Self-Calibration

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Calibration

Flux density calibration

visibilities --> flux density units

primary calibrator (~ once during the observing run)

Gain (bandshape) calibration of antennas

complex (amplitude and phase)

secondary calibrator (a few times during the observing run)

Why is this inadequate ?

approximate image ......

gain varies with time and direction

electronics,

atmosphere (high freq), ionosphere (low freq)

residual phase and amp errors limit the dynamic range

dynamic range (peak / rms, fidelity)

Phase and Amplitude Errors and Dynamic Range

N antennas – N(N-1) / 2 independent visibilities,
In an ideal situation, calibrated visibilities (vectors) add up in phase
Otherwise, there are residual phase (amp) errors

Phase error Φ on every baseline, the final image will be limited
to a dynamic range ~ N / Φ (single 'snapshot')

Amplitude error ε on each baseline, DR ~ N / ε

So, for a phase error ~ 10^0 , dynamic range (peak / rms) for VLA / GMRT is ~ 200
grossly inadequate (bright sources, e.g., DR vs confusion limit)
need dynamic ranges ~ 10,000

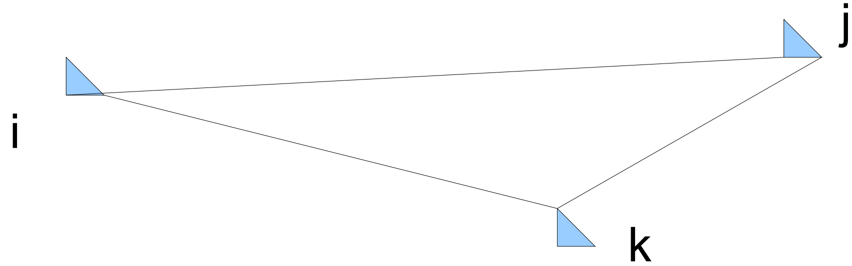
Phase error (Φ) results in asymmetric residuals, with an amplitude ~ Φ

Amplitude error (ε) results in symmetric residuals, with an amplitude ~ ε

(Perley 1985, for more details)

Closure Quantities

An appropriate sum of visibility phases around a closed loop of baselines Is free of antenna-based errors (Jennison 1958)



$$V_{ij,\ obs}(t) = G_i(t)\, G_j^*(t)\, V_{ij,\ true}(t) + \text{noise term}$$

$$\Phi_{ij,\ obs}(t) = \Phi_{ij,\ true}(t) + \Theta_i(t) - \Theta_j(t) + \text{noise term}$$

Antennas i, j, and k

$$C_{ijk,\ obs}(t) = \Phi_{ij,\ obs}(t) + \Phi_{jk,\ obs}(t) + \Phi_{ki,\ obs}(t)$$

$$= \Phi_{ij,\ true}(t) + \Phi_{jk,\ true}(t) + \Phi_{ki,\ true}(t) + \text{noise term}$$

$$= C_{ijk,\ true}(t) + \text{noise term}$$

Closure phases and amplitudes

For an array of N antennas, there are (N(N-1) / 2 – (N-1)) “good” phases
Similarly, there are (N(N-1) / 2 – N) “good” amplitudes

So, for every set of 4 baselines,

$$\Gamma_{ijkl}\ (t) = A_{ij}\,(t)\, A_{kl}\,(t) \,/\, A_{ik}\,(t)\, A_{jl}\,(t)$$

The A's are amplitudes of complex gains G's
The closure amplitude is free of antenna based errors $g_i g_j g_k g_l$

(Cornwell 1985, for more details)

Iterative Schemes

(Readhead & Wilkinson 1978, Cotton 1979, Cornwell & Wilkinson 1981)

Produce CLEAN images consistent with closure quantities

Iterative Scheme

$$V_{ij,\,obs}(t) = G_i(t)\, G_j^*(t)\, V_{ij,\,true}(t) + \varepsilon_{ij}(t)$$

Aim is to obtain a
Model sky, I ---> model visibilities, V_{mod} ---corrupted by gains --> V_{obs}

Constraints on the model sky – positive brightness, confined structures

Minimize the following quantity (subject to closure) :

$$S = \sum \sum \left(V_{ij,\,obs}(t_k) - G_i(t_k)\, G_j^*(t_k)\, V_{mod,\,ij}(t_k) \right)^2$$

First sum is over k, the second sum is over i,j (i ≠ j)

Iterative Scheme (in practice)

Observed calibrated visibilities

FT, dirty image, CLEAN (not too deep !)

