Calibration II

Dharam V. Lal

with due thanks to several friends / collaborators at

IDIA-UCT (SA), NCRA-TIFR (India) and NRAO (USA)

CALIBRATION AND IMAGING

- Standard calibration and imaging
  - (DI instrumental effects)
- w/ DD instrumental + propagation effects
  - correction for *w*-term and for PB
    - image plane correction
    - Fourier plane correction
    - pointing self-calibration
  - Mosaicing
- w/ advanced image parameterisation
  - multi-scale CLEAN (deconvolution)
  - multi-frequency synthesis (imaging)
  - full polarisation (Stokes) calibration and imaging

TELESCOPE SENSITIVITY

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\rm sys}}{A_{\rm eff} \times \sqrt{(\Delta\nu \times \Delta t)}}$$

- Sensitivity improvements achieved by
  - wide band receivers,
  - long integration times
  - more antennas
  - long baselines

$$\sigma_{\rm confusion} \propto (\nu^{-2.7}/B^2_{\rm max})$$

    - B_{max} ~100 km @200 MHz, the confusion noise is ~1 μJy beam^{-1}.

IMAGING CHALLENGES AT LOW FREQ.

- Wide-field imaging
  - account for direction dependent (DD) effects
  - PB: time, frequency and polarisation dependence
  - *w*-term
- Wide-band imaging
  - … plus frequency dependence of the sky brightness
- Data volume $\propto N_{\rm ant}^2 \times N_{\rm channel} \times t$
  - Sky brightness $\Longrightarrow$ multi-scale deconvolution
  - Ionospheric effects $\Longrightarrow$ need for DD solvers

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

$$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\ dl\ dm$$

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

MEASUREMENT EQUATION

$$\Gamma(\vec{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \vec{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \vec{b}\cdot\vec{s}/\lambda} d\Omega$$

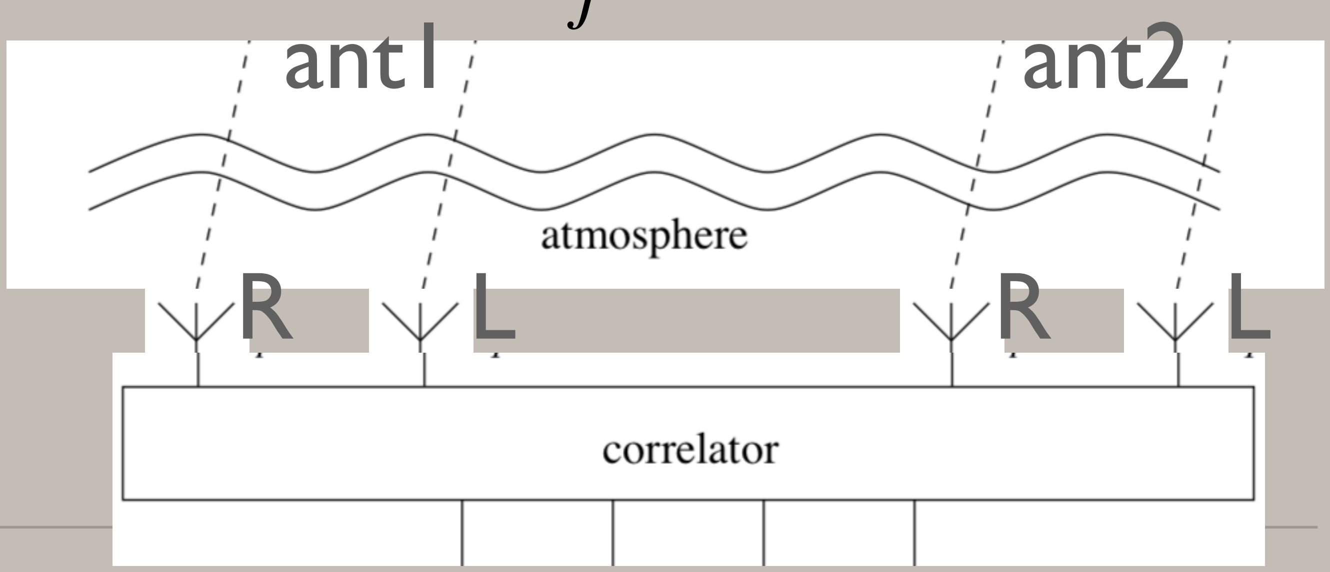
- $\Gamma(\vec{b})$: mutual coherence function
- $E(\vec{s}, t)$: complex amplitude of the radiation emanating from the source in the direction $\vec{s}$
- $\vec{s} = \vec{s}_0 + \vec{\sigma}$: point near the phase centre
- $\vec{b} \cdot \vec{s}/c$: time difference between the incoming radiation collected at two antennas separated by $\vec{b}$
- $d\Omega = \frac{d\vec{s}}{R^2}$

MEASUREMENT EQUATION

- $$\Gamma(\vec{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \vec{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \vec{b}\cdot\vec{s}/\lambda} d\Omega$$
- $$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))}\, dl\, dm$$
  - (for $w \simeq 0, n \simeq 1$)
    - $$V(u, v) = \int I(l, m)\, e^{-2\pi i (ul + vm)}\, dl\, dm$$
    - (this is van-Cittert Zernike theorem)

MEASUREMENT EQUATION

- $$\Gamma(\vec{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \vec{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \vec{b}\cdot\vec{s}/\lambda} d\Omega$$
- $$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \; dl \; dm$$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \; E^l]_i^T$
  - (four cross-correlation products, $\langle \overrightarrow{E_i} \otimes \overrightarrow{E_j^*} \rangle$ per baseline)
  - $\overrightarrow{V_{ij}} = [V^{rr} \; V^{rl} \; V^{lr} \; V^{ll}]_{ij}^T$
  - $\vec{I} = [I^{rr} \; I^{rl} \; I^{lr} \; I^{ll}]^T$



MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
    - (a 2×2 matrix product)
      - complex gains, G,
      - polar'n leakage, D and
      - feed config'n, C.
  - DD: $J_i^{sky} = [EPF]$
    - (a 2×2 matrix product)
      - AIPs, E,
      - PA effects, P and
      - tropospheric / ionospheric effects, and Faraday R'n, F.

MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r\ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r\ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
  - DD: $J_i^{sky} = [EPF]$
    - $K_{ij}^{\{vis,\ sky\}} = [J_i \otimes J_j^\dagger]^{\{vis,\ sky\}}$
    - (effect on each baseline ij is described by the outer-product of these antenna-based Jones matrices, a 4×4 matrix!)
- $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b}\cdot\vec{\sigma}/\lambda} d\Omega$

MEASUREMENT EQUATION

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}} . \vec{s}}\ d\vec{s}$$

DATA observed visibility

DI gains

Weights

DD & Ionospheric Corrections

Antenna PB

MT-MFS

non-coplanar baselines *w*-projection

- $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma} / \lambda} d\Omega$$

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    - **image plane** correction
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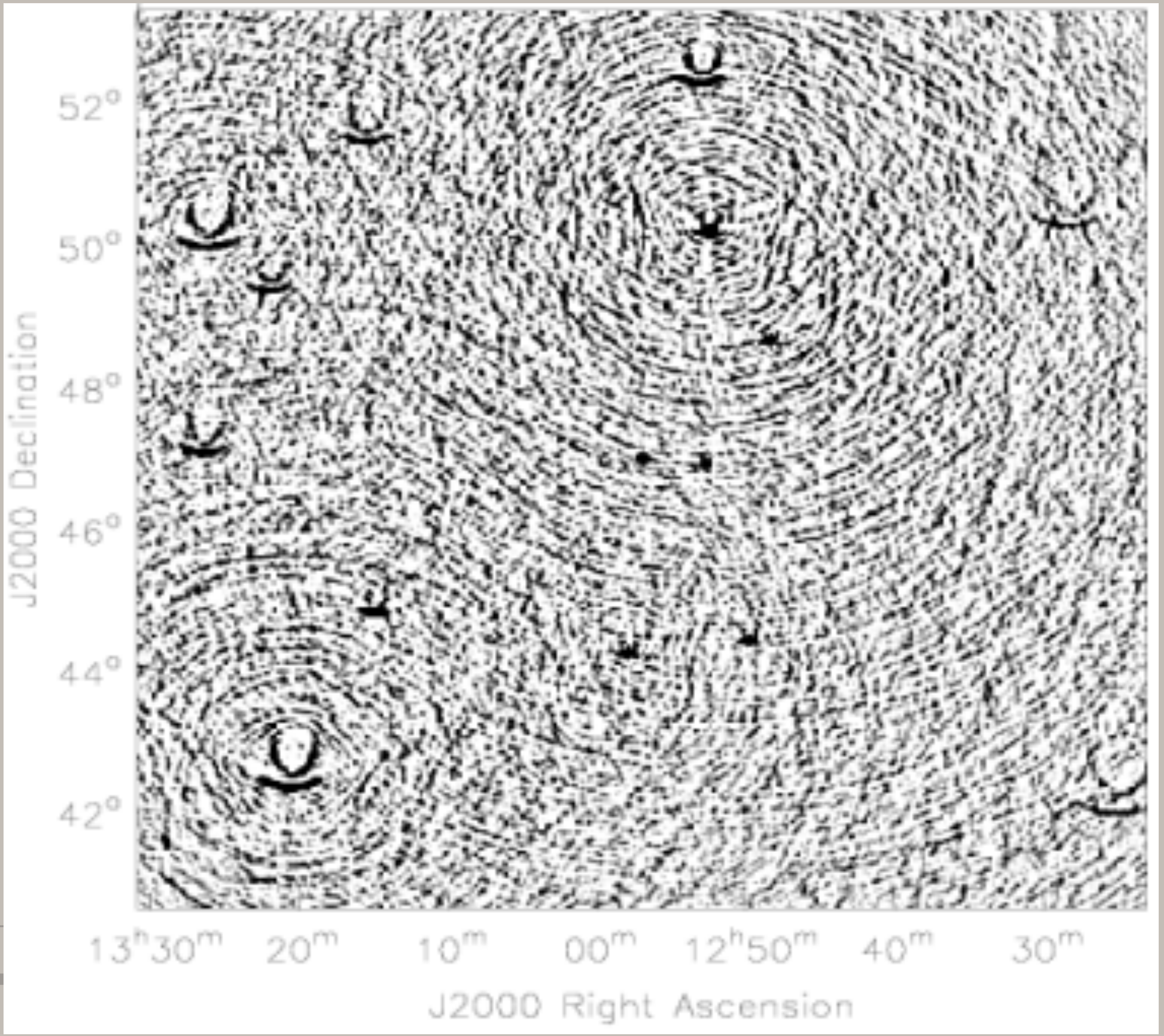
    - pointing self-calibration
  - Mosaicing
- w/ advanced image parameterisation
  - **multi-scale CLEAN** (deconvolution)
  - **multi-frequency synthesis** (imaging)
  - **full polarisation** (Stokes) calibration and imaging

