Sensitivity - in Radio Astronomy

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References

- Synthesis Imaging in Radio Astronomy II - Ed: Taylor, Carilli, Perley
- Low Frequency Radio Astronomy - Ed: Chengalur, Gupta, Dwarakanath
- Interferometry and Synthesis in Radio Astronomy - Thompson, Moran, Swenson
- Lectures by Scott Ransom, Joeri van Leeuwen, Rick Perley.

Why is sensitivity?

- What is the weakest source one can detect in an image.
- A key to write a technically sound observing proposal (e.g. 10 uJy with the uGMRT in 3 hours!!!) and carrying out error analysis for your image.
- The astrophysical signals are incredibly weak. Almost all the power we measure by individual antennas is noise.

Why is sensitivity crucial in Radio Astronomy?

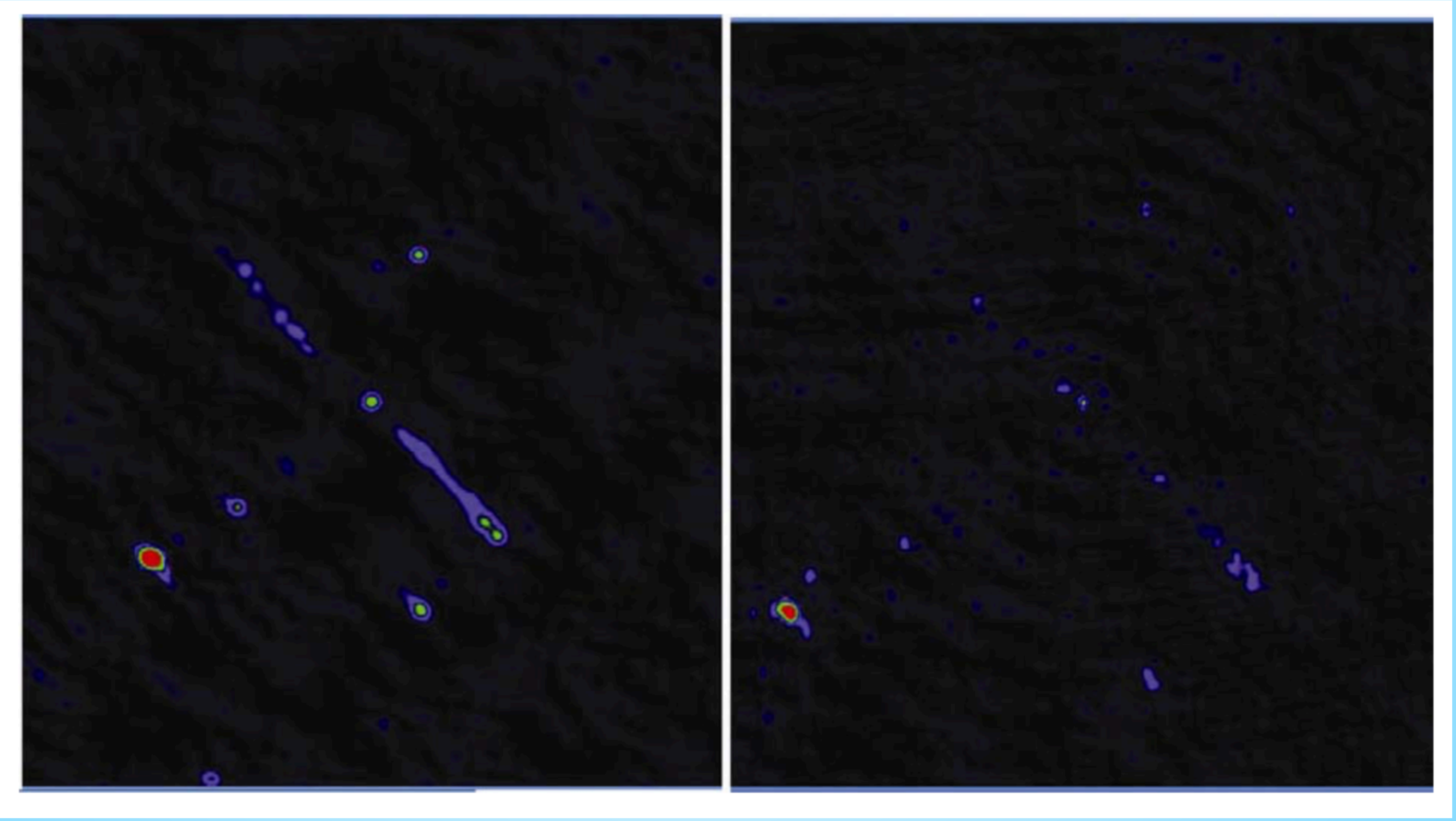
- Because of the low energies involved in radio astronomy.

