Calibration II

Dharam V. Lal

with due thanks to several friends / collaborators at

UCT, IDIA (SA), NCRA-TIFR (India) and NRAO (USA)

and CASA / AIPS (NRAO)

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

Discuss several topics:

- **Standard calibration and imaging**
  - **(DI instrumental effects)**
  - **w/ DD instrumental and propagation effects**
  - **advanced image parameterisations**

TELESCOPE SENSITIVITY

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\rm sys}}{A_{\rm eff} \times \sqrt{(\Delta\nu \times \Delta t)}}$$

- Sensitivity improvements achieved by
  - wide band receivers,
  - long integration times
  - more antennas
  - long baselines $\sigma_{\rm confusion} \propto (\nu^{-2.7}/B^2_{\rm max})$
    - B_{max} ~100 km @200 MHz, the confusion noise is ~1 μJy beam^{-1}.

IMPLICATIONS FOR IMAGING

- Long baselines $B_{\max} > 2\ \text{km} \implies \text{DR} > 10^4$
- Wide-field effects:
  - **w**-term, PB effects and ionosphere effects
- Larger data volume
  - $N_{\text{ant}}^2 \times N_{\text{channel}} \times t$
- Wide-field, wide-band, high resolution, high dynamic range imaging using large data sizes
  - a natural consequence of low frequency and high sensitivity imaging.

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\; W_{ij}(\nu, t) \int P_{ij}\; M_{ij}(s, \nu, t)\; \vec{I}(s, \nu)\; e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\; d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

MEASUREMENT EQUATION …

- $$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s},t)\cdot E^*(\vec{s},t-\overrightarrow{b}\cdot\vec{s}/c)\right\rangle e^{-2\pi i\overrightarrow{b}\cdot\vec{s}/\lambda}d\Omega$$

mutual coherence function

complex amplitude of the radiation emanating from the source in the direction $\vec{s}$

$\vec{s} = \vec{s}_0 + \vec{\sigma}$

point near the phase centre

time difference between the incoming radiation collected at two antennas separated by $\overrightarrow{b}$

$$d\Omega = \frac{d\vec{s}}{R^2}$$

MEASUREMENT EQUATION ...

- $\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$
- $V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \, dm$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (four cross-correlation products, $\langle \overrightarrow{E_i} \otimes \overrightarrow{E_j^*} \rangle$ per baseline)
  - $\overrightarrow{V_{ij}} = [V^{rr} \ V^{rl} \ V^{lr} \ V^{ll}]_{ij}^T$
  - $\vec{I} = [I^{rr} \ I^{rl} \ I^{lr} \ I^{ll}]^T$

MEASUREMENT EQUATION …

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
    - (a 2×2 matrix product)
      - complex gains, G,
      - polar'n leakage, D and
      - feed config'n, C.
  - DD: $J_i^{sky} = [EPF]$
    - (a 2×2 matrix product)
      - AIPs, E,
      - PA effects, P and
      - tropospheric / ionospheric effects, and Faraday R'n, F.

MEASUREMENT EQUATION ...

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
  - DD: $J_i^{sky} = [EPF]$
  - $K_{ij}^{\{vis,\ sky\}} = [J_i \otimes J_j^{\dagger}]^{\{vis,\ sky\}}$
  - (effect on each baseline *ij* is described by the outer-product of these antenna-based Jones matrices, a 4×4 matrix!)
- $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma}/\lambda} d\Omega$

MEASUREMENT EQUATION

- $$\vec{V}_{ij}^{Obs}(\nu, t) = G_{ij}\; W_{ij}(\nu, t) \int P_{ij}\; M_{ij}(s, \nu, t)\; \vec{I}(s, \nu)\; e^{\imath \vec{b}_{ij}.\vec{s}}\; d\vec{s}$$

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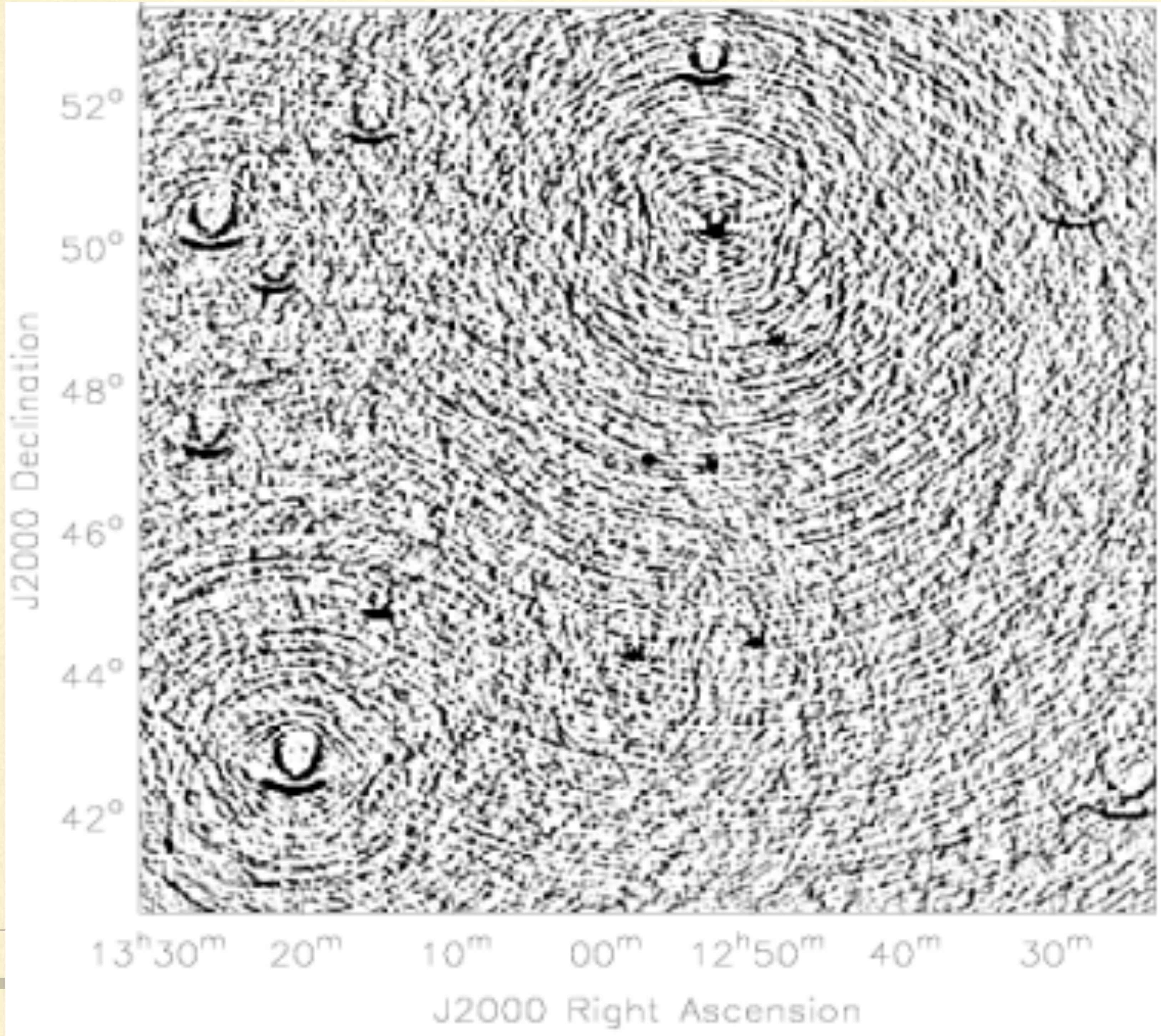
CALIBRATION AND IMAGING

- Standard calibration and imaging
    - (DI instrumental effects)
- w/ DD instrumental + propagation effects
    - correction for **w term**
    - correction for **PB**
        - image plane correction
        - Fourier plane correction
        - pointing self-calibration
    - Mosaicing
- w/ advanced image parameterisation
    - **multi-scale CLEAN** (deconvolution)
    - **multi-frequency synthesis** (imaging)
    - **full polarisation** (Stokes) calibration and imaging

w-TERM

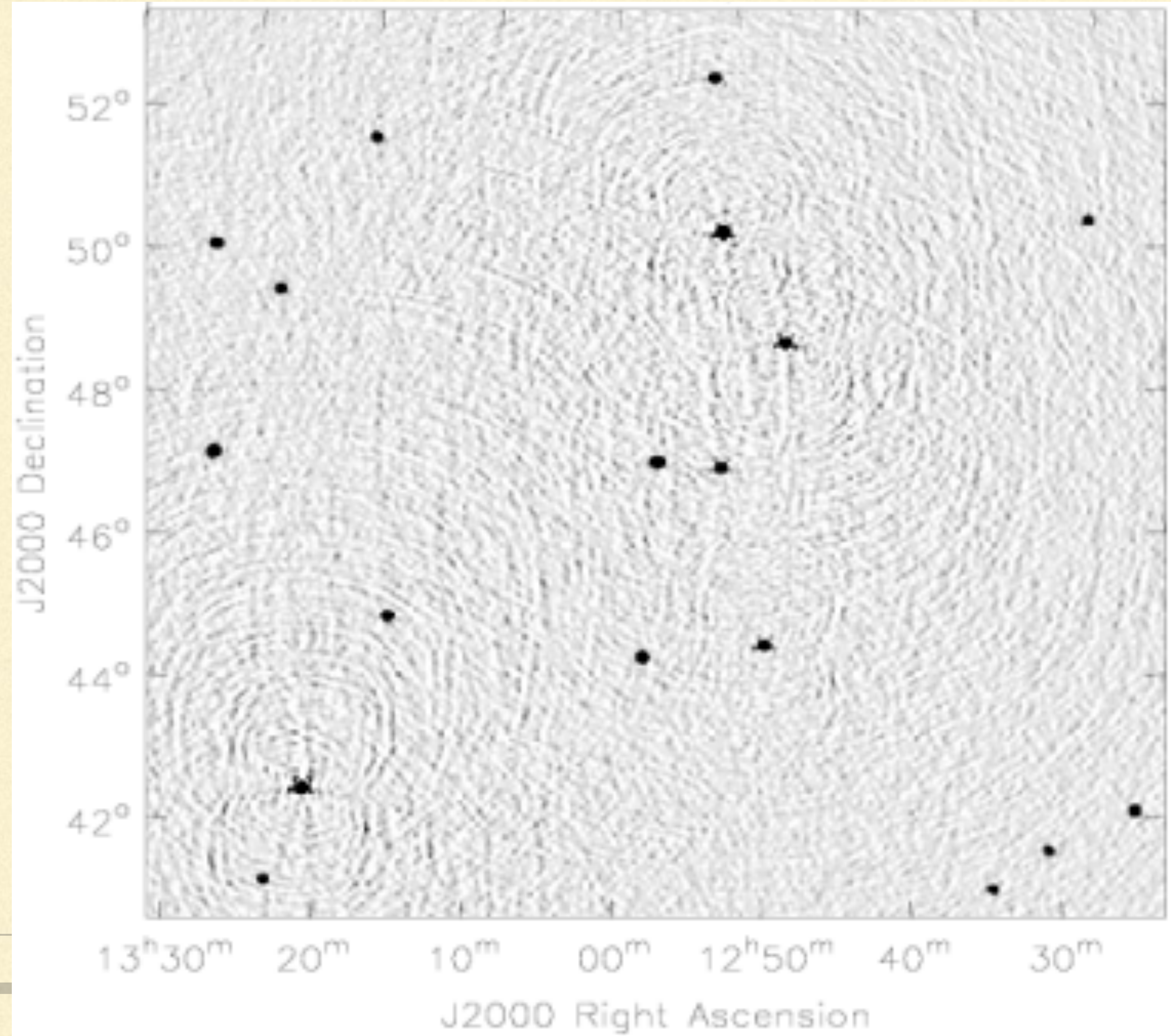
- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$e^{\iota w\sqrt{1-l^2-m^2}}$$