W-TERM

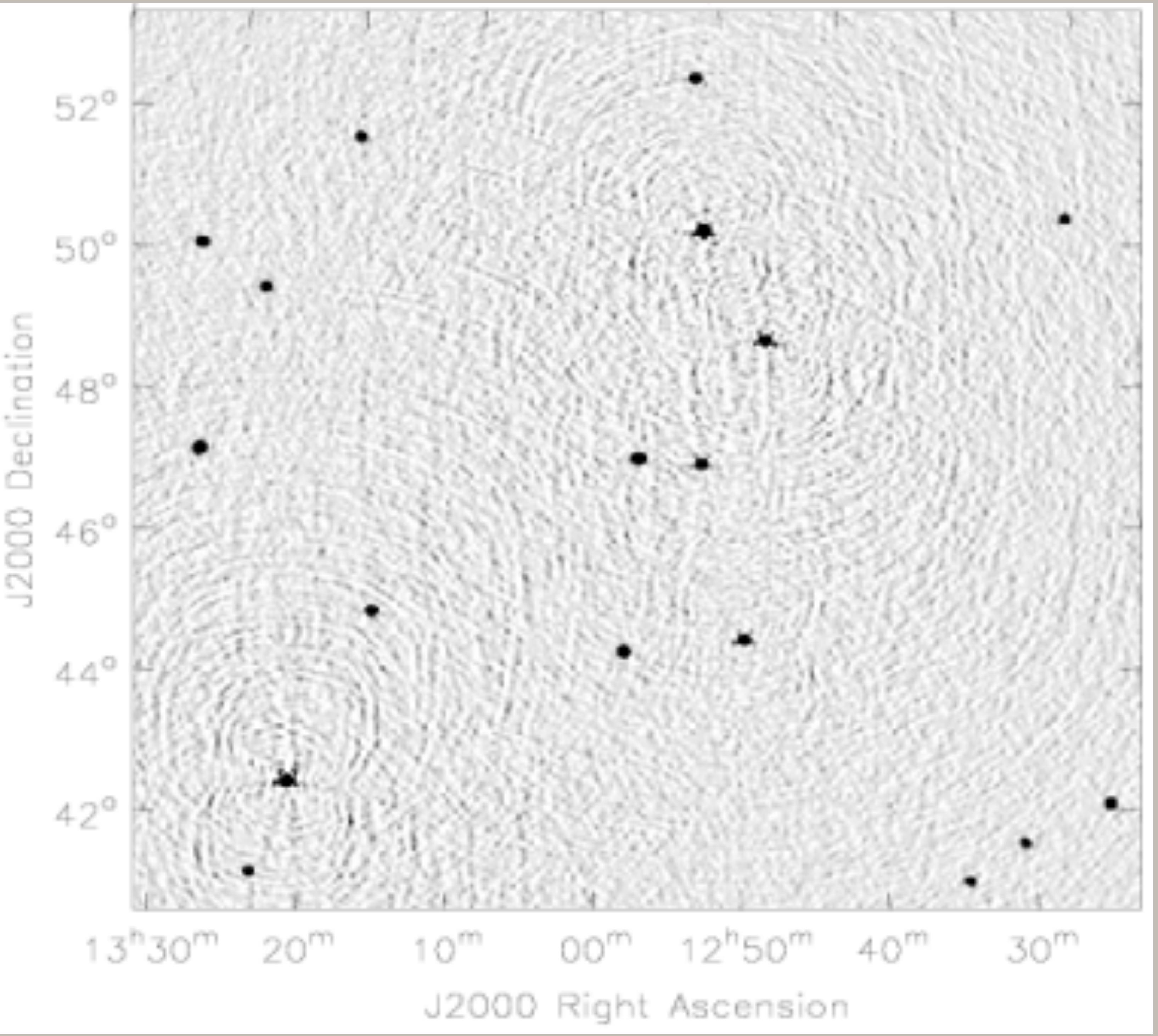
- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$e^{\iota w\sqrt{1-l^2-m^2}}$$



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

w-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $e^{\iota w\sqrt{1-l^2-m^2}}$
- divide the FoV into a no. of FACETS



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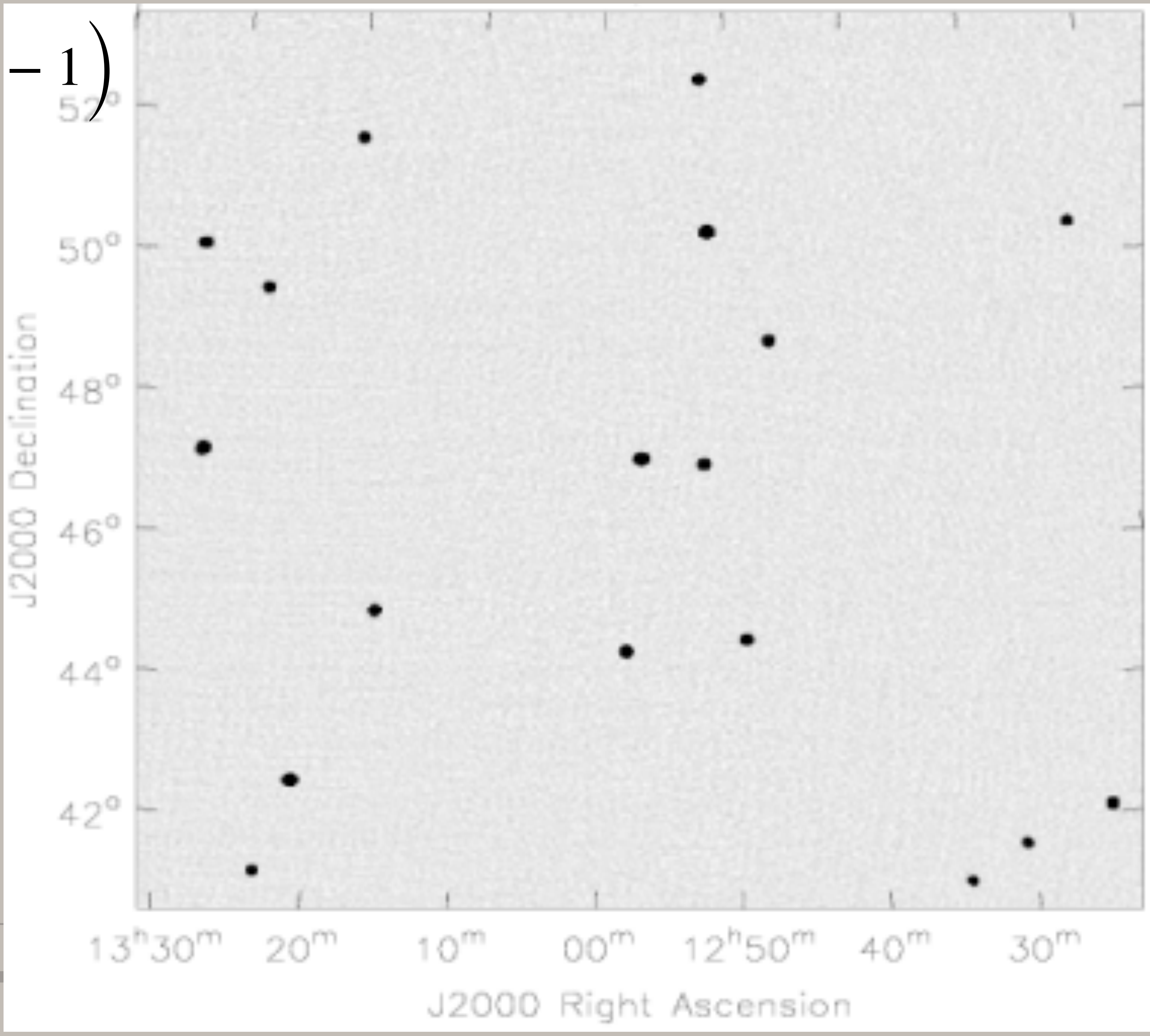
W-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$K_{ij}^{Sky} = e^{w_{ij}\left(\sqrt{1-l^2-m^2}-1\right)}$$
- An order-of magnitude faster than FACETing, and
- for the same amount of computing time provides higher DR images.



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b}\cdot\vec{\sigma}/\lambda} d\Omega$$
  - $$\overrightarrow{V}_{cn\times 1}^{obs} = [K_{cn\times cn}^{vis}][S_{cn\times cm}][F_{cm\times cm}][K_{cm\times cm}^{sky}]\vec{I}_{cm\times 1}^{sky}$$

vector of n visibilities

projection operator describing the uv-coverage

Fourier transfer operator

Pixelated image of sky

CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b}\cdot\vec{\sigma}/\lambda} d\Omega$$
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s, \nu, t)\ I(s, \nu)\ e^{i(ul+vm+w(\sqrt{(1-l^2-m^2)}-1))}\ ds$$

DATA PB SKY GEOMETRY

- Visibility depends on time and frequency!

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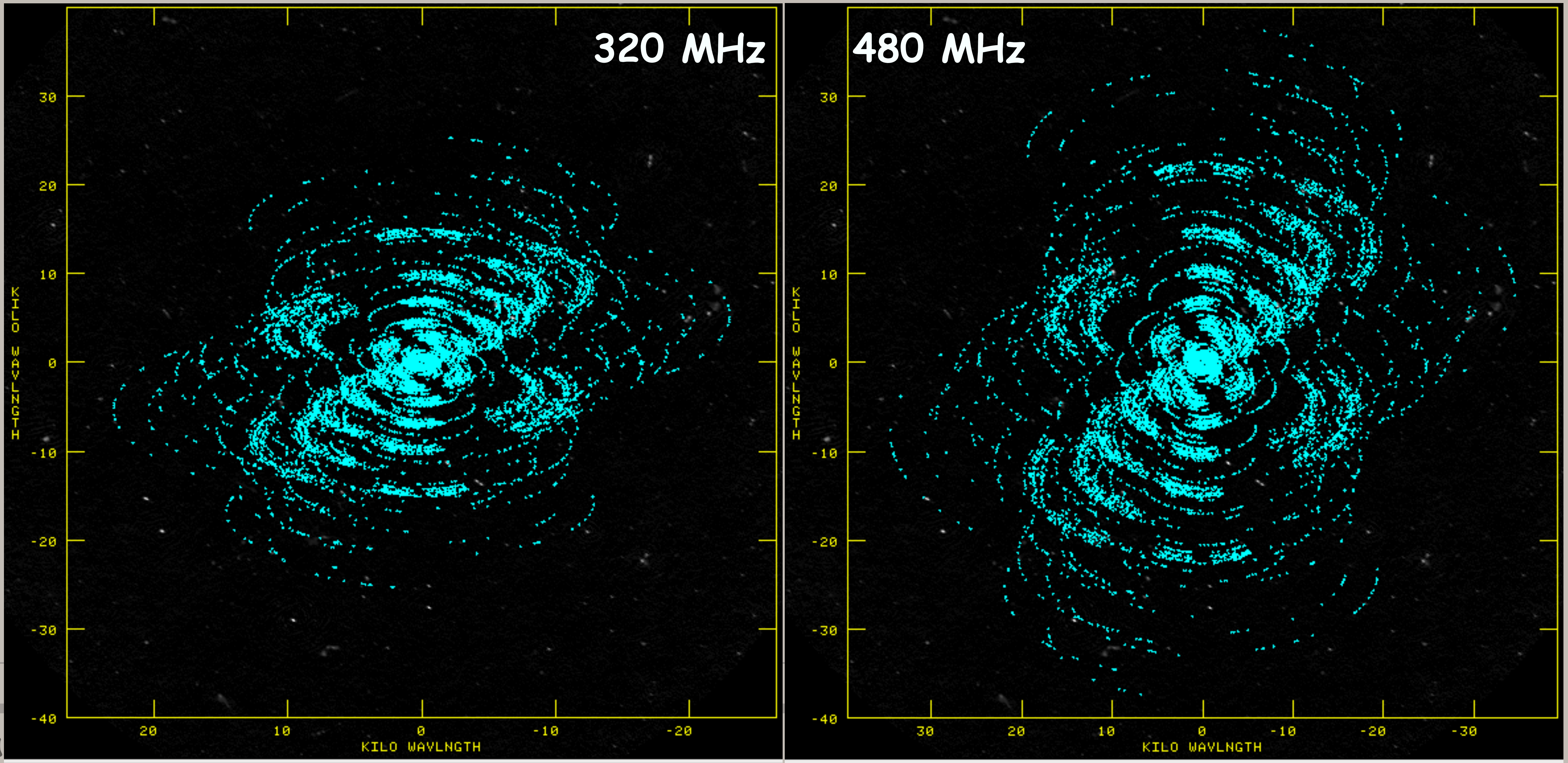
CORRECTION FOR **PB**