Initial Sky Model (approx, could also come from a different freq)

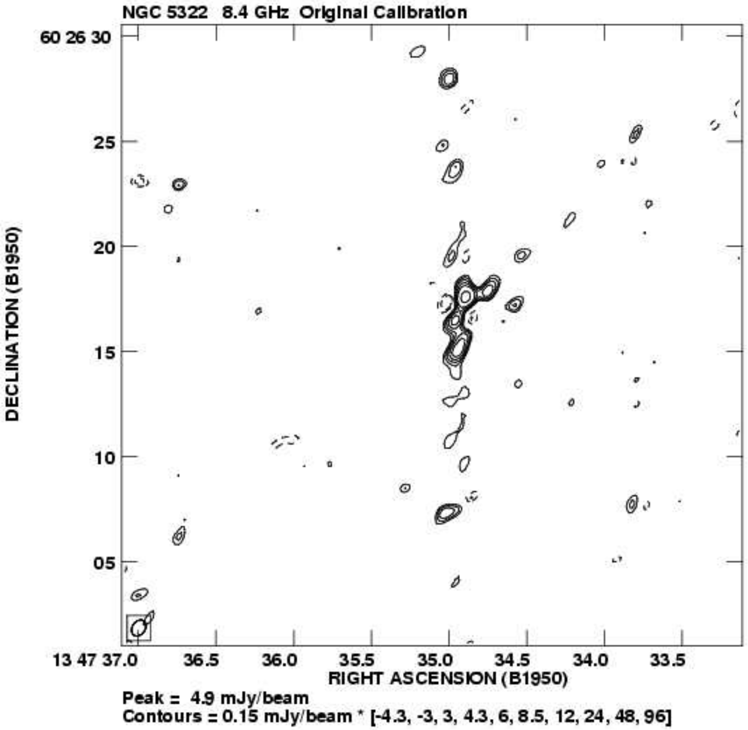
*Estimate new antenna gains at shorter time intervals
with closure amp / phase constraints
and
Minimising the residual between model and observed visibilities*

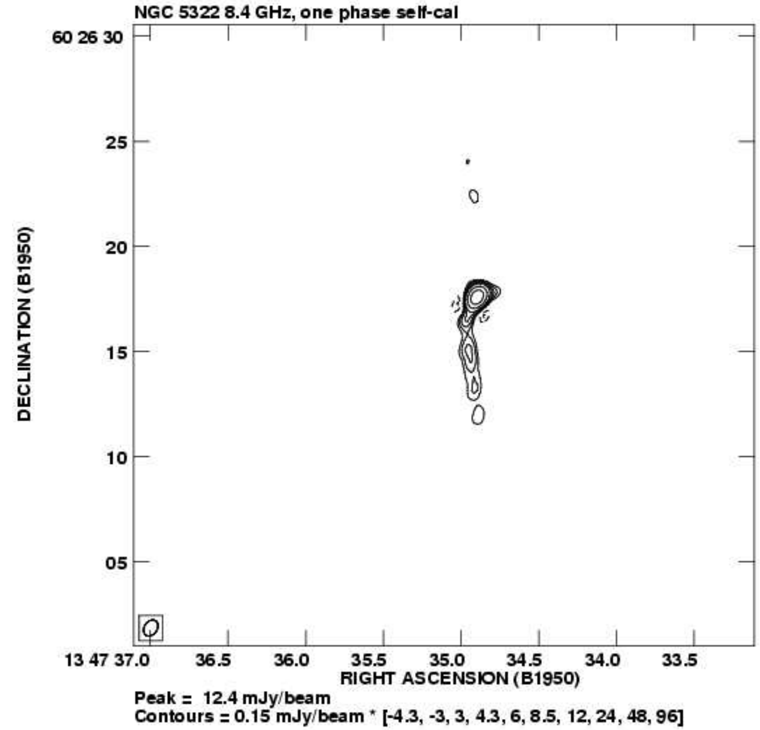
Apply the new gain solutions to obtain modified visibilities