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

w-TERM

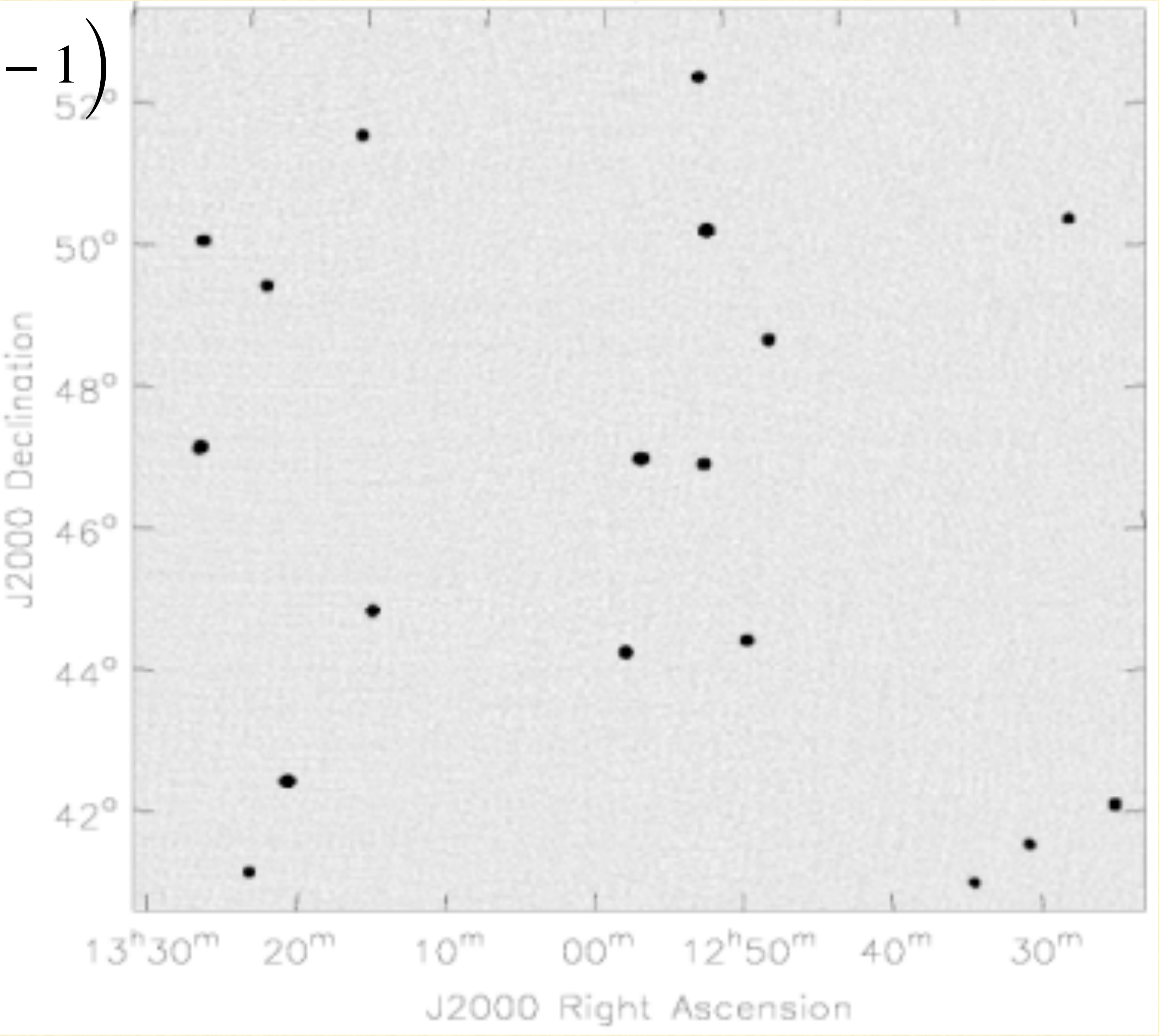
- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$e^{\imath w\sqrt{1-l^2-m^2}}$$
- divide the FoV into a no. of FACETS



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

w-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$K_{ij}^{Sky} = e^{w_{ij}\left(\sqrt{1-l^2-m^2}-1\right)}$$
- An order-of magnitude faster than FACETing, and
- for the same amount of computing time provides higher DR images.



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CORRECTION FOR **PB**

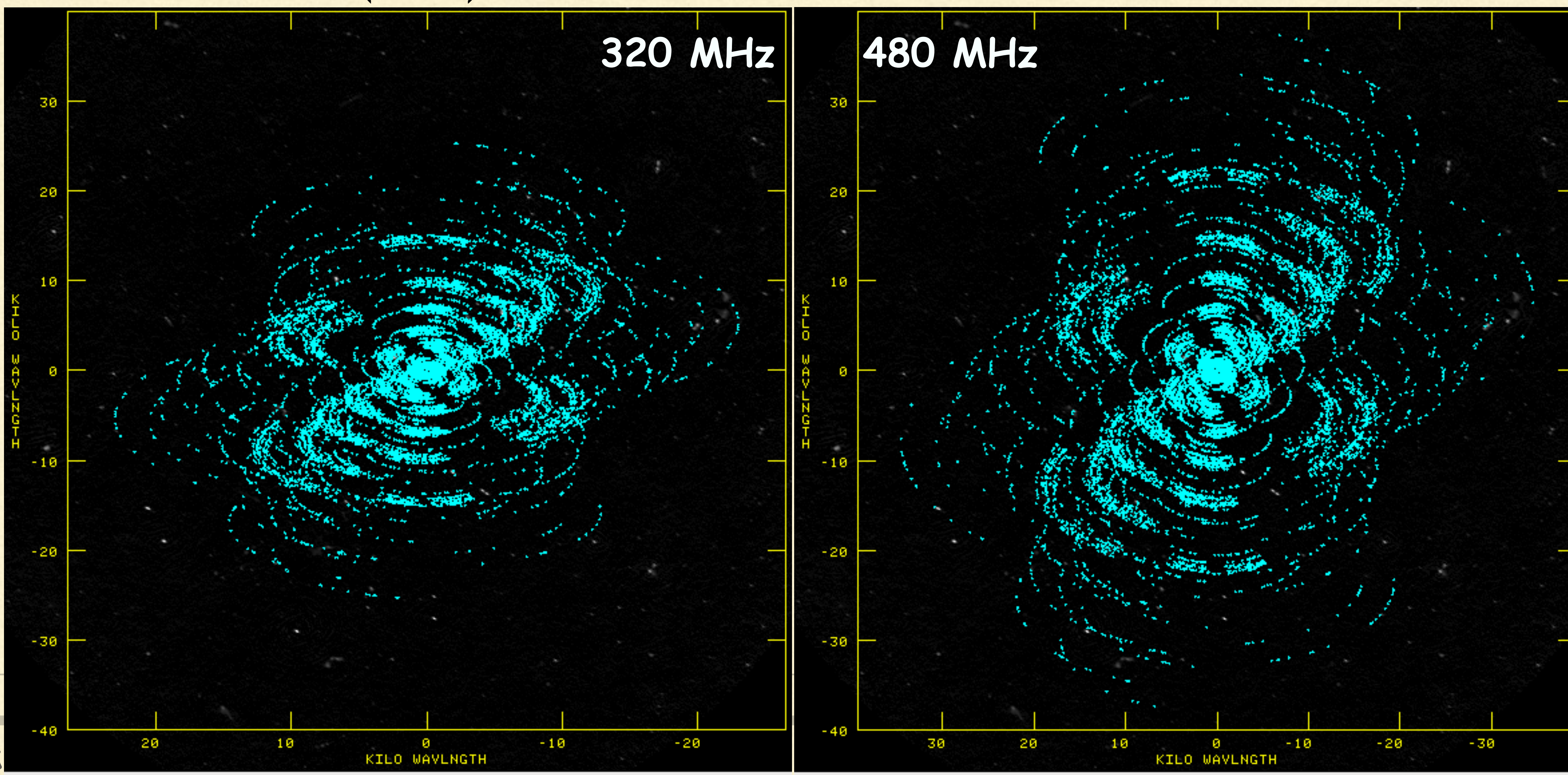
- **A**-projection
  - $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma}/\lambda} d\Omega$
  - $\overrightarrow{V}_{ij}^{obs}(\nu) = \int \; J_{ij}^{S}(s, \nu, t) \; I(s, \nu) \; e^{i(ul+vm+w(\sqrt{(1-l^2-m^2)}-1))} \; ds$

DATA PB SKY GEOMETRY

  - Visibility depends on time and frequency!