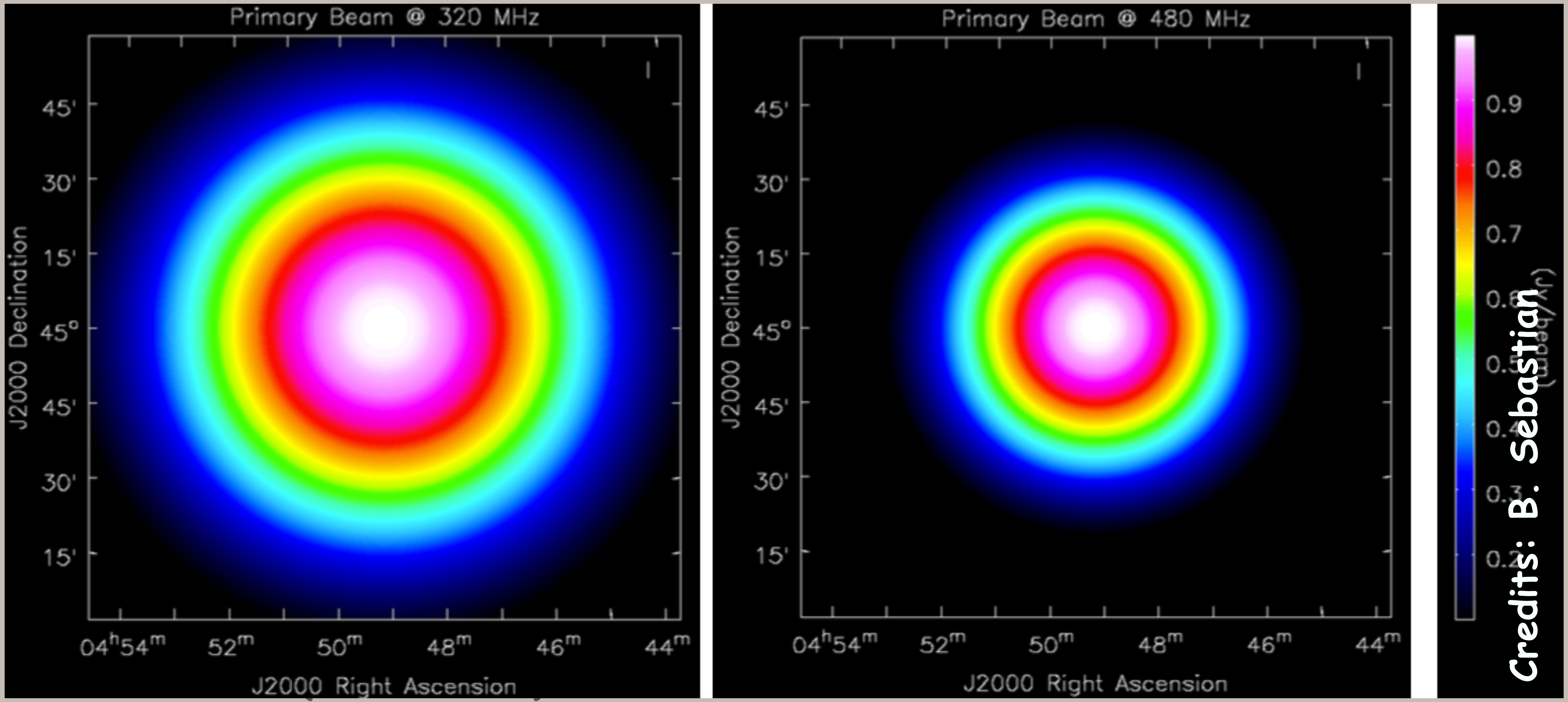
- **multi-frequency** synthesis
- $$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0}\right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$



Dha

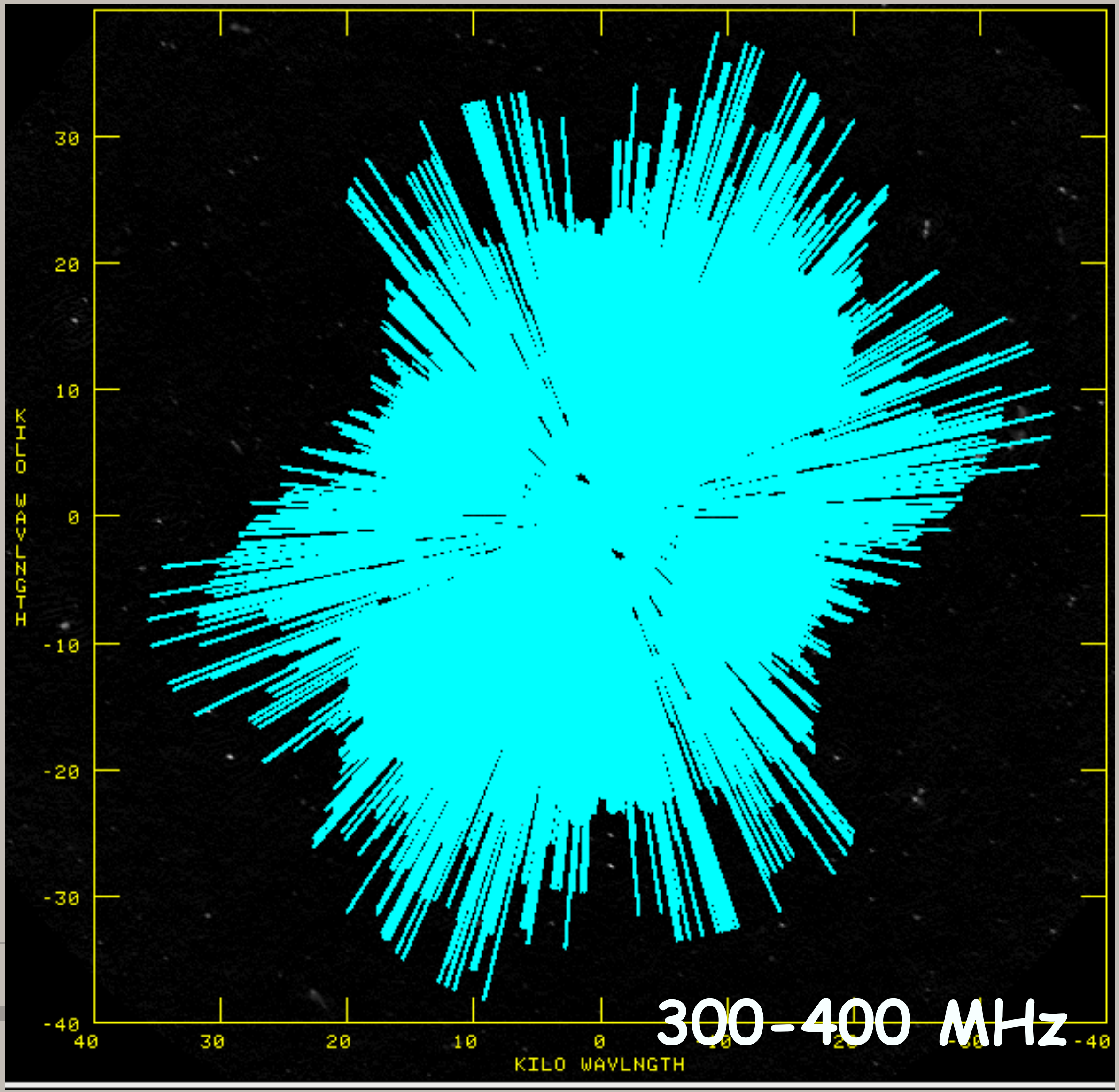
CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\; I(s,\nu)\; e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\, ds$$

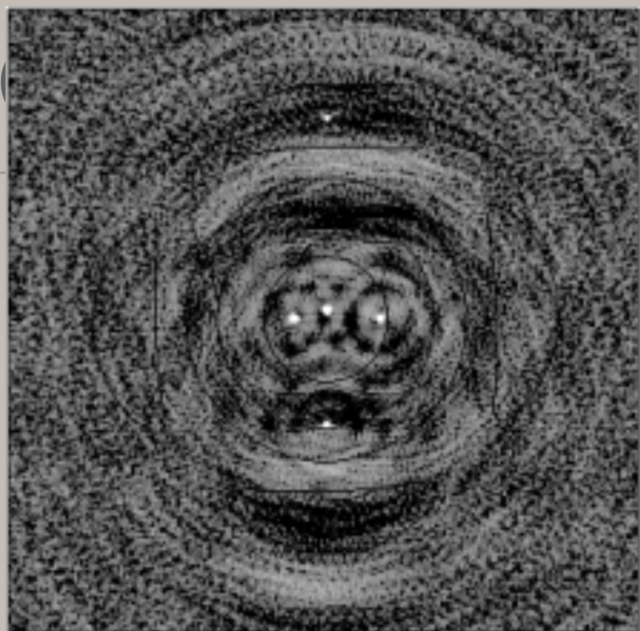


CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s, \nu, t)\ I(s, \nu)\ e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\ ds$$
- **multi-frequency** synthesis
  - $$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0}\right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$
  - $$I_0 = I_{\nu_0}$$
  - $$I_1 = I_{\alpha} \times I_{\nu_0}$$
  - $$I_2 = (I_{\alpha}(I_{\alpha} - 1)/2 + I_{\beta}) \times I_{\nu_0}$$

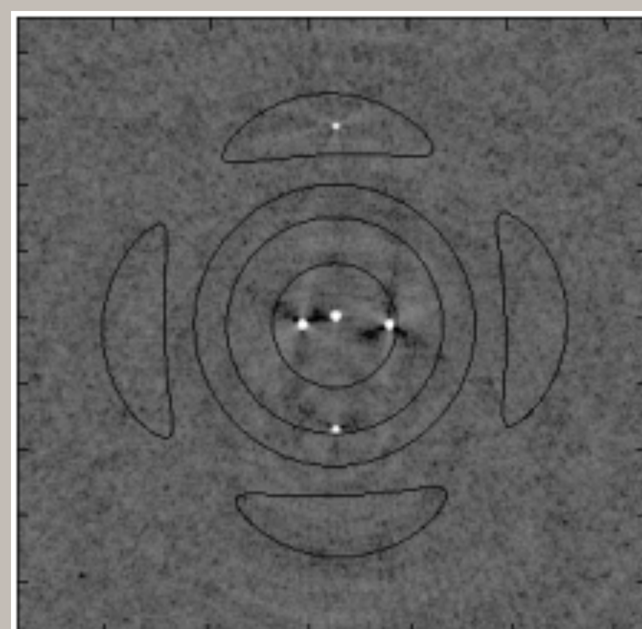


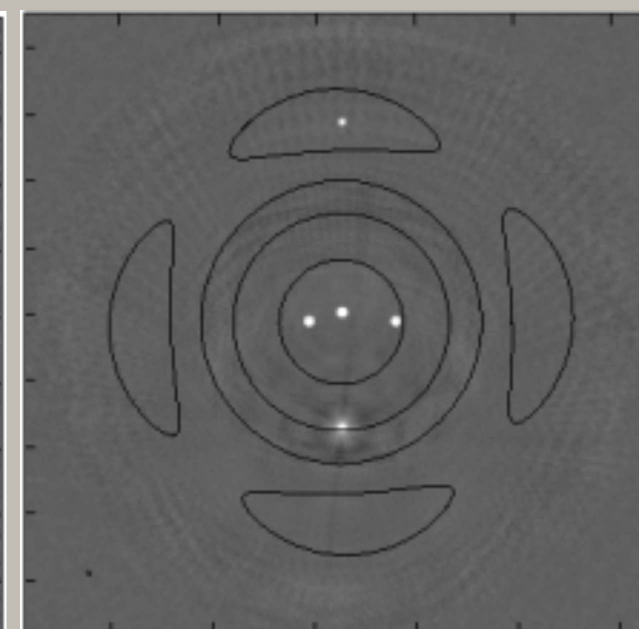
ON FOR **PB**



FT

(standard imaging)