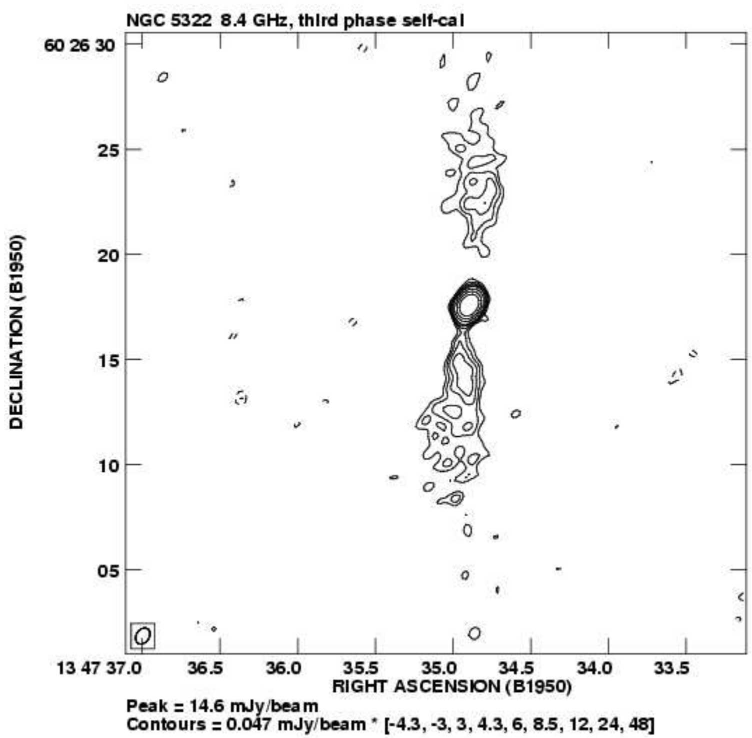
Make another Image

*Stop, if satisfied; otherwise, go back to estimating gains
from this image*

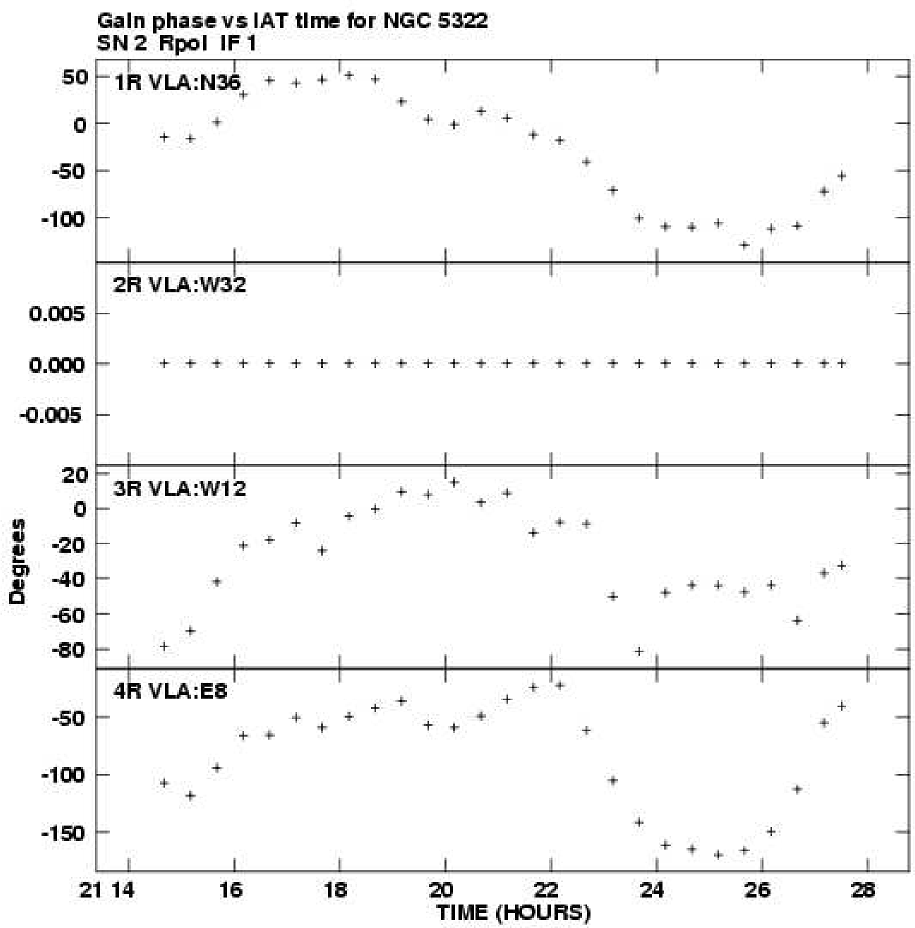
Can perform the self-cal + CLEAN loop until satisfied