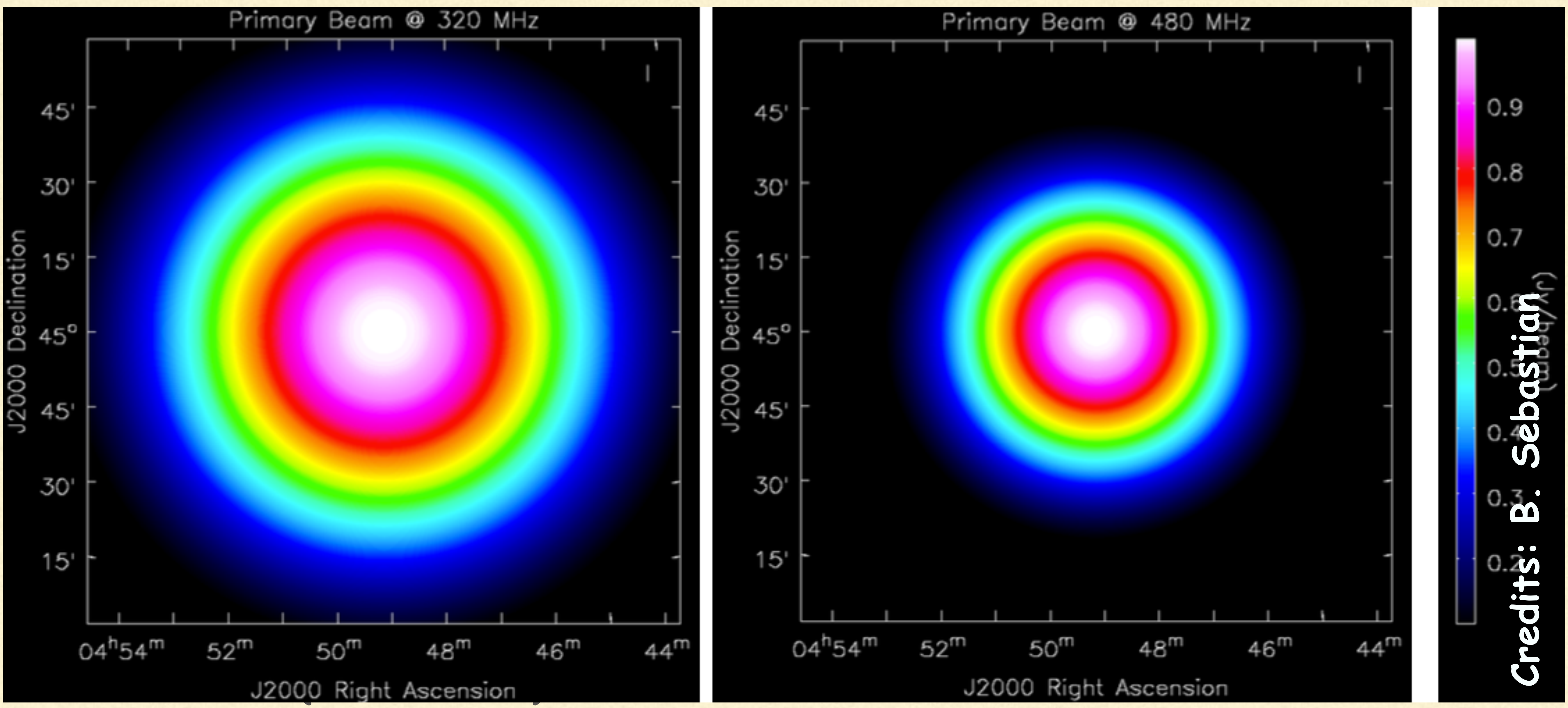
CORRECTION FOR **PB**

- **multi-frequency** synthesis
  - $$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0} \right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$



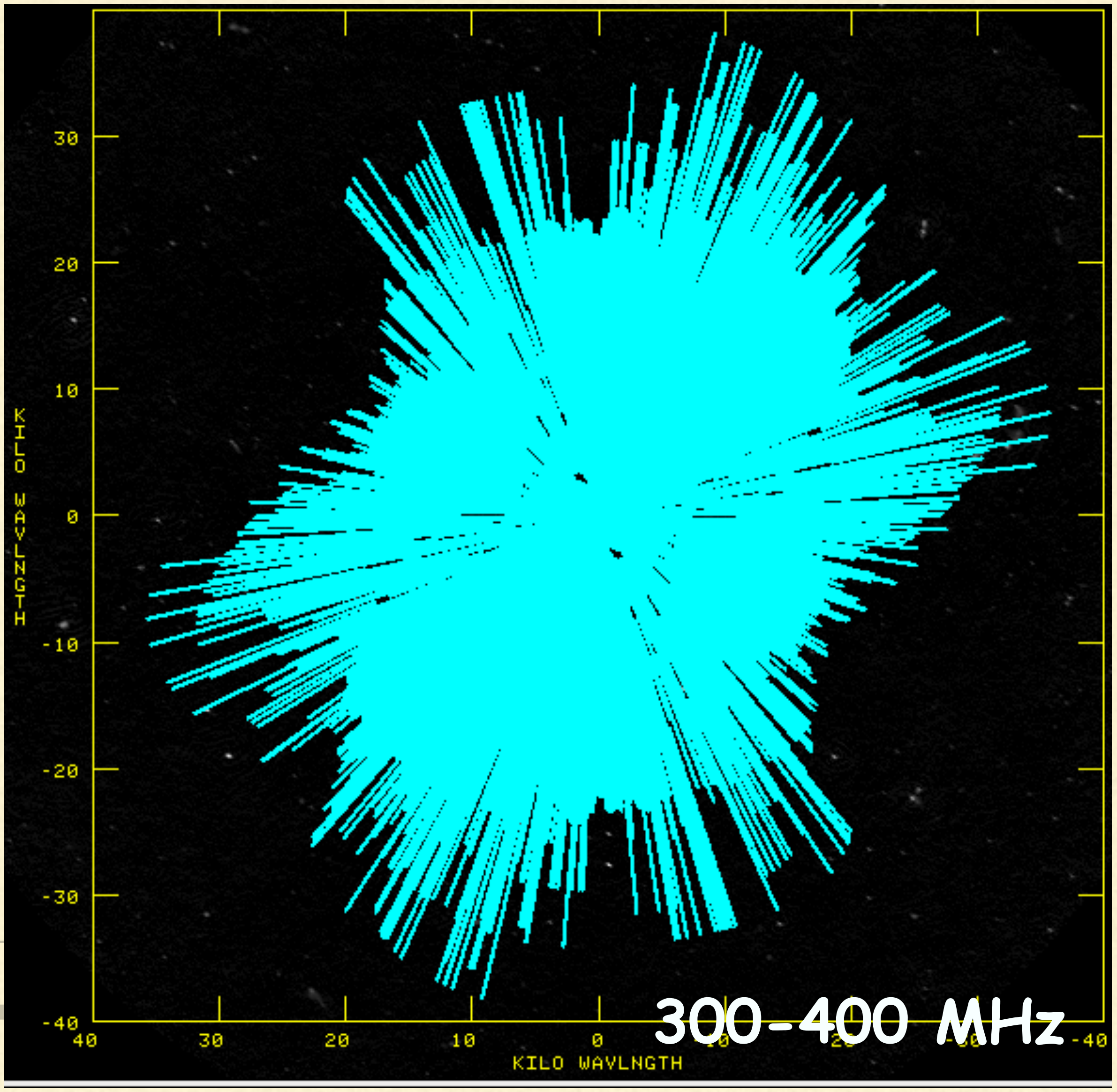
CORRECTION FOR **PB**

- **A**-projection
  - $\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\; I(s,\nu)\; e^{i(ul+vm+w(\sqrt{(1-l^2-m^2)}-1))}\; ds$



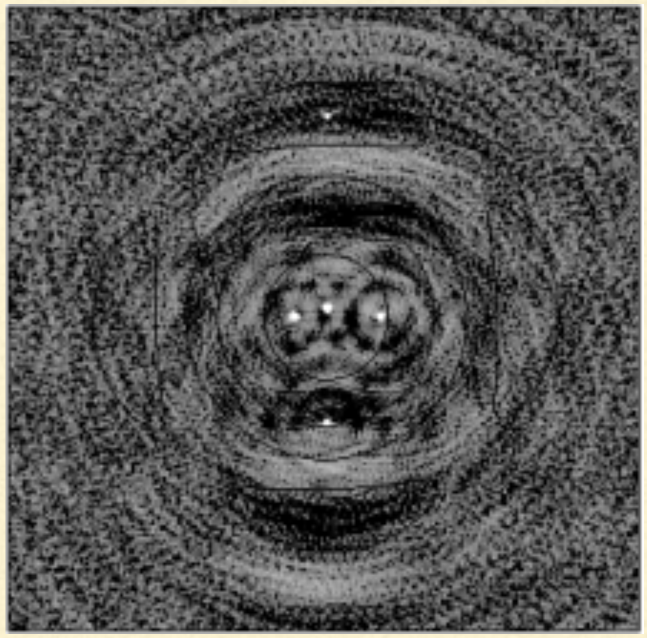
CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{i(ul+vm+w(\sqrt(1-l^2-m^2)-1))}\ ds$$
  - **multi-frequency** synthesis
  - $$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0}\right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$



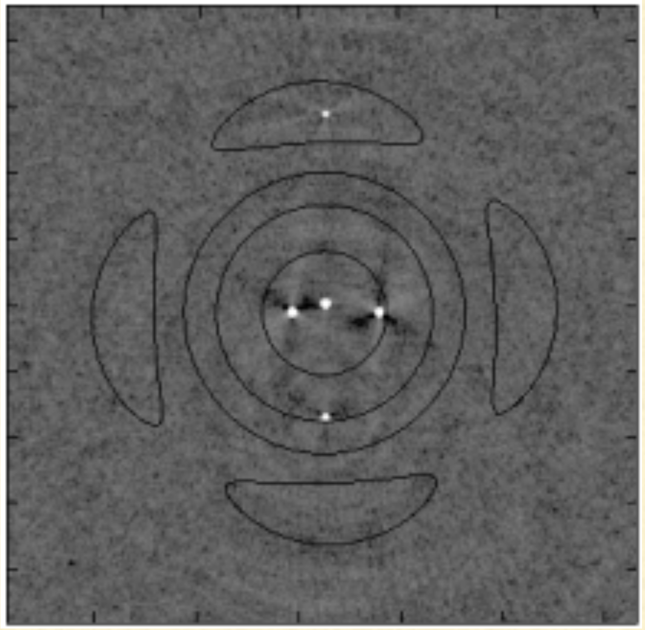
CORRECTION FOR EVERYTHING(?)