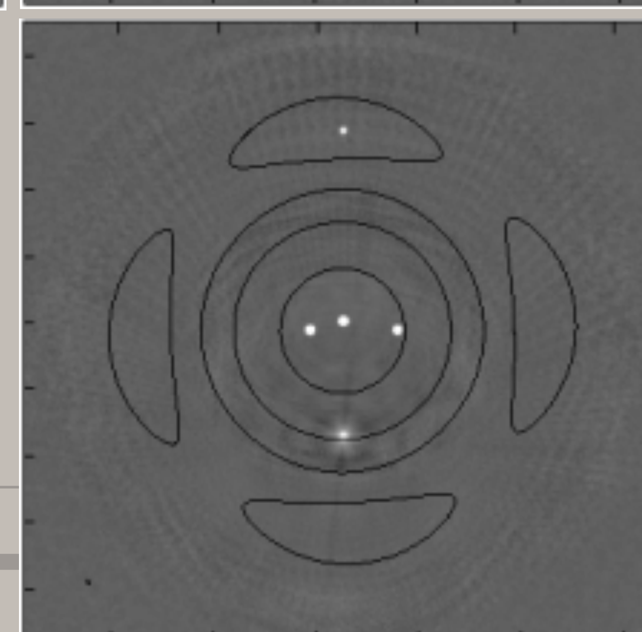
FT

+ MT-MFS

+ A-projection

FT

+ MT-MFS



FT

+ MT-MFS

+ WB A-projection

Credits: S. Bhatnagar (NRAO, USA)

PEELING: DD CALIBRATION

- antenna based gains are determined in the direction of each compact source.
  - subtract these gains (contribution of compact sources from the observed data using a DFT) and the residual visibilities are imaged again.
- drawbacks of peeling…

PEELING: DD CALIBRATION

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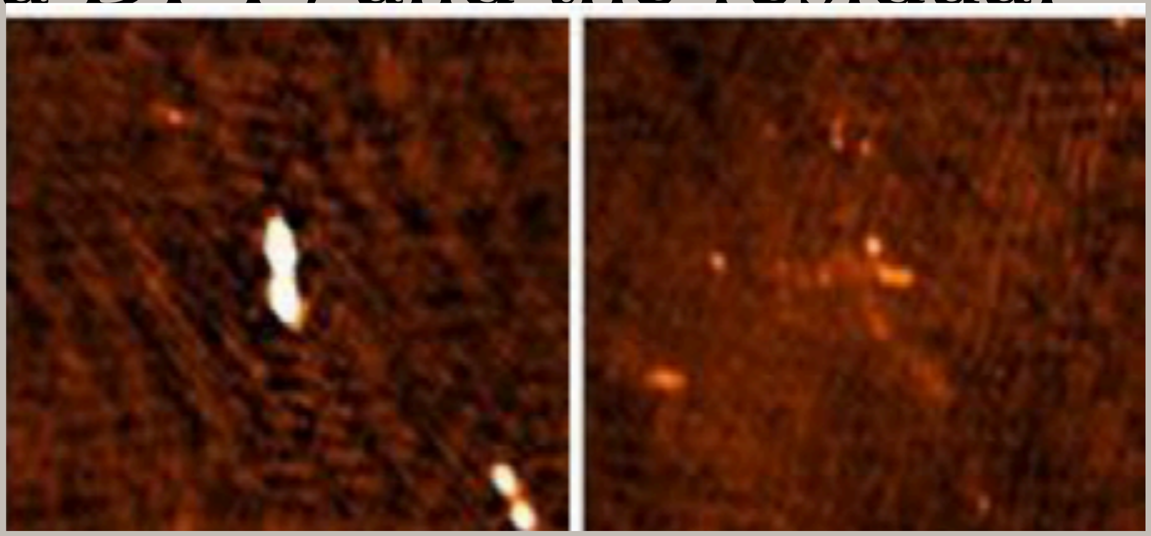
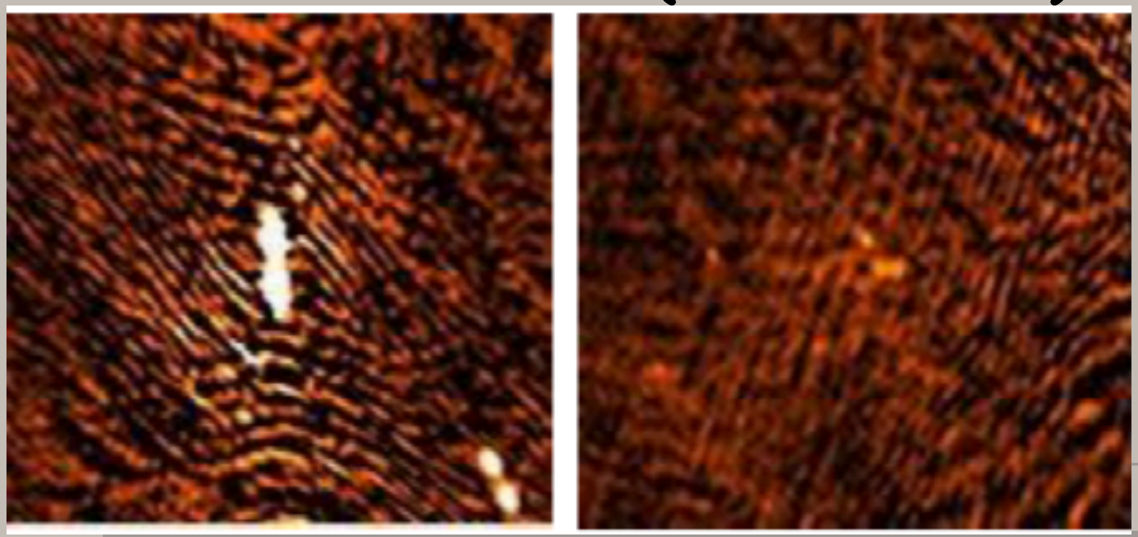
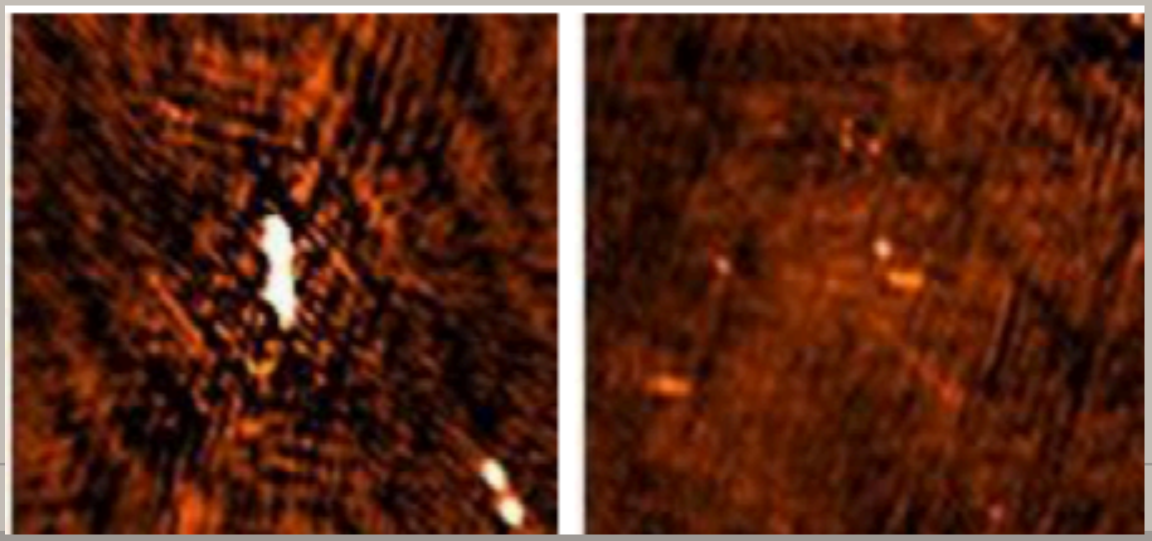


Credits: H. Intema (Leiden Obs.)

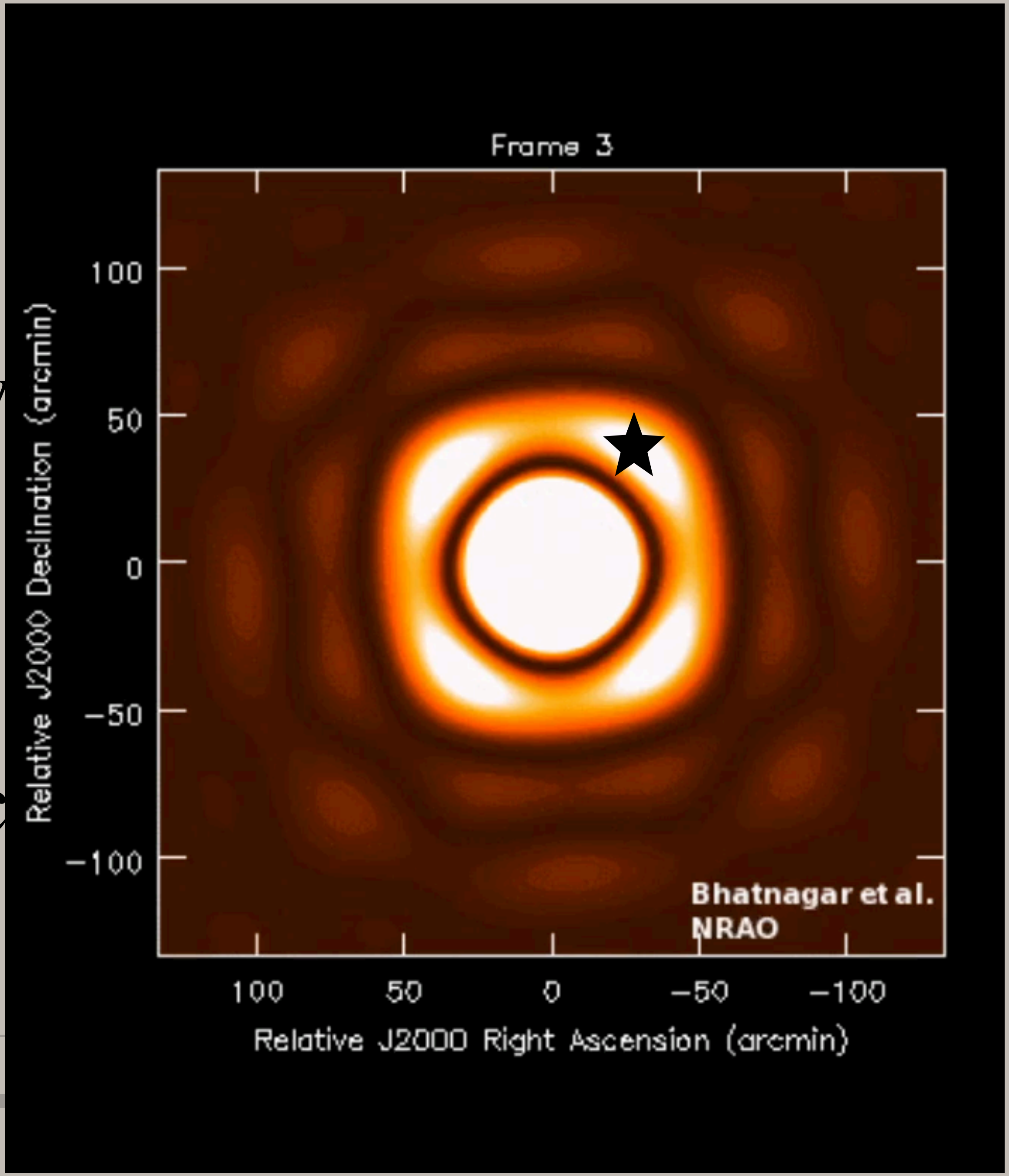




MORE DD ISSUES: CORRECTION FOR **PB**

- **A**-projection
  - $$\vec{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b}\cdot\vec{\sigma}/\lambda} d\Omega$$

