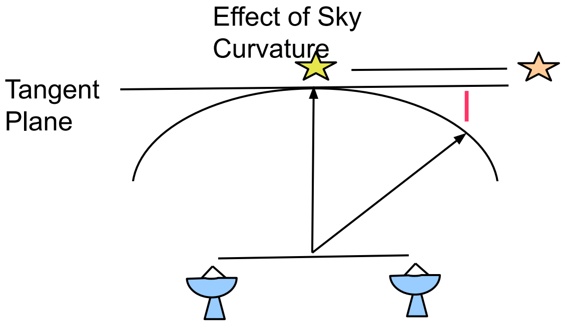
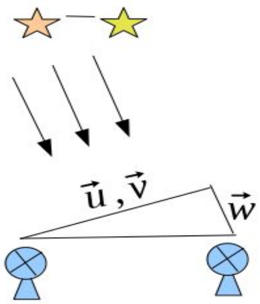


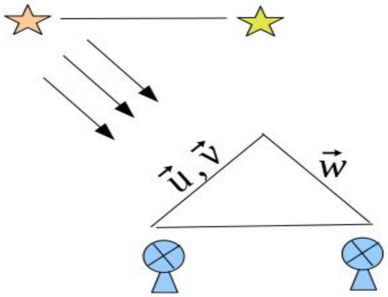
FT => Shift the Primary Beam

W Term - Effects

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+\boxed{w(n-1)})}dldmdn$$







A known geometric effect

Algorithms

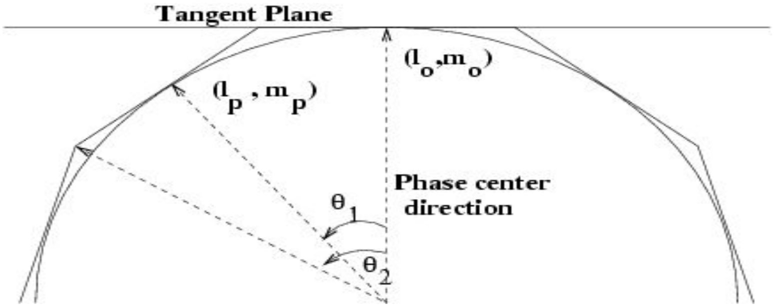
3D Imaging,
W-stacking,
Faceting,
W-Projection

For a field-of-view given by the Primary Beam of an antenna of diameter D, at wavelength λ and with a maximum baseline length of B

The w-term becomes relevant if
λB/(D*D) > 1

W Term - Faceting

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+\boxed{w(n-1)})}dldmdn$$



- Approximate the celestial sphere by a set of tangent planes (facets) such that 2D geometry is valid per facet

- Image each facet with its own phase reference center and re-project to the tangent plane

Algorithm Variants:

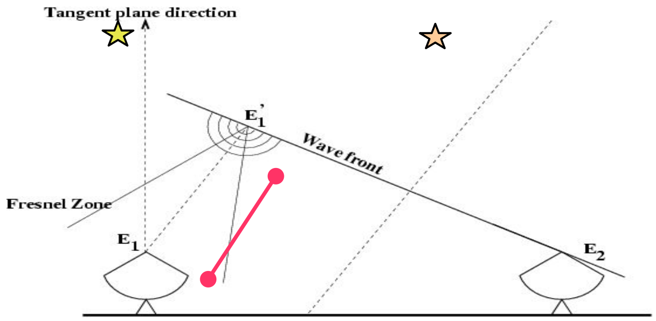
Deconvolve facets separately before re-projecting and stitching

(or)

Image all facets onto the same tangent plane grid and perform a joint deconvolution.

W Term - Wprojection

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+\boxed{w(n-1)})}dldmdn$$



W-term appears as a convolution in the UV-domain

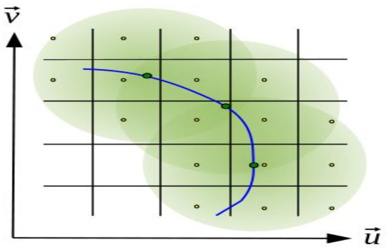
For ideal 2D imaging we need to measure E,
Instead, we measure E'

E and E' are related by a Fresnel diffraction/propagation kernel.