Credits: S. Bhatnagar NRAO, USA)



FT

(standard imaging)



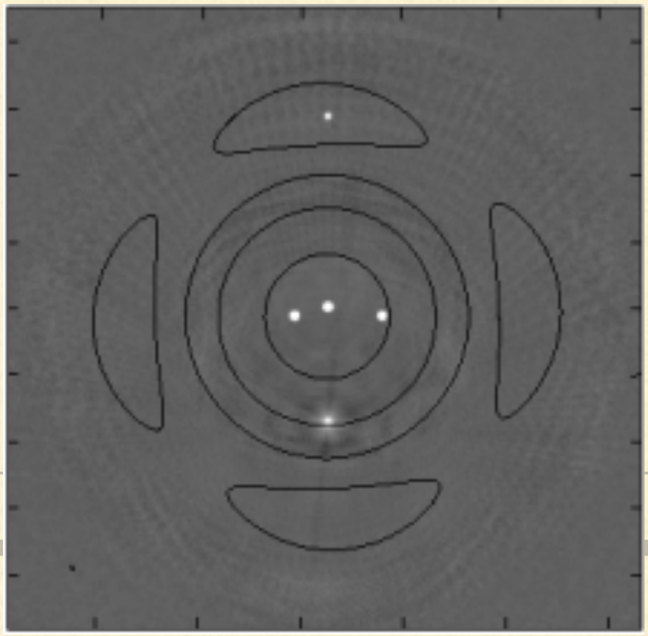
FT

+ MT-MFS

FT

+ MT-MFS

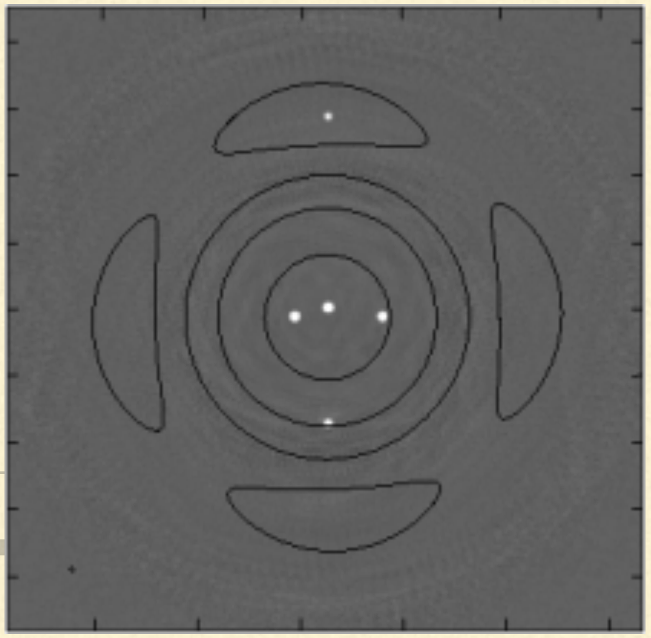
+ A-projection



FT

+ MT-MFS

+ WB A-projection

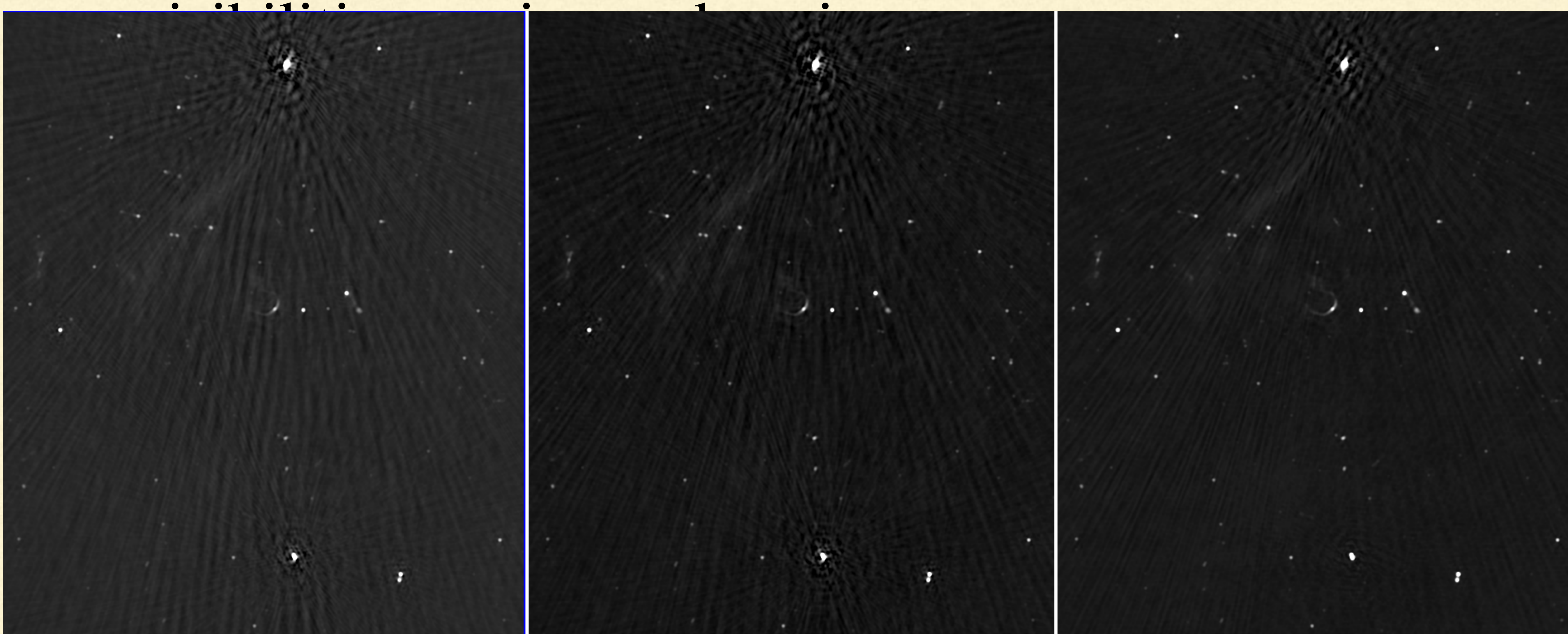


PEELING: DD CALIBRATION

- antenna based gains are determined in the direction of each compact source.
  - subtract these gains (contribution of compact sources from the observed data using a DFT) and the residual visibilities are imaged again.
- drawbacks of peeling…

PEELING: DD CALIBRATION

- antenna based gains are determined in the direction of each compact source.
    - subtract these gains (contribution of compact sources from the observed data using a DFT) and the residual



STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
  - $$I = \frac{(RR + LL)}{2}$$
  - $$\frac{Q}{I} = \frac{\mathrm{Re}(RL + LR)}{RR + LL}$$
  - $$\frac{V}{I} = \frac{RR - LL}{RR + LL}$$
  - $$\frac{U}{I} = \frac{\mathrm{Im}(RL - LR)}{RR + LL}$$
    - Leakages: the total intensity can leak into the polarised components (I into $\{Q,U,V\}$).

MUELLER MATRIX

Hans Mueller

NCRA • TIFR

- The leakage of each polarisation into the other can be measured and quantified in a 4×4 matrix (Mueller 1943).

- $$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

- $$\begin{bmatrix} RR + LL \\ RL + LR \\ RL - LR \\ RR - LL \end{bmatrix} = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix} \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
- $I \Leftrightarrow V$ leakage
- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- *RL* phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