- Remember, 200 / 400 MHz bandw
- Assumption:
  - sky is (not) variable, and
  - Antenna power pattern is (not) c



Credits: S. Bhatnagar

Sir George Gabriel Stokes

STOKES PARAMETERS

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
  - $RR = \mathscr{A}(RR)e^{i\psi RR} = I + V$
  - $LL = \mathscr{A}(LL)e^{i\psi LL} = I - V$
  - $RL = \mathscr{A}(RL)e^{i\psi RL} = Q + iU$
  - $LR = \mathscr{A}(LR)e^{i\psi LR} = Q - iU$

Sir George Gabriel Stokes

STOKES PARAMETERS

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
  - $$I = \frac{(RR + LL)}{2}$$
  - $$\frac{Q}{I} = \frac{\mathrm{Re}(RL + LR)}{RR + LL}$$
  - $$\frac{V}{I} = \frac{RR - LL}{RR + LL}$$
  - $$\frac{U}{I} = \frac{\mathrm{Im}(RL - LR)}{RR + LL}$$

STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
- Is it really that simple?
  - No, there are leakages…
    - The total intensity can leak into the polarised components (I into $\{Q,U,V\}$).

Hans Mueller

MUELLER MATRIX

- The leakage of each polarisation into the other can be measured and quantified in a 4×4 matrix (Mueller 1943).

- $$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

- $$\begin{bmatrix} RR + LL \\ RL + LR \\ RL - LR \\ RR - LL \end{bmatrix} = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix} \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
- $I \Leftrightarrow V$ leakage
- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- RL phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

Stokes V ~ 0 for most calibrators so no need to worry too much unless you require very high precision

PUTTING THIS ALL TOGETHER

- In the end what we are trying to do is relate products from our correlator to the intrinsic polarised radiation from the source.
- So we need to correct the raw correlator outputs for
  - imperfections in the receiver (leakages).
    - The orientation of the receiver with respect to the telescope structure.
      - a.k.a. the changing parallactic angle.
  - Any measured propagation related polarisation effects (e.g. Faraday rotation).

BEAM EFFECTS

- For point sources, all of the previous is fine.
- What if the source you are looking at is extended compared to the telescope beam?
  - There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
    - Squint
    - Squash

BEAM EFFECTS

- For point sources, al
- What if the source y compared to the tele
  - There are instrum measurement of e
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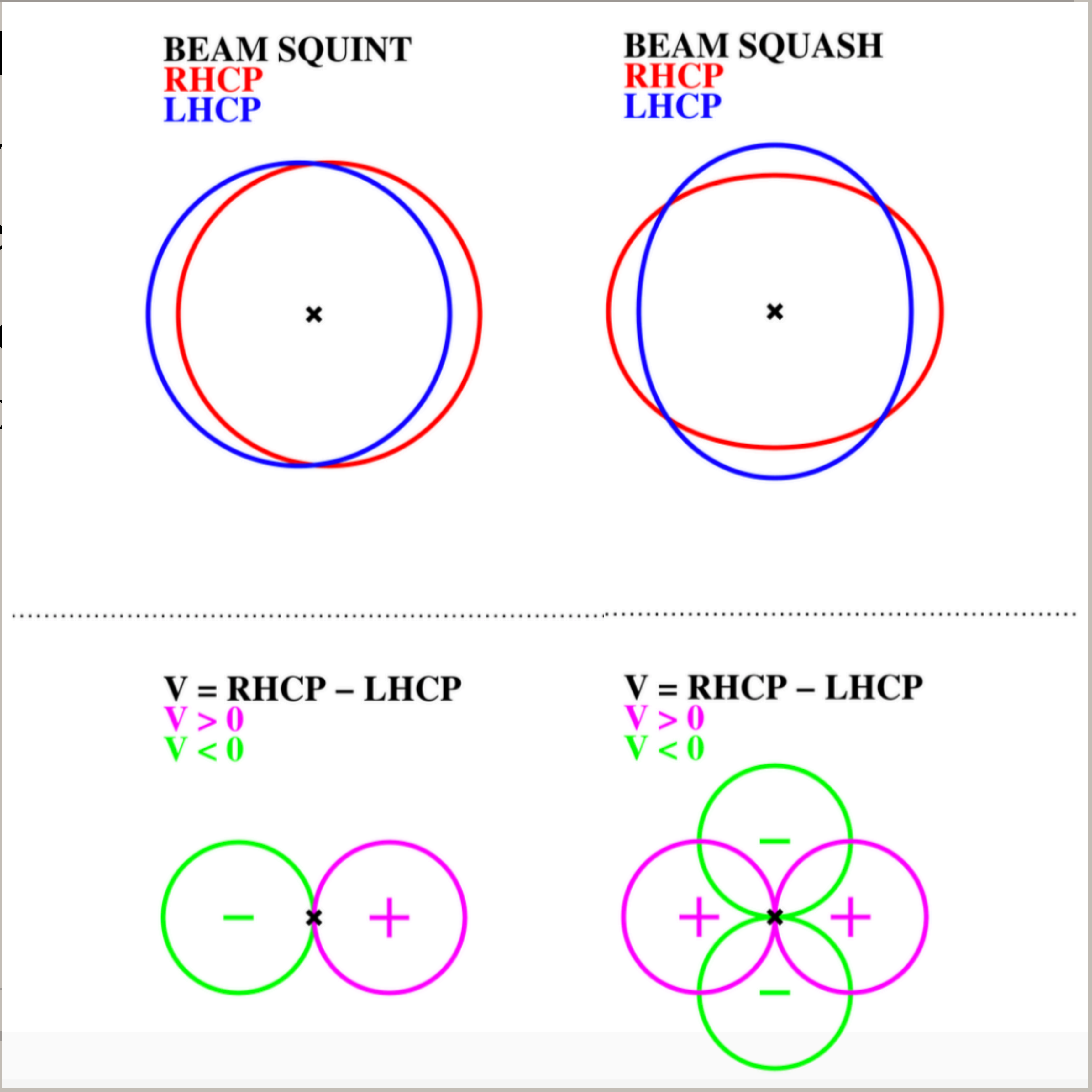


BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t)\int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)e^{\imath\overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA

Weights

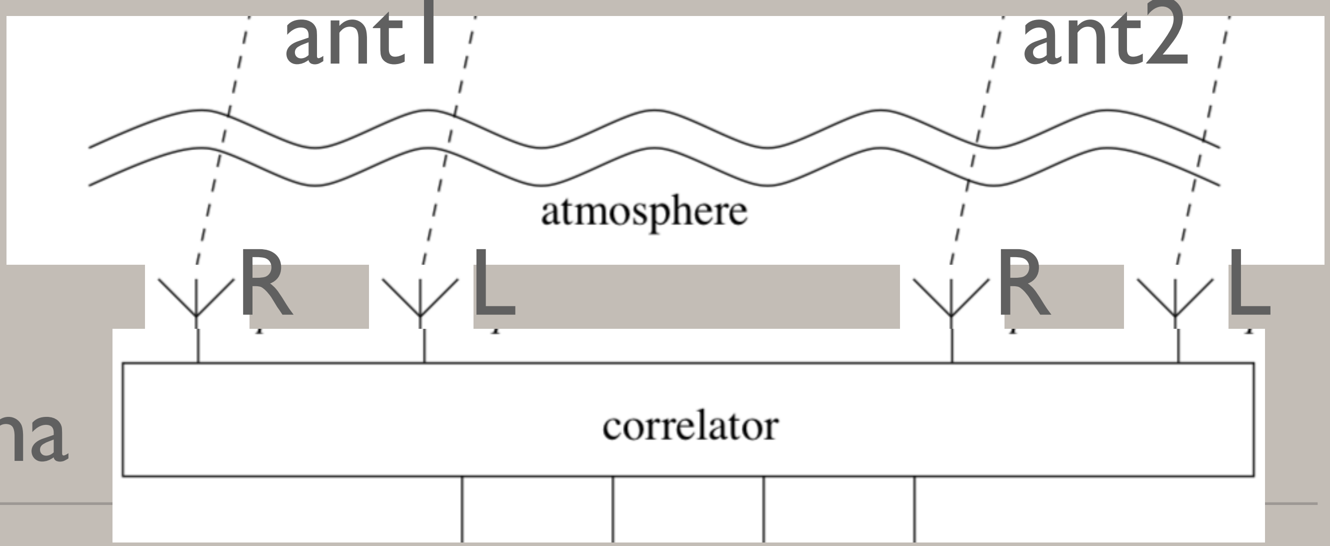
Mueller matrix

full-polarization vector of the sky brightness distribution

BEAM EFFECTS

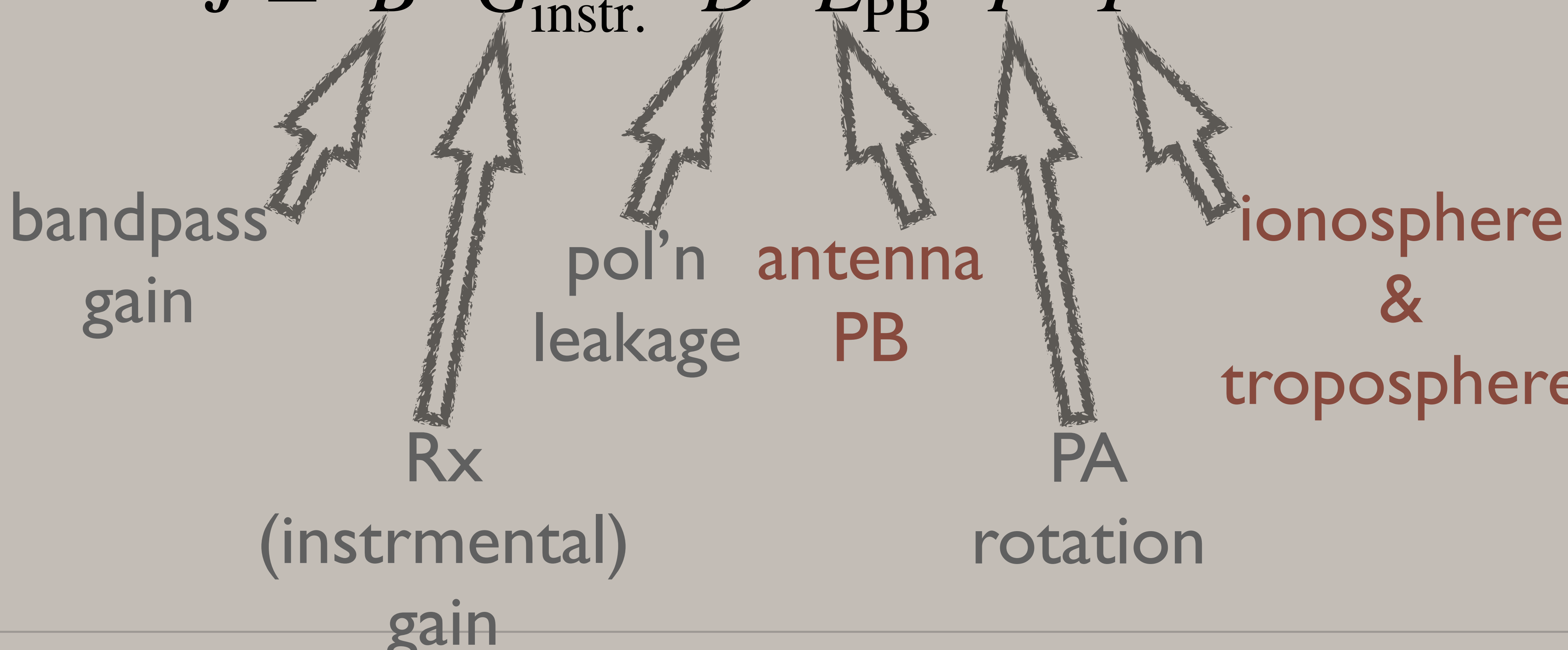
- $$\vec{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \vec{b_{ij}}.\vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$
  - $$\vec{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \mathscr{F} \left[\left(E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t) \right) \cdot \vec{I}(\vec{s}, \nu) \right]$$
    - $$= W_{ij}(\nu, t) \left[A_{ij} \star \vec{V}_{ij} \right]$$
    - $$\text{where}, A_{ij} = A_i \otimes A_j^*$$