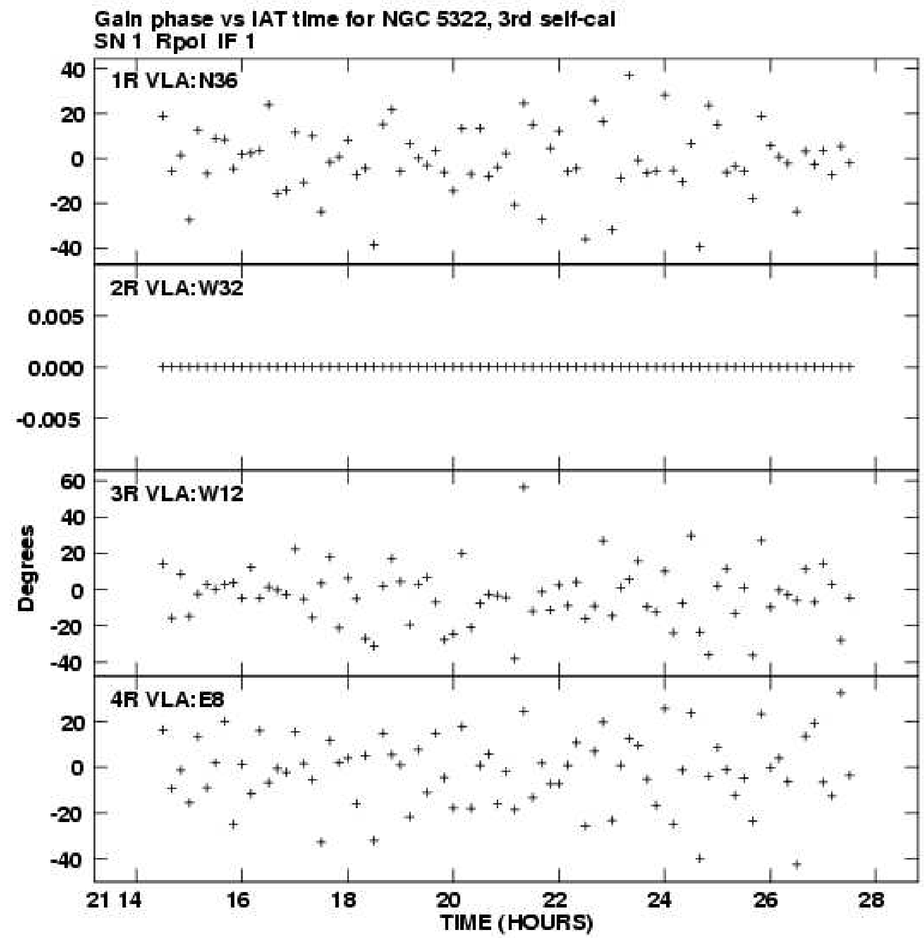






(example taken from Cornwell 2004)


Gain phase vs IAT time for NGC 5322
SN 2 Rpol IF 1
1R VLA:N36
2R VLA:W32
3R VLA:W12
4R VLA:E8
Degrees
TIME (HOURS)


Gain phase vs IAT time for NGC 5322, 3rd self-cal
SN 1 Rpol IF 1
1R VLA:N36
2R VLA:W32
3R VLA:W12
4R VLA:E8
Degrees
TIME (HOURS)

Self-cal works if the amplitude and phase errors are antenna based

*2N unknowns, while there are ~ N**2 knowns*

For large N (like GMRT/VLA) this is a well constrained problem

Many parameters to play with

No of CLEAN comps, type of solution,

Integration time, uv range restriction, phase only, phase &amp

Self-cal will not work if the initial model is totally wrong, includes RFI,

If there are non-isoplanicity conditions, or, baseline based errors

References

Cornwell 1985, NRAO Synthesis Imaging Summer School

Cornwell 2004, NRAO Synthesis Imaging Summer School

Cornwell & Wilkinson 1981, MNRAS, 196, 1067

Cotton 1979, AJ, 84, 1122

Jennison 1958, MNRAS, 118, 276

Perley 1985, NRAO Synthesis Imaging Summer School

Readhead & Wilkinson 1978, ApJ, 223, 25

Consider observing a source of unit amplitude strength at the phase center

$N(N-1) / 2$ complex visibilities are recorded at any given instant of time

Ideal case : $V(u) = \delta(u-u_k)$

Discrepant : $V(u) = \delta(u-u_o)\, e^{-i\varphi}$

Image : $I(l) = \int V(u)\, e^{i2\pi ul}\, du$

Each 'good' baseline : $2\cos(2\pi u_k l)$

'bad' baseline : $2\cos(2\pi u_o l - \varphi) \rightarrow 2[\cos(2\pi u_o l) + \varphi \sin(2\pi u_o l)]$ (small φ)

Image : $I(l) = 2\varphi\sin(2\pi u_o l) + 2\sum \cos(2\pi u_k l)$
(summation over $k = 1$ to $N(N-1) / 2$)

Synth beam is just the second term, leaving (an odd) residual $\sim \varphi$

Hence, $DR \sim N^2 / \varphi$, but if all baselines have random errors, $DR \sim N / \varphi$

Amplitude error, discrepant : $V(u) = (1+\varepsilon)\delta(u-u_o)$, obtain the same result
But, with $\varphi \rightarrow \varepsilon$ and sin --> cos