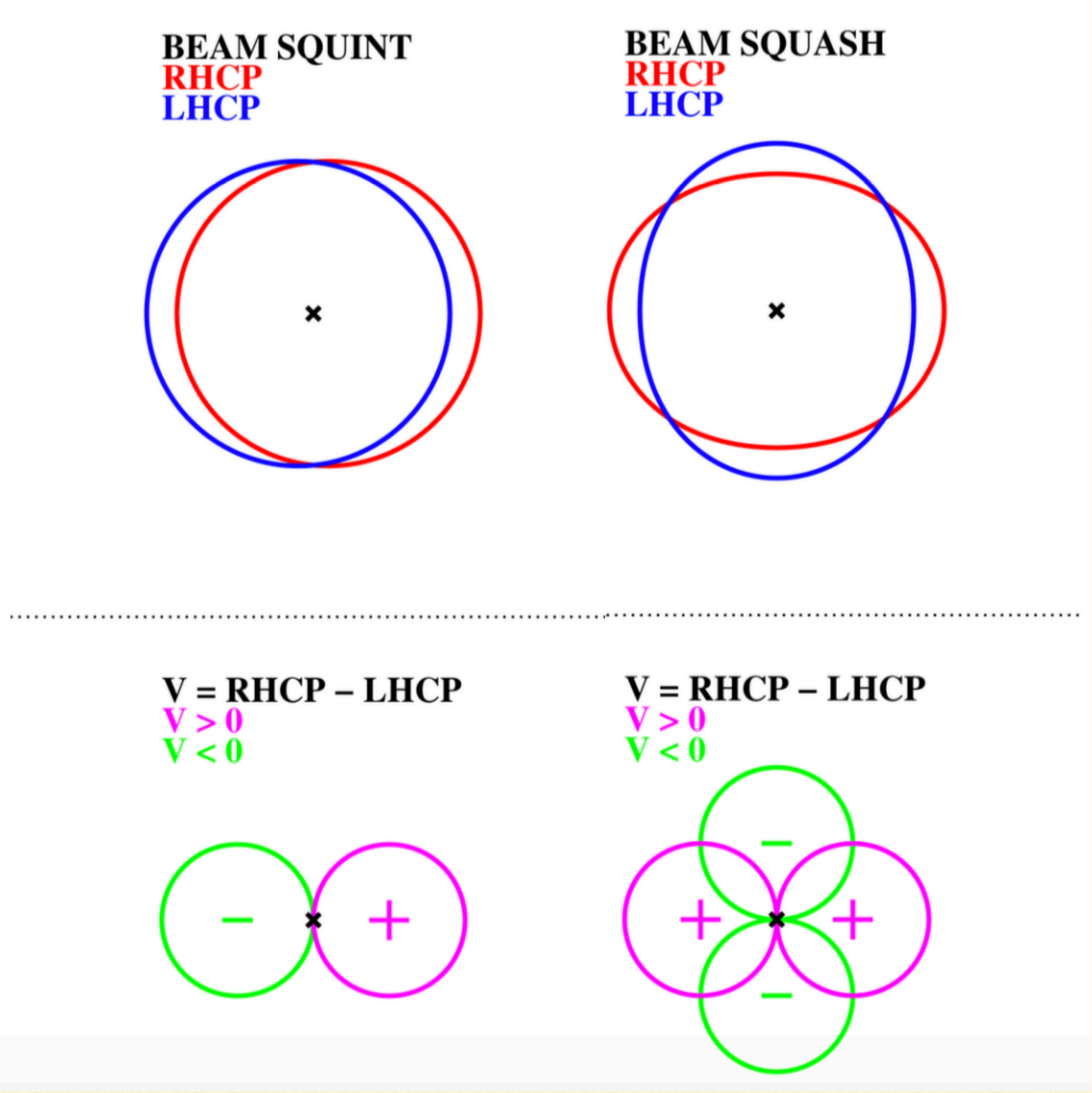
Stokes V ~ 0 for most calibrators so no need to worry too much unless you require very high precision

BEAM EFFECTS

- For point sources, all of the previous is fine.
- What if the source you are looking at is extended compared to the telescope beam?
  - There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
    - Squint
    - Squash

BEAM EFFECTS

- For point sources, al
- What if the source y compared to the tele
  - There are instrum measurement of e
    - Squint
    - Squash



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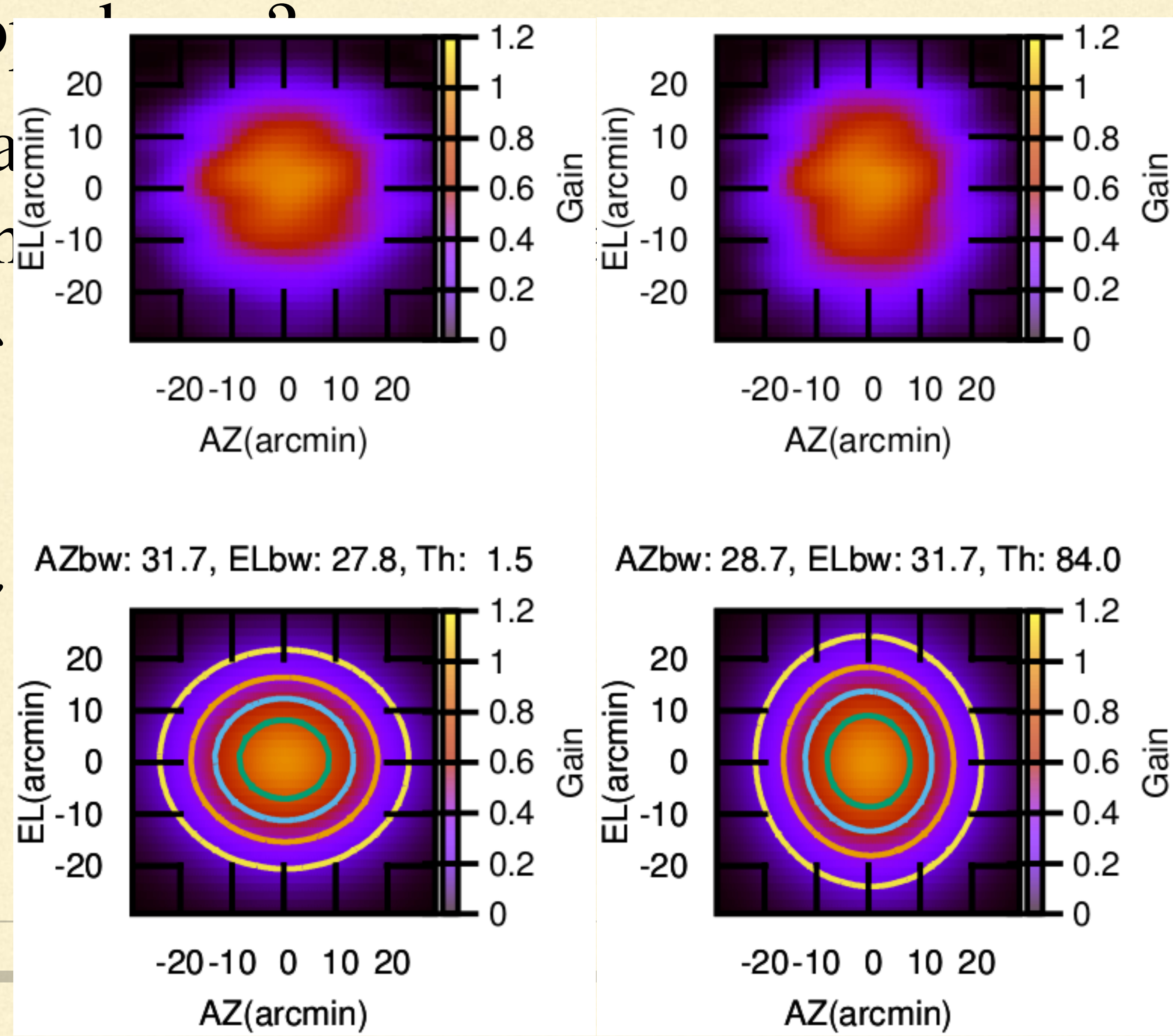


Credits: S.N. Katore (NCRA-TIFR)

BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA

Weights

Mueller matrix

full-polarization vector of the sky brightness distribution

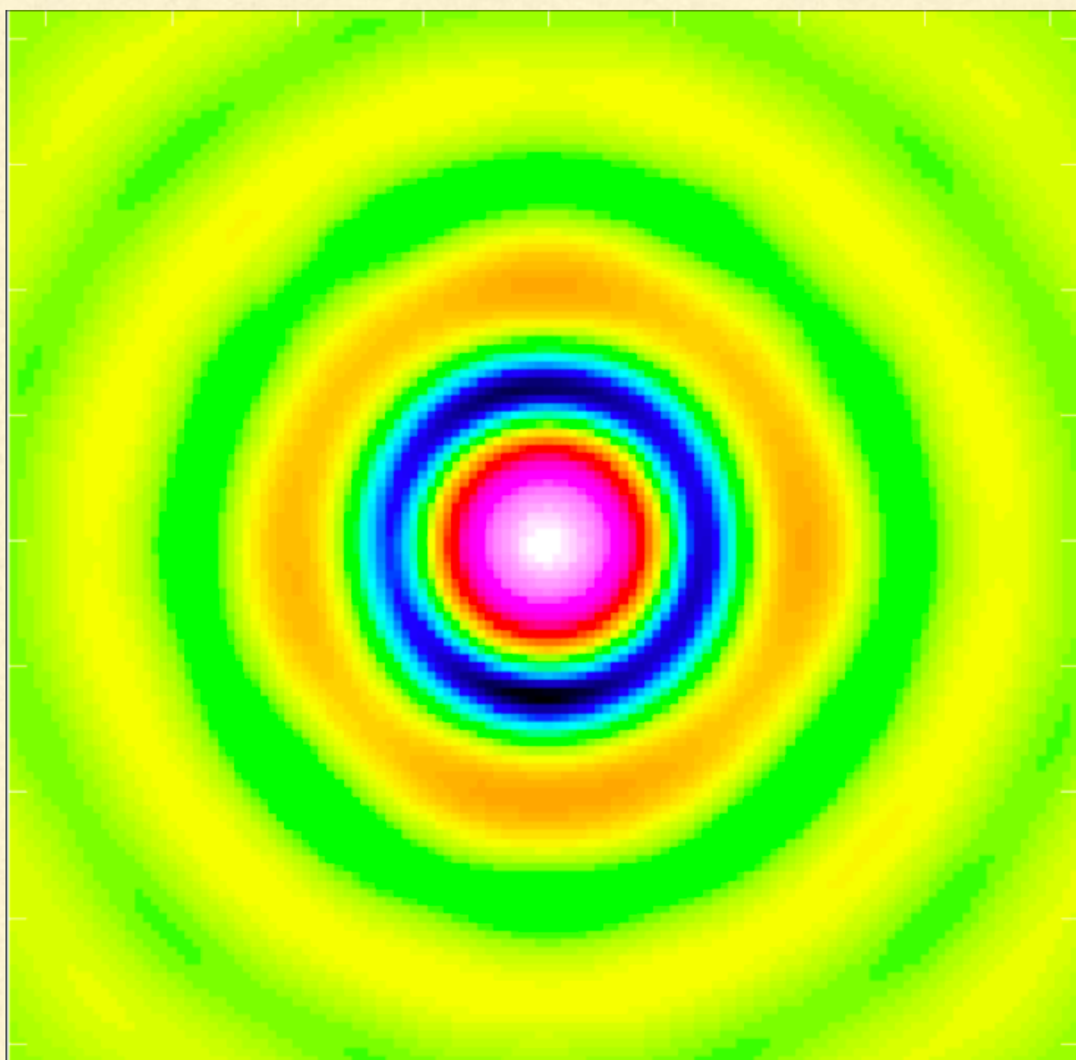
BEAM EFFECTS

- $$\vec{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \vec{b}_{ij} \cdot \vec{s}}\ d\vec{s}$$
  - $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$
  - $$\vec{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \mathscr{F} \left[\left(E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t) \right) \cdot \vec{I}(\vec{s}, \nu) \right]$$
    - $$= W_{ij}(\nu, t) \left[A_{ij} \star \vec{V}_{ij} \right]$$
    - $$\text{where}, A_{ij} = A_i \otimes A_j^*$$