AIPs for two antenna



DD & DI EFFECTS

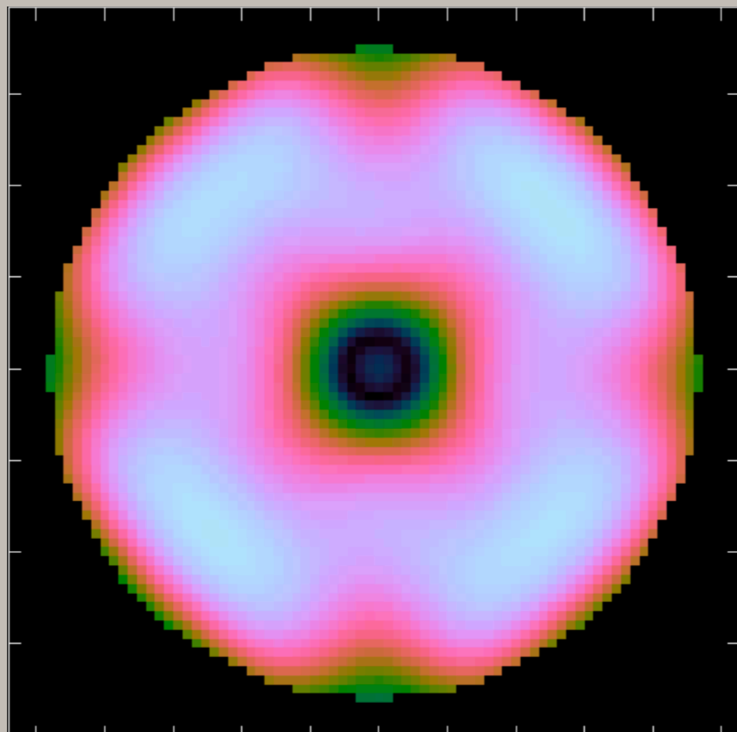
- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}} . \vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$
- Jones matrix formulation:
- $$J = B \ \ G_{\text{instr.}} \ \ D \ \ E_{\text{PB}} \ \ P \ \ T$$

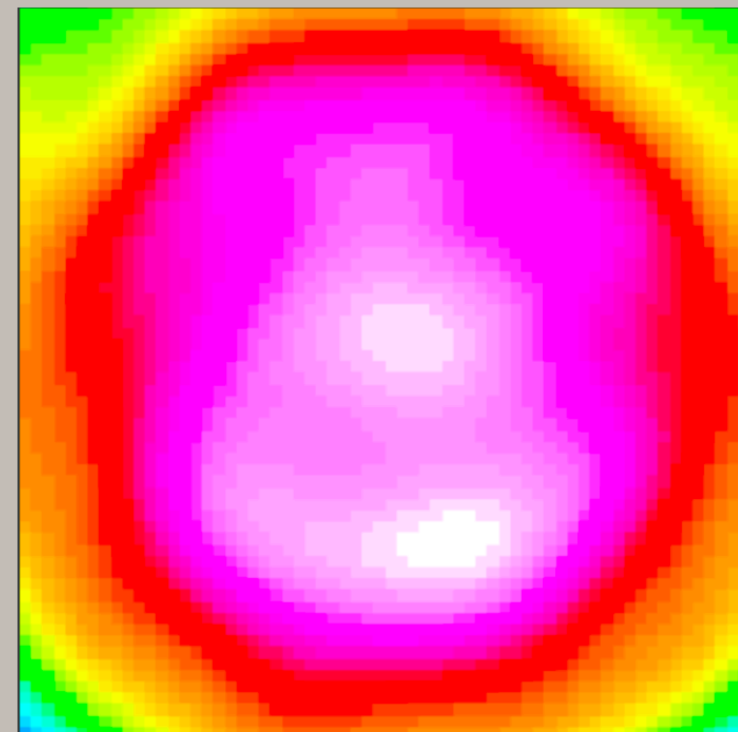


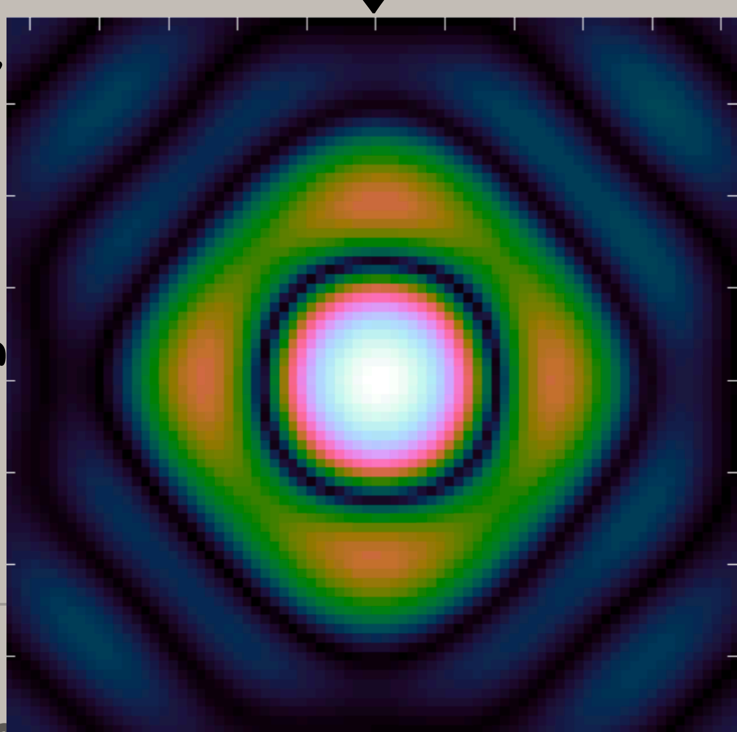
A-TO-Z SOLVER

- Use Zernike polynomials to directly model the complex aperture
  - it is a natural domain to model optical aberrations that cause PB weirdness
  - (Telescope agnostic - does not require ray traced model for different antennas/telescopes, only Holography)
  - Aperture size is fixed, independent of number of measured sidelobes.

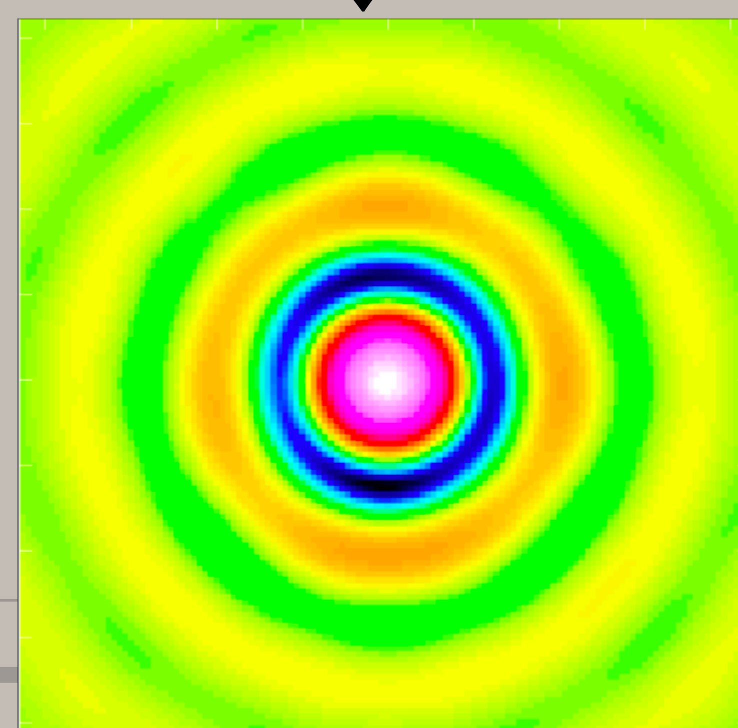
APERTURE ILLUMINATION PATTERN



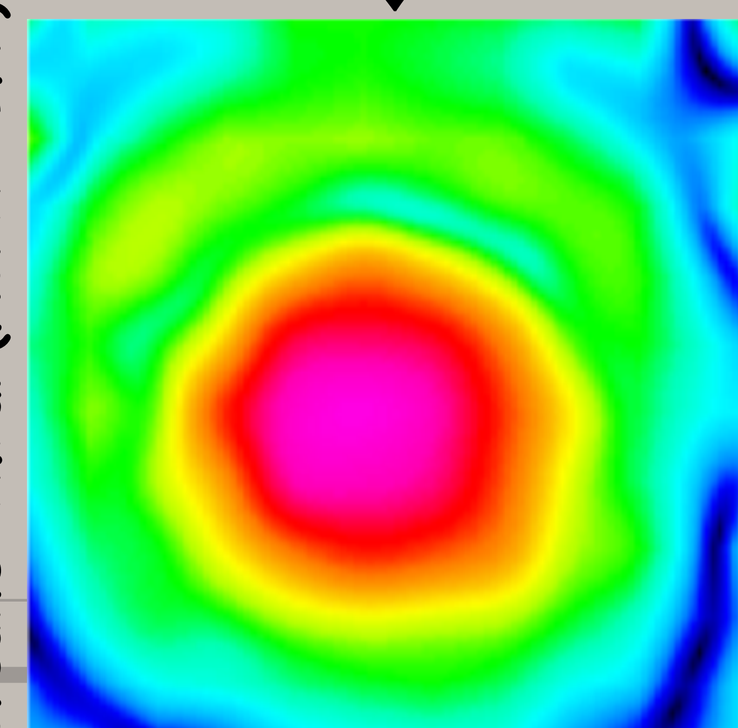




Credits: P. Jagannathan (NRAO)