=> Correct it by another convolution with the inverse/conjugate kernel (during the gridding step)

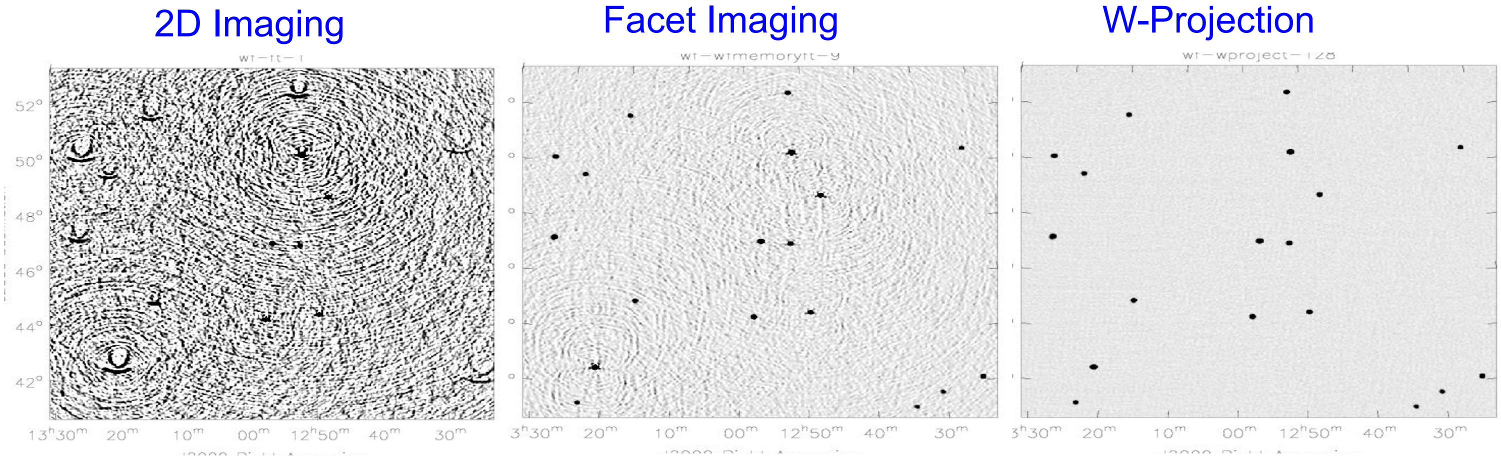
=> Use different kernels for different W values (appropriately quantized)

=> Quantize equally in zero to sqrt(Wmax)



W Term - W Correction

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$



References

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Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency (time)

W-Term

Primary Beam (A-Proj) : Bhatnagar, S., Cornwell, T. J., Golap, K., and Uson, J. M., “Correcting direction-dependent gains in the deconvolution of radio interferometric images,” Astron. & Astrophys. 487, 419–429 (Aug. 2008).

Full-pol A-Projection : Tasse, C., van der Tol, S., van Zwieten, J., van Diepen, G., and Bhatnagar, S., “Applying full polarization A-Projection to very wide field of view instruments: An imager for LOFAR,” AAP 553, A105 (May 2013).

Full-pol PB models : Jagannathan, P., Bhatnagar, S., Brisken, W., and Taylor, A. R., “Direction-dependent corrections in polarimetric radio imaging. ii. a-solver methodology: A low-order solver for the a-term of the a-projection algorithm,” The Astronomical Journal 155(1), 3 (2018).

Wideband Sky Model : Rau, U. and Cornwell, T. J., “A multi-scale multi-frequency deconvolution algorithm for synthesis imaging in radio interferometry,” AAP 532, A71 (Aug. 2011).

W-Term : Cornwell, T. J., Golap, K., and Bhatnagar, S., “The non-coplanar baselines effect in radio interferometry: The w-projection algorithm,” IEEE Journal of Selected Topics in Sig. Proc. 2, 647–657 (Oct 2008).

Mosaicing : Sault, R. J., Staveley-Smith, L., and Brouw, W. N., “An approach to interferometric mosaicing.,” AAPS 120,375–384 (Dec. 1996).

Pointing Self-Cal : Bhatnagar, S. and Cornwell, T. J., “The pointing self-calibration algorithm for aperture synthesis radio telescopes,” The Astronomical Journal 154(5), 197 (2017).

DD-cal + wideband : Tasse, C., Hugo, B., Mirmont, M., Smirnov, O., Atemkeng, M., Bester, L., Hardcastle, M. J., Lakhoo,R., Perkins, S., and Shimwell, T., “Faceting for direction-dependent spectral deconvolution,” AAP 611, A87 (Apr. 2018).