AIPs for two antenna

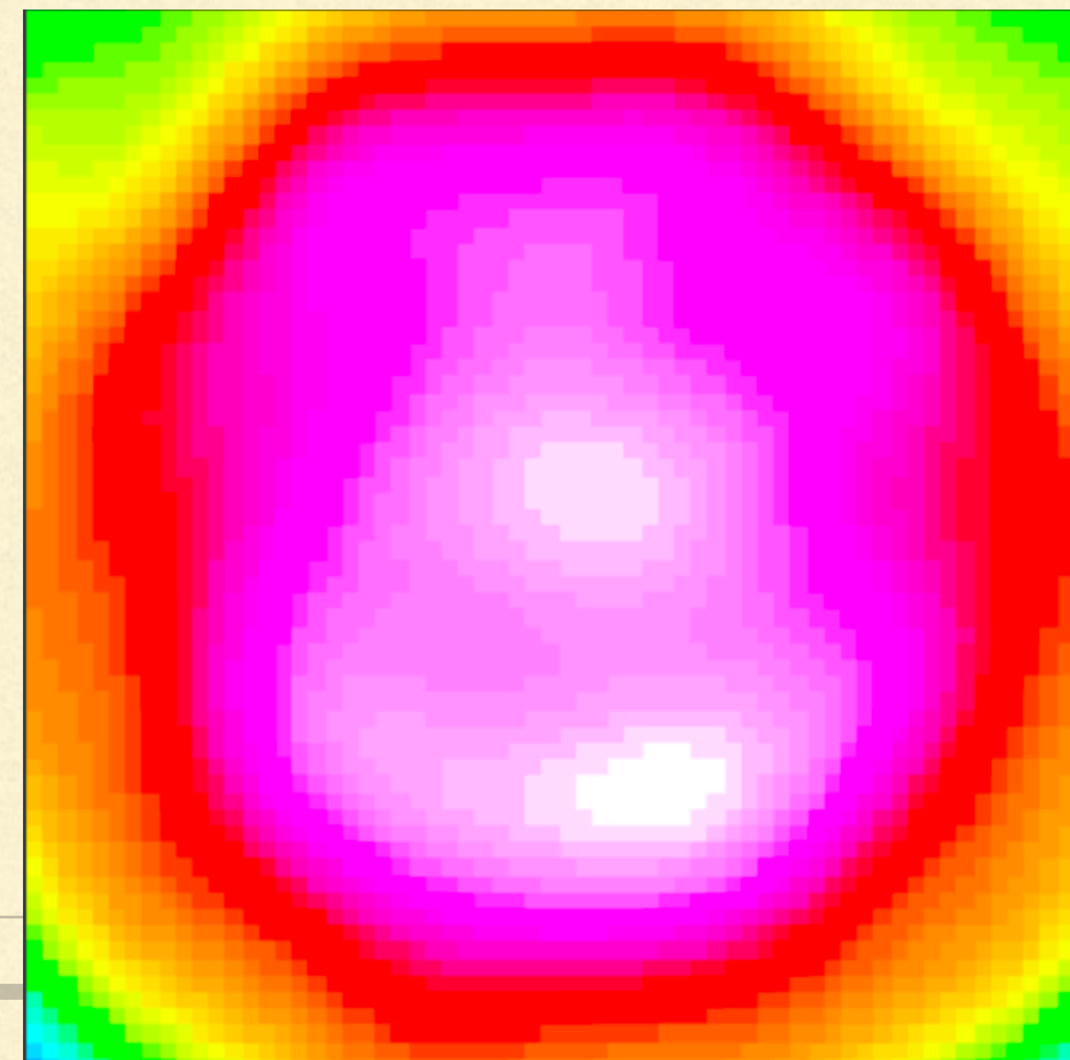
APERTURE ILLUMINATION PATTERN

- Holography data: MeerKAT
  - Obtained by Fourier Transforming the PB Holography measurements



⇔

Credits: S. Sekhar (UCT-IDIA)



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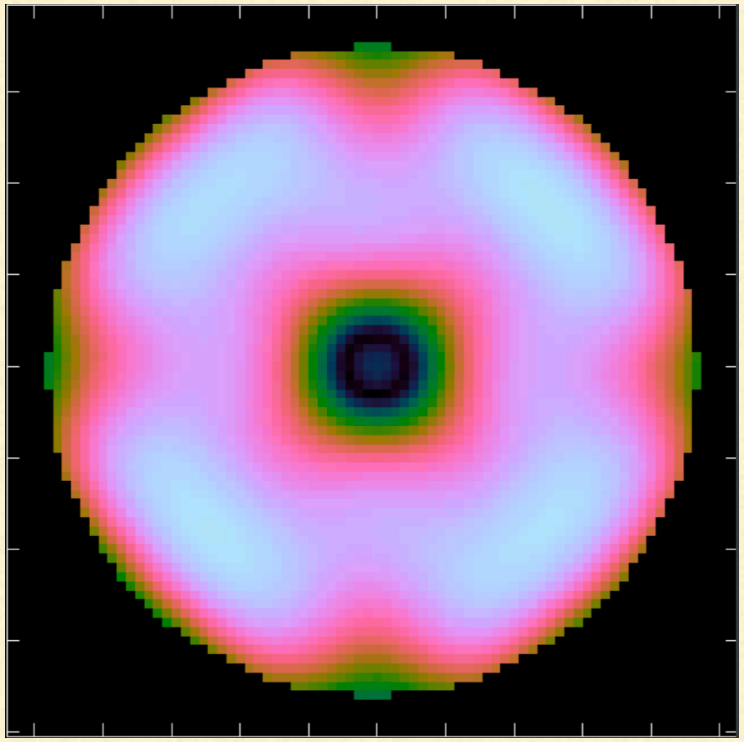
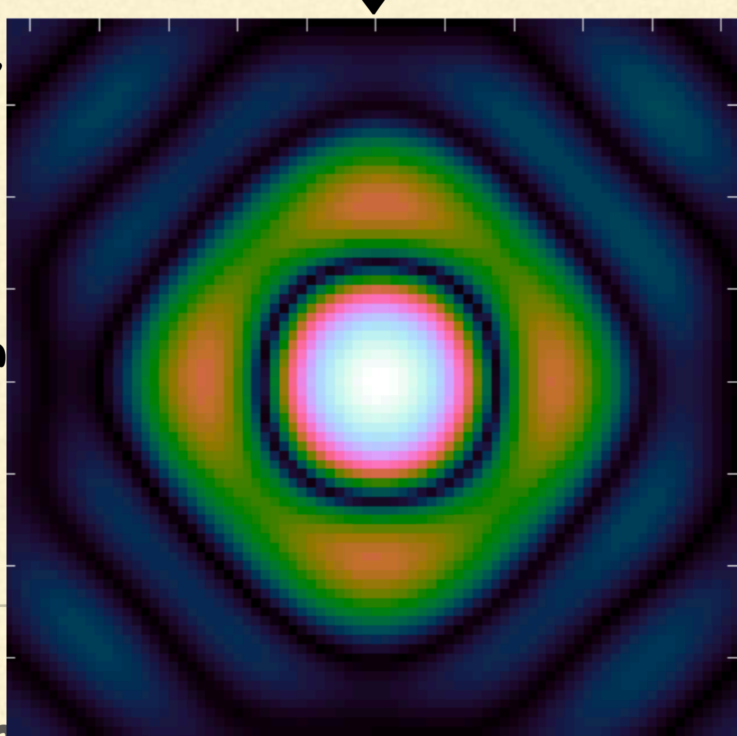
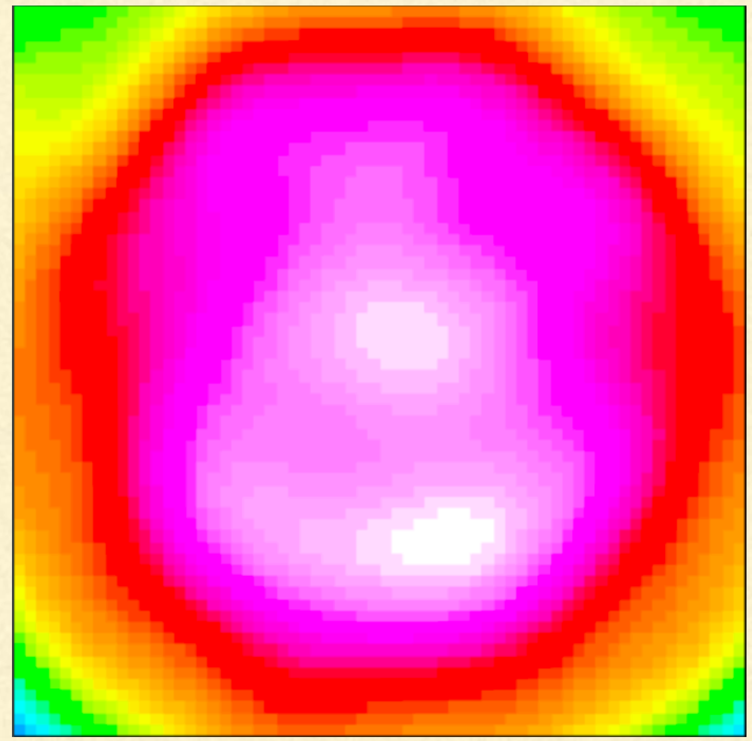
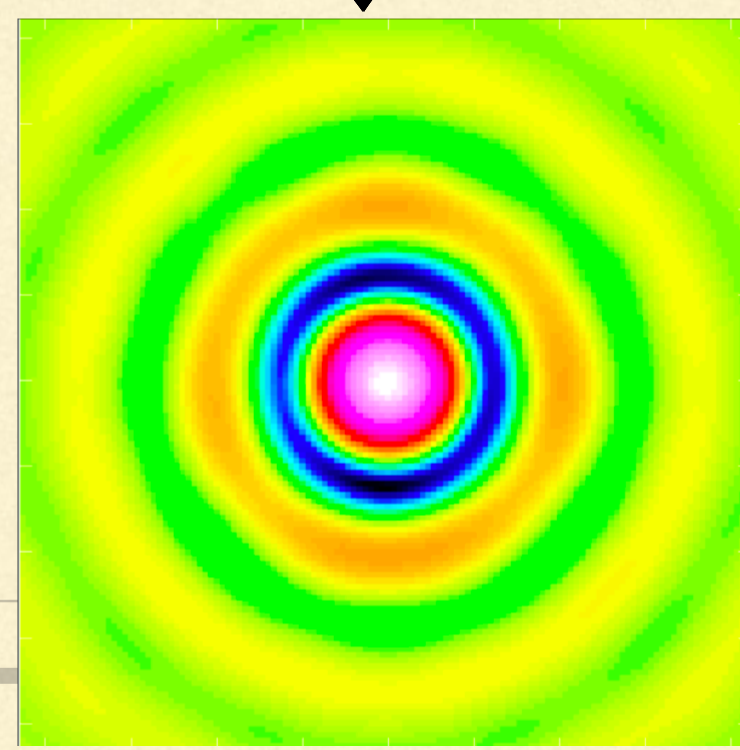