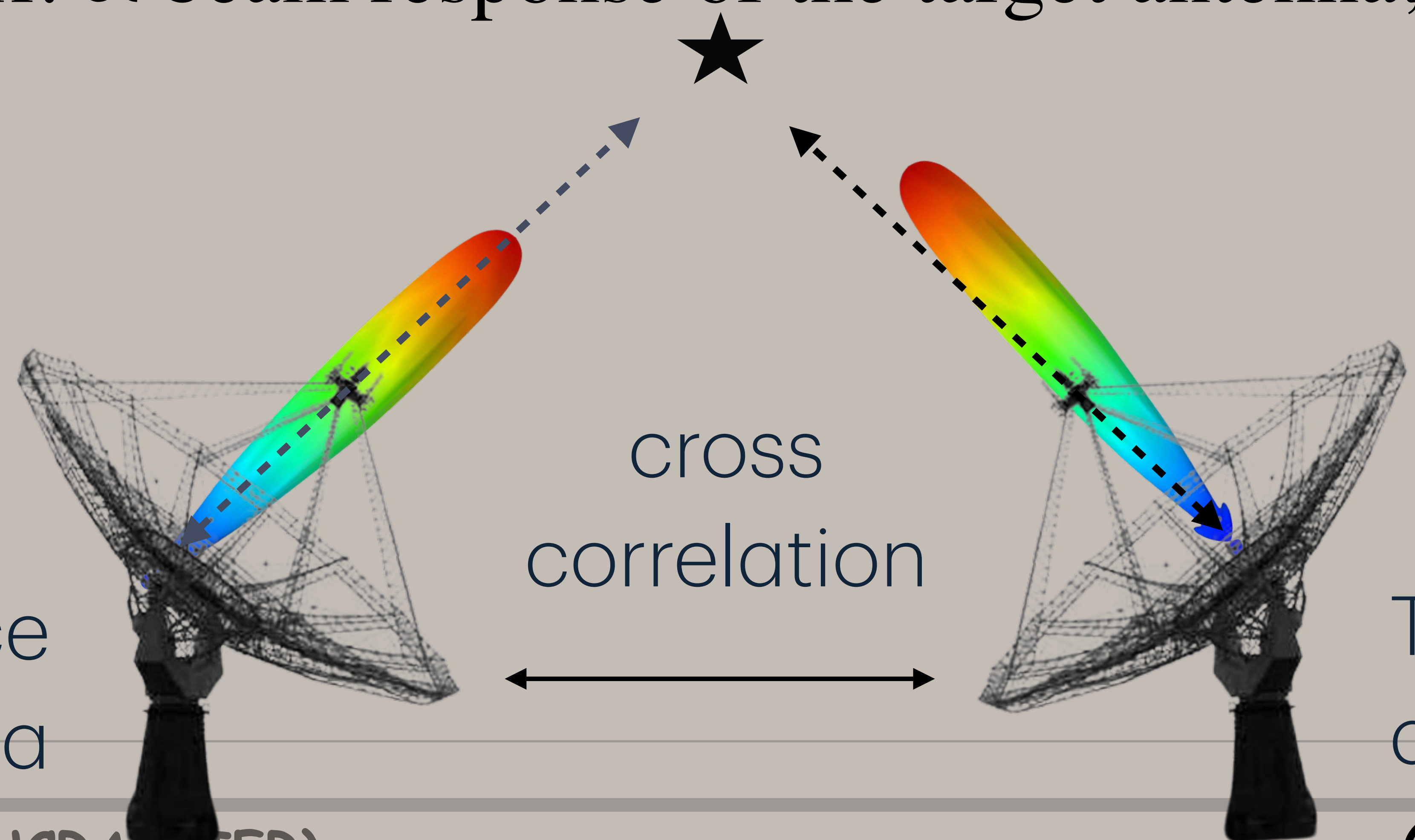
Credits: S. Sekhar (UCT-IDIA)



Credits: A.Pal (NCRA-TIFR), S. Sekhar (UCT-IDIA)

ANTENNA HOLOGRAPHY

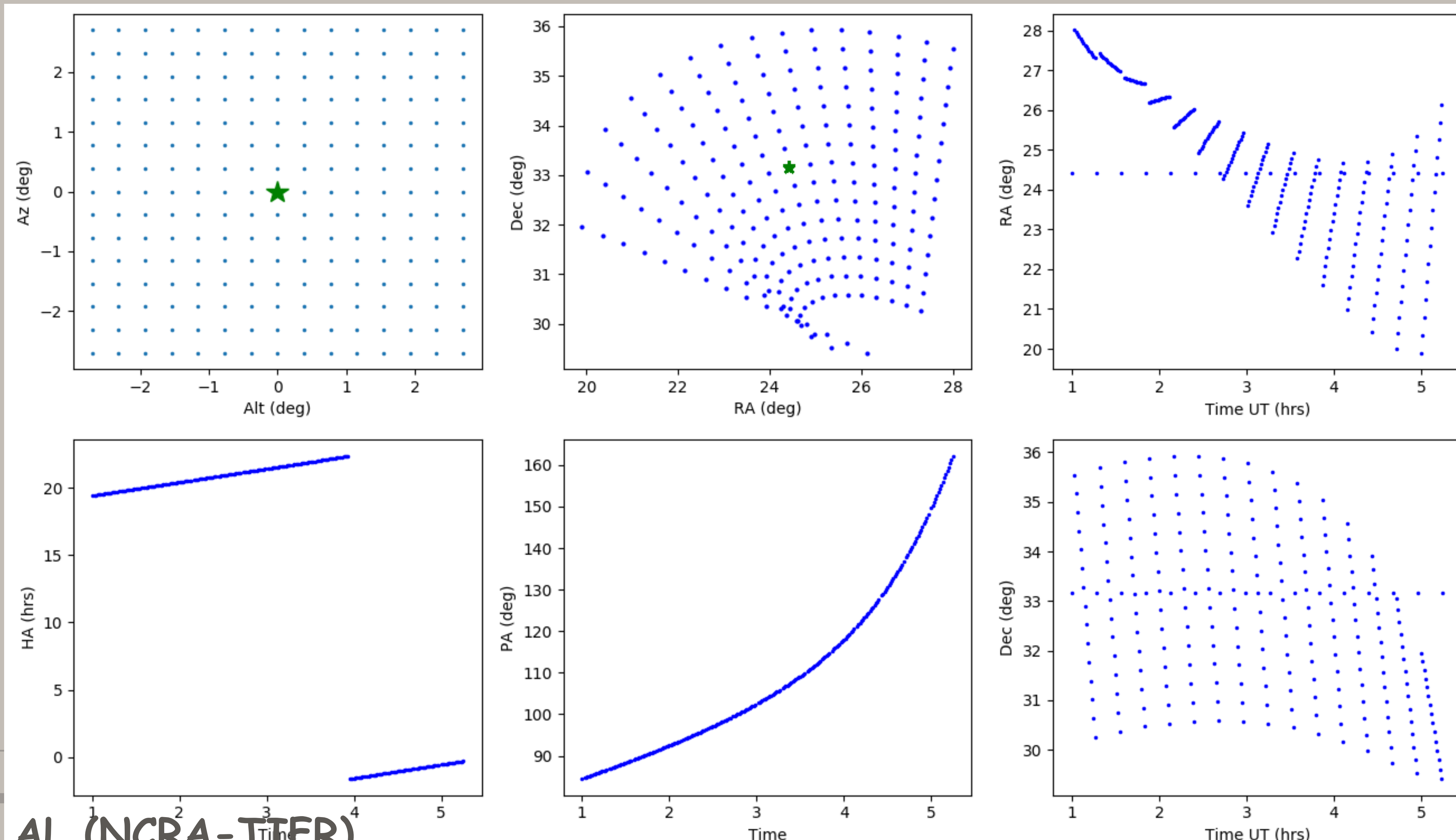
- DD effects: getting the beams
  - Ref. ant. will track the bright point source ($P(0,0) = 1$)
  - Target ant. will scan the target, but with an offset,
    - response $\propto P(l, m)$
  - cross-corr. $\propto$ beam response of the target antenna, $P(l, m)$