Credits: P. Jagannathan (NRAO)





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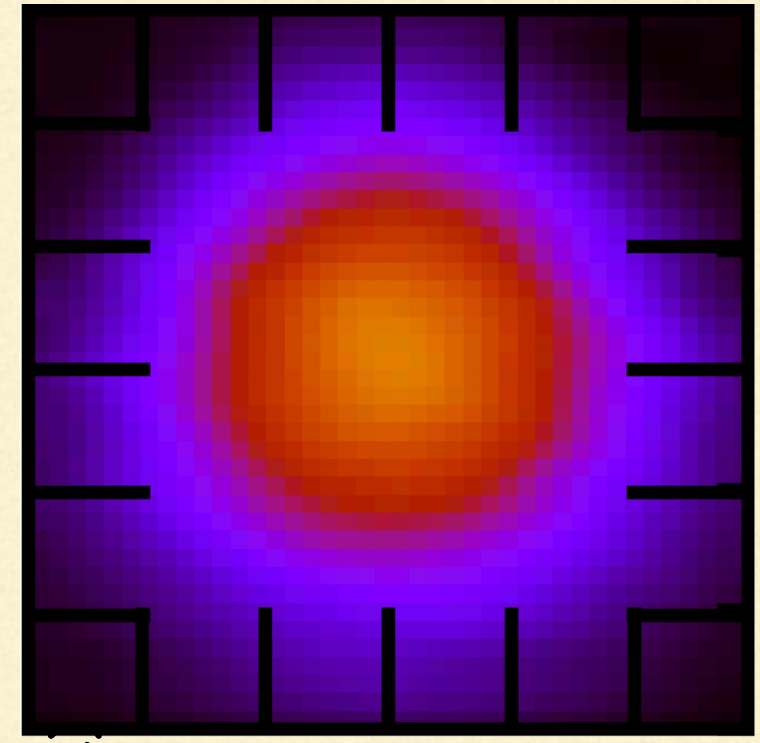
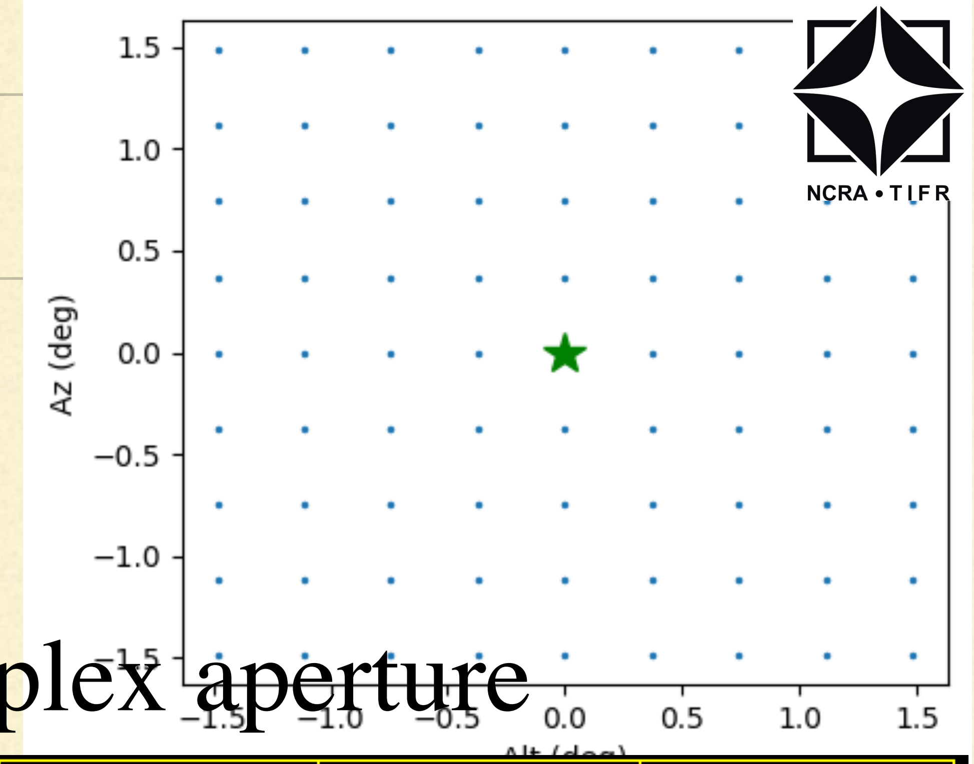
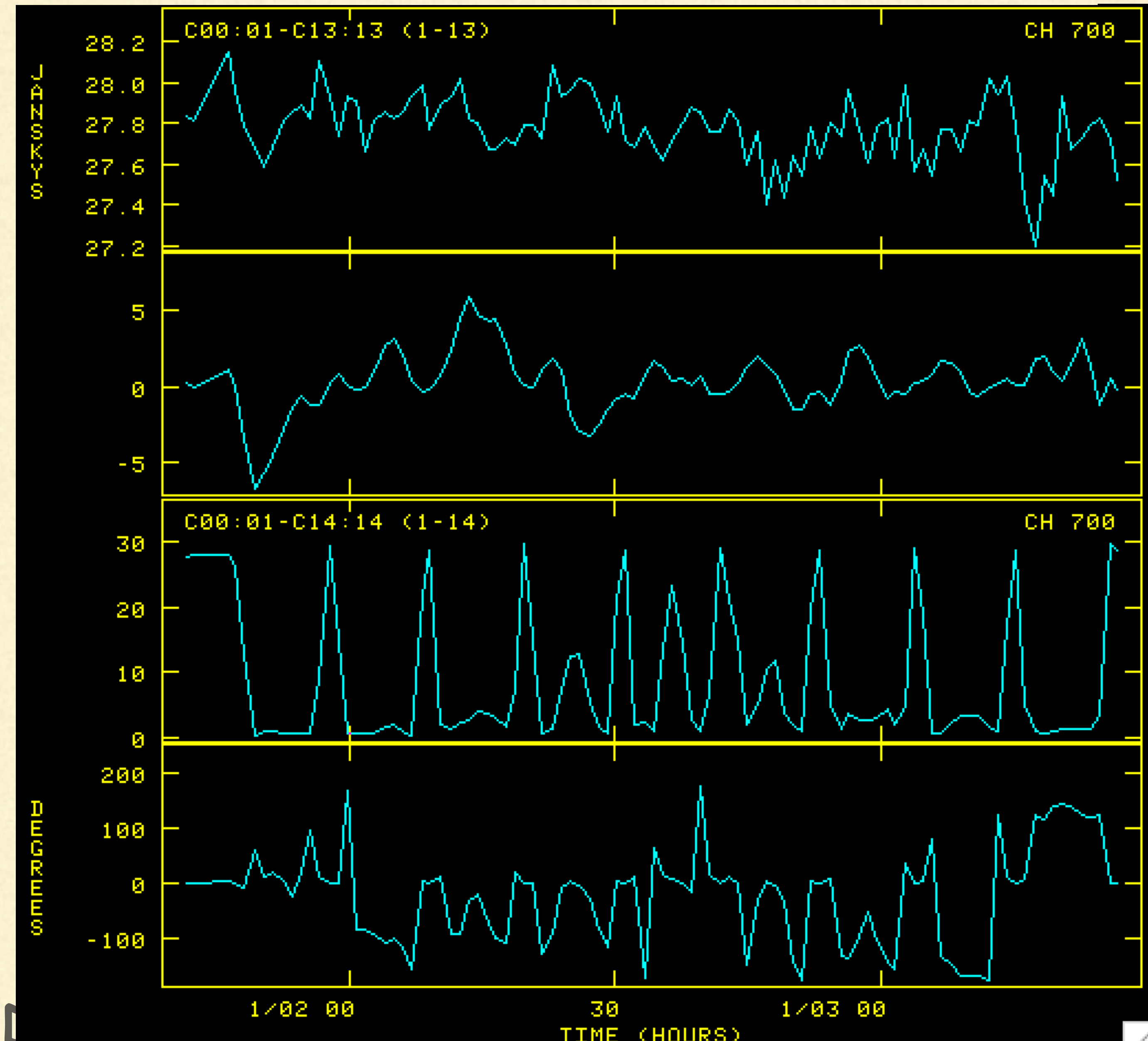