Credits: A. Pal

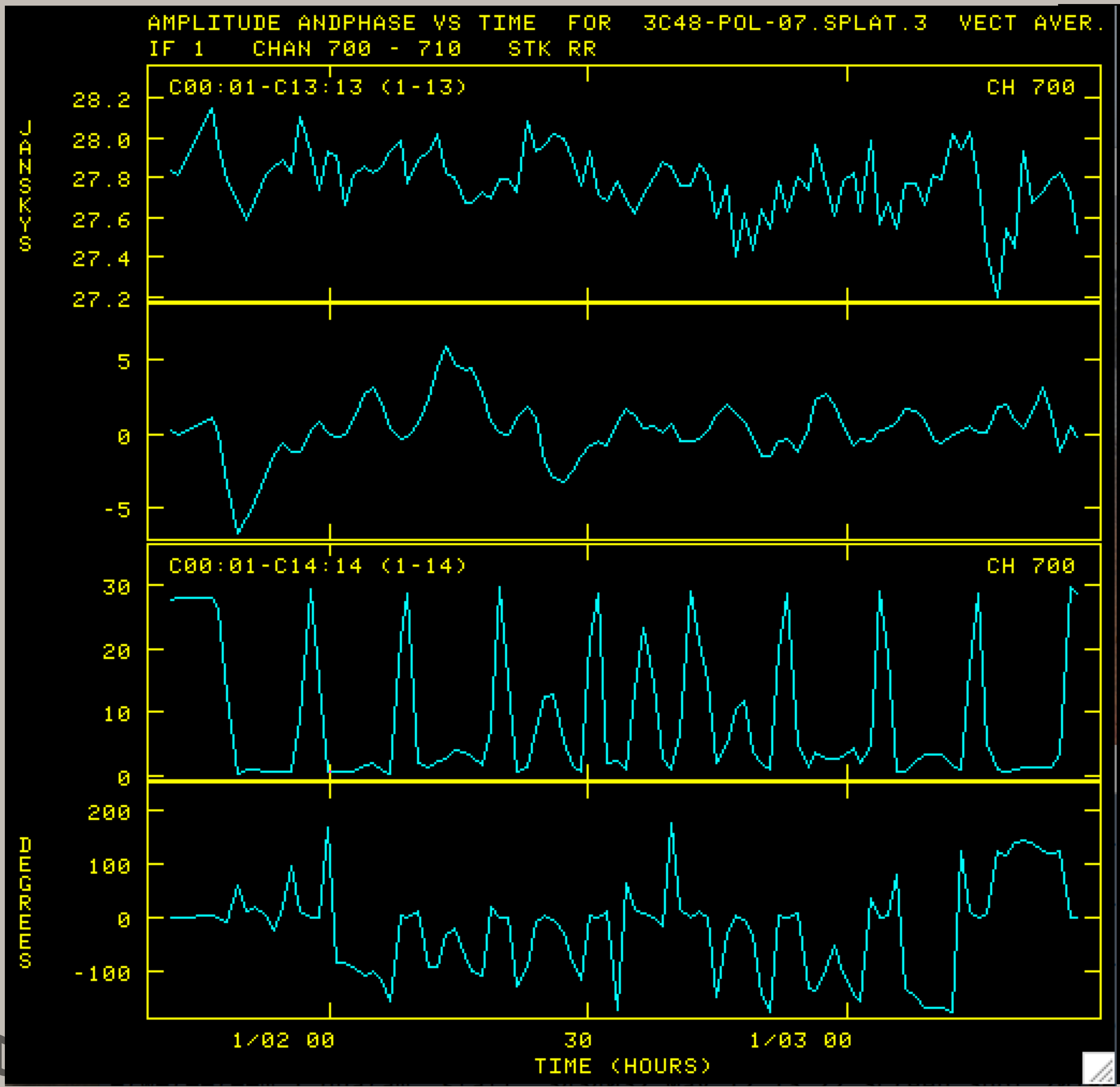
UGMRT DATA

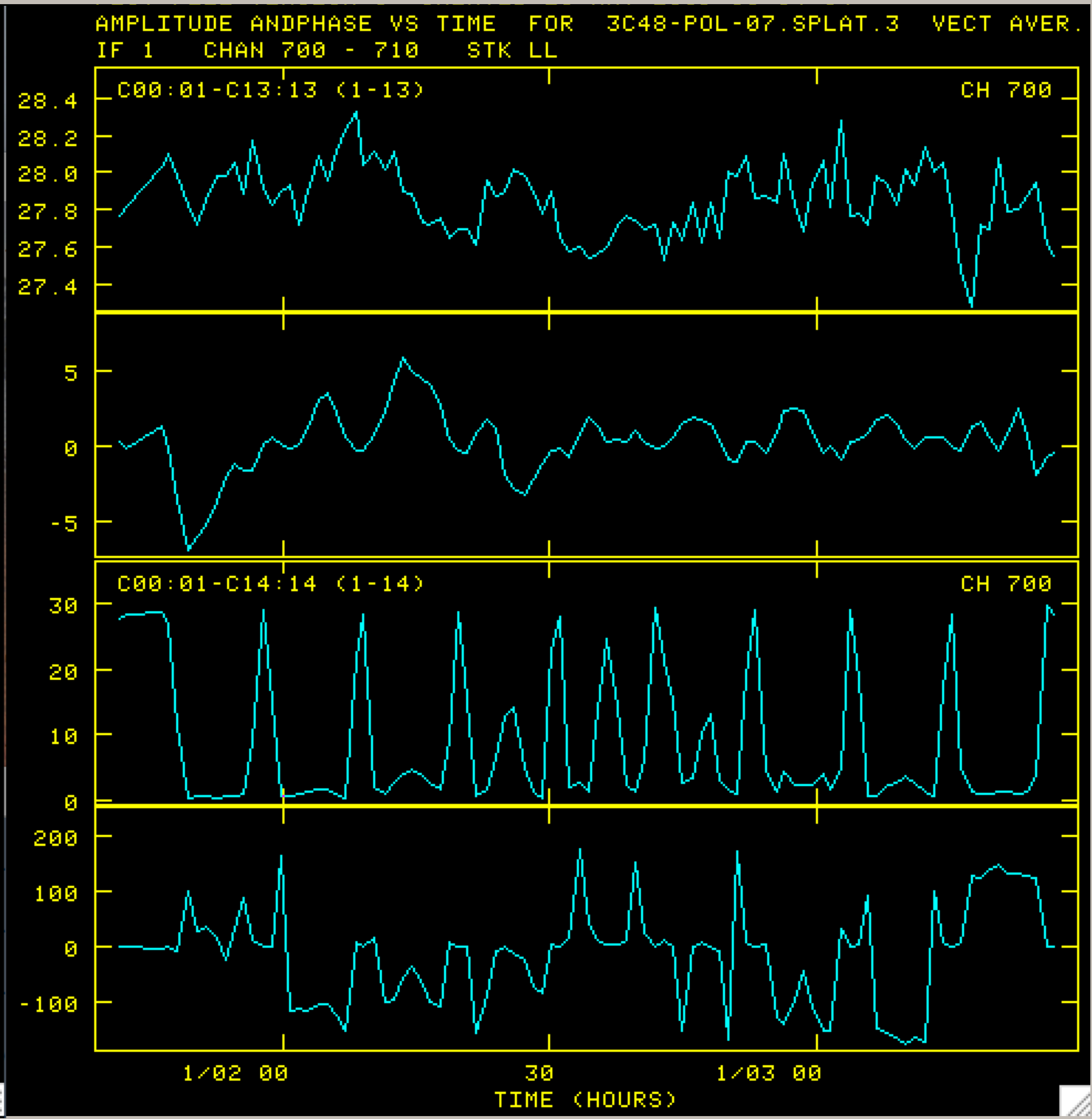
- Holography data



UGMRT DATA

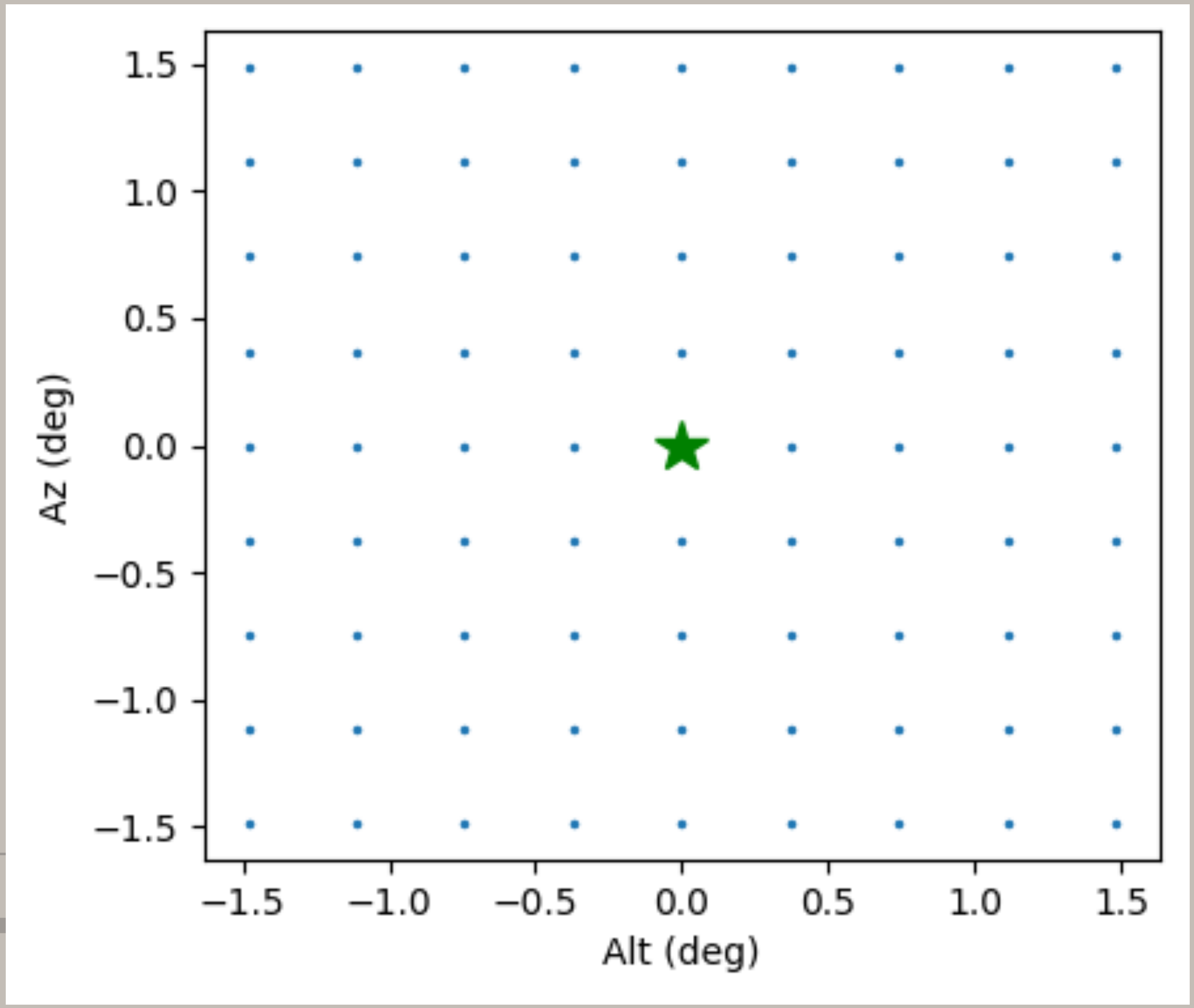
- Holography data
  - scans/data as a function of time





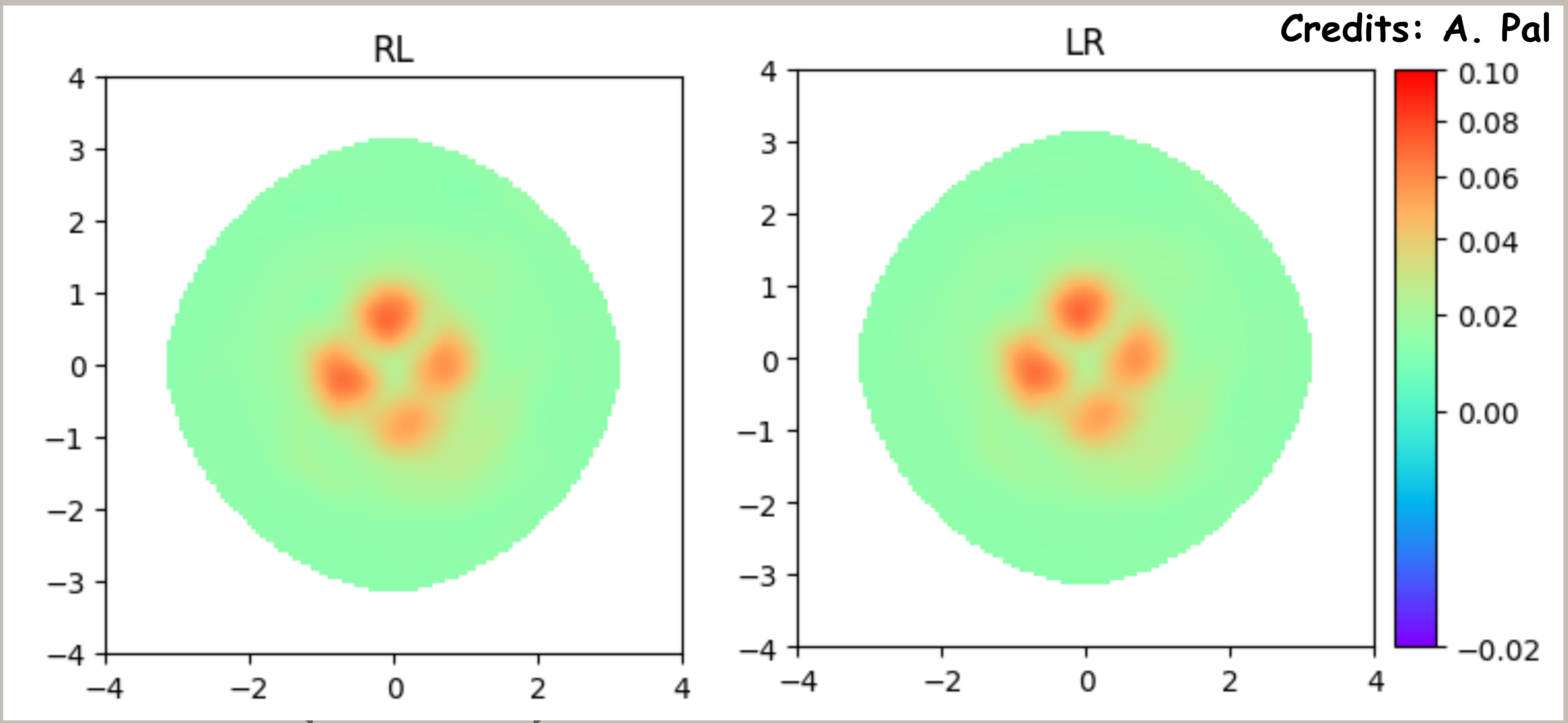
UNDERSTANDING DATA

- Holography data
  - scans/data as a function of time
- construct (Stokes) image/beam
- Understanding the polarisation properties of the GMRT dish is fundamental …
- need for
  - “accurate aperture model”
  - CASA implementation



DD EFFECTS: GETTING THE BEAMS

- Beams:
  - Once the beams are available, turn-ON AW-projection in CASA to take care of the beam rotation.



MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\iota \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

Acknowledgements:
Rau+ 2009, Jagannathan+ 2019,
ASP Conf. Ser. Vol. 180 (1999)

Thank you all for your attention!