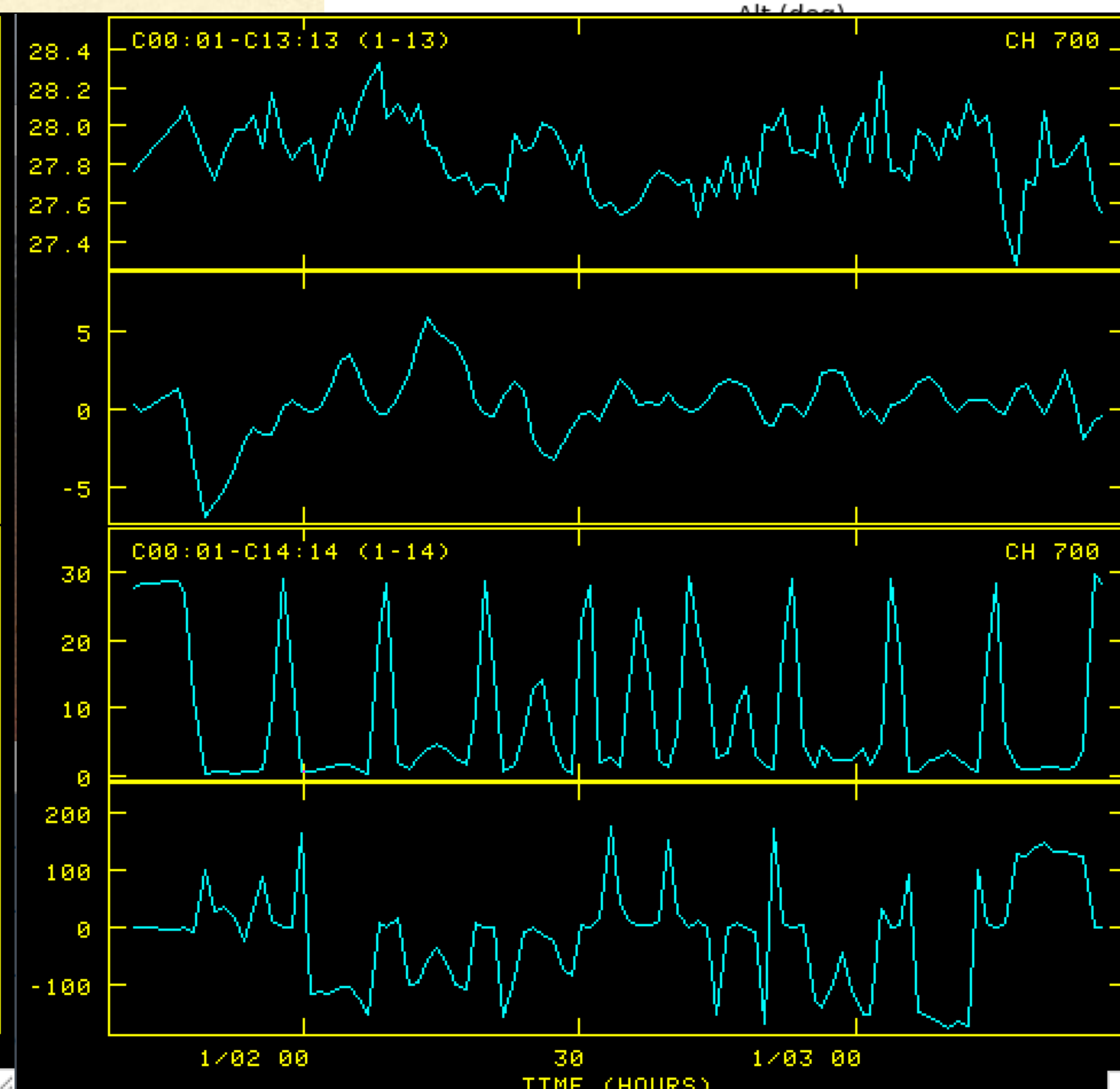
Credits: S.N. Katore (NCRA-TIFR)

UGMRT DATA



- Holography data
  - scans/data as a function of time
- Zernike polynomials to model the complex aperture

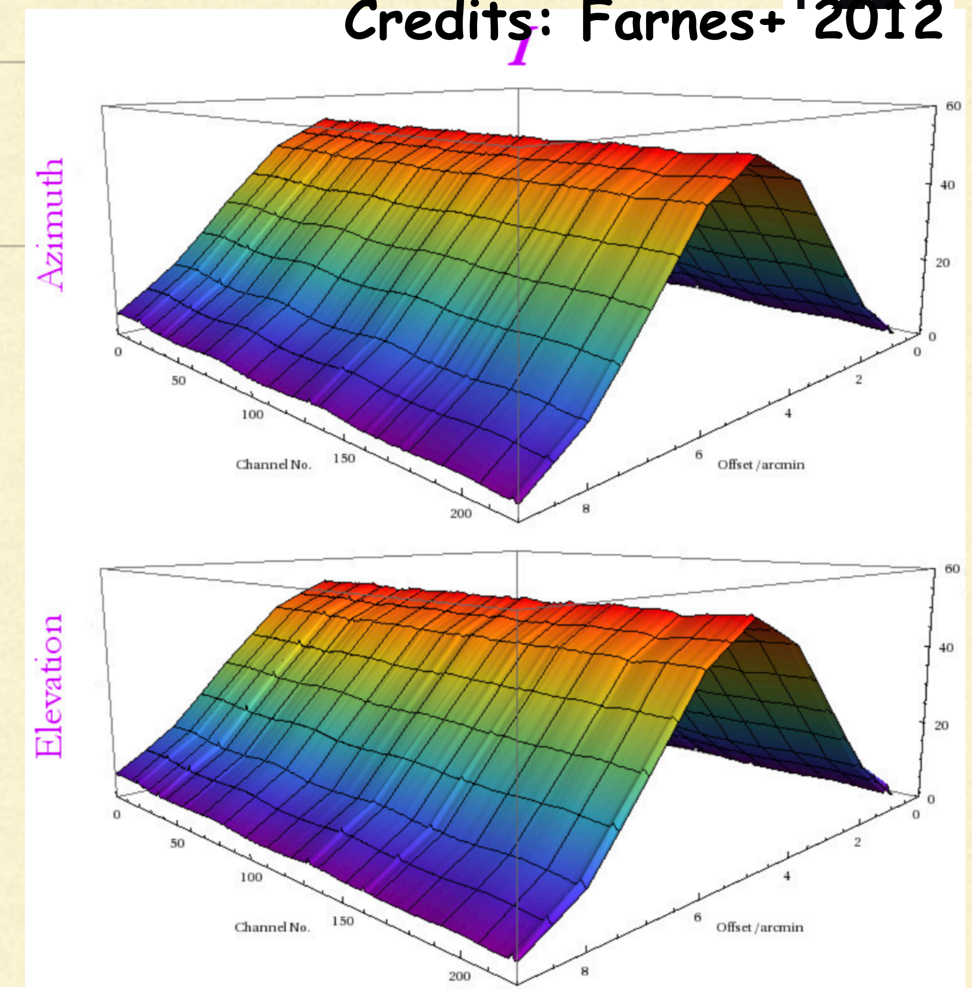




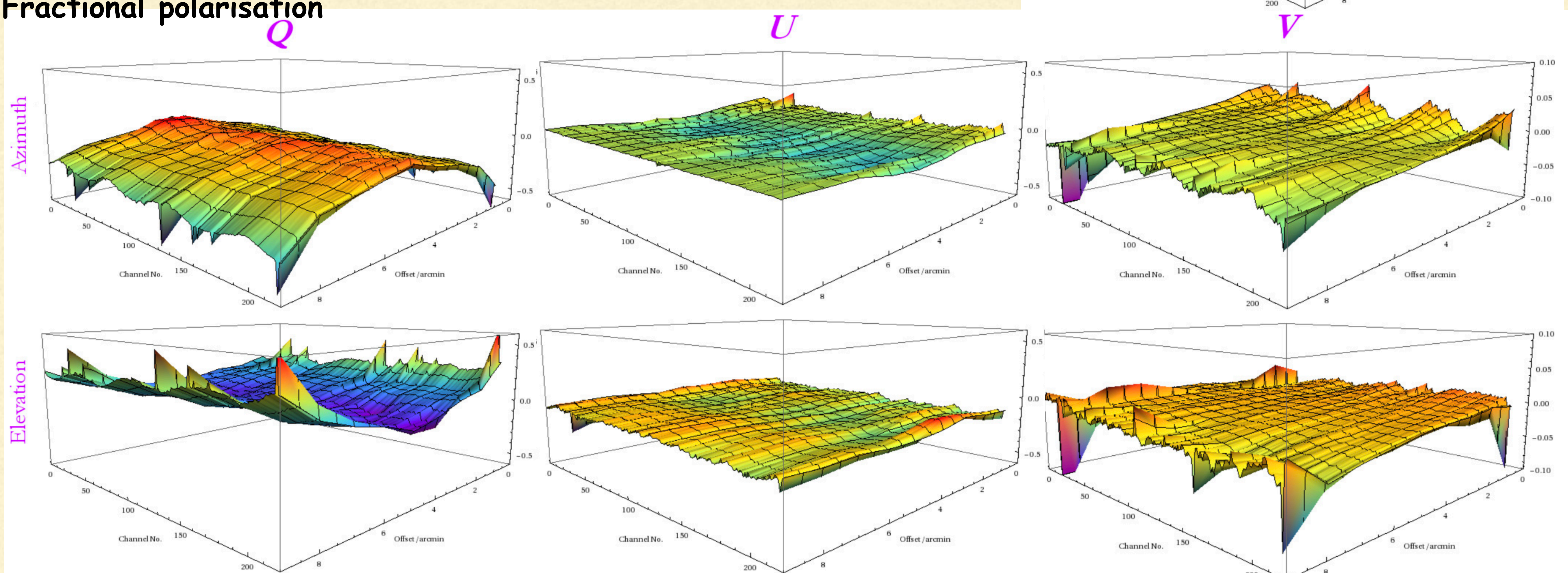
NEXT: BEAM PROFILES

- 325 MHz:
  - X-axis offset from phase-centre
  - Y-axis beam response
  - Z-axis channel/frequency

Credits: Farnes+ 2012



Fractional polarisation



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Acknowledgements:
Rau+ 2009, Jagannathan+ 2019,
ASP Conf. Ser. Vol. 180 (1999)

Thank you all for your attention!