- Consider a large radio telescope with a total collecting area of 10^4 m^2 pointed toward a radio source of flux density 1 mJy and accepting signals over a bandwidth of 50 MHz. In 1000 years, the total energy accepted is about 10^{-7} J (1 erg), which is comparable to a few percent of the kinetic energy in a single falling snowflake.

Astrophysical Applications of Radio Astronomy

- Discrete cosmic radio sources, at first, supernova remnants and radio galaxies (1948).
- The 21cm line of atomic hydrogen (1951).
- Quasi Stellar Objects "Quasars" (1963).
- The Cosmic Microwave Background (1965).
- Inter stellar molecules and pronto-planetary discs (1968).
- Pulsars (1968); Gravitational lenses (1979);
- The Sunyaev-Zeldovich effect (1983);
- Distance determinations using source proper motions determined from Very Long Baseline Interferometry (1993); and
- Molecules in high-redshift galaxies (2005).

Which image has higher sensitivity



Sensitivity

- Building block of radio telescope
- Antenna sensitivity
- Coupling two antennas - 2-element interferometer sensitivity
- Going to image domain - performance of several 2-element interferometers

Rayleigh-Jeans in Radio wavelengths

- For thermal radiation from a black body, the intensity is related to Physical temperature by Planck's radiation law.

$$I_\nu = \frac{2kT\nu^2}{c^2}\left[\frac{\frac{h\nu}{kT}}{e^{h\nu/kT} - 1}\right]$$

- For $h\nu \ll kT$,

$$I_\nu = \frac{2kT\nu^2}{c^2}$$

- Rayleigh-Jeans approximation

Brightness temp is physical temp only for blackbody

- Rayleigh –Jeans approximation requires ν (GHz) << 20 T (K) and is violated at high frequencies and low temperatures

Rayleigh-Jeans approximation

- Possible because $h\nu/kT << 1$
- E.g. Sun at 1 GHz

$$\frac{h\nu}{kT} = \frac{6.63 \times 10^{-27}\,\mathrm{erg\,s}.10^{9}\,\mathrm{Hz}}{1.38 \times 10^{-16}\mathrm{erg\,K}^{-1}.5800\,\mathrm{K}} \approx 10^{-5}$$

- Rayleigh Jeans law - Brightness . For small angular sized source- flux density

$$S_\nu = \Omega B_\nu$$

Jansky

- For small sources of small angular sizes Ω, flux density S_ν,
- Unit Jansky
- 1 Jansky $\equiv$ 1 Jy
  - $= 10^{-26}$ W m^{-2} Hz^{-1}
  - $= 10^{-23}$ erg sec^{-1} cm^{-2} Hz^{-1}

1 Watt=10^7 erg/s

‘Sensitivity’ for a single antenna

Nyqvist Approximation

- In the radio regime Power per unit bandwidth $P = kT$.
- Rayleigh-Jeans approximation.
- Hence antenna temperature $T_A = P/k$
- T_{sys}=Total Power referred to receiver inputs/k
- Signal to noise $S/N = T_A/T_{sys}$

System temperature

- The system temperature when looking at blank sky is a measure of the total random noise in the system.
- It is desirable to make the system temperature as low as possible.
- Noise from the various sub systems that make up the radio telescope are uncorrelated and hence add up linearly. The system temperature can be very generally written

$$T_{sys} = T_{sky} + T_{spill} + T_{loss} + T_{rec}$$

- T_{sys}: system temperature- contribution from receiver noise, feed losses, spill over, atmospheric emission, Galactic background

System temperature

- T_{sky} is the contribution of the background sky brightness. - the galaxy is a strong emitter of non thermal continum radiation, which at low frequencies usually dominates the system temperature. At all frequencies the sky contributes at least 3K from the cosmic background radiation.

- The feed antenna is supposed to collect the radiation focused by the reflector. Often the feed antenna also picks up stray radiation from the ground (which radiates approximately like a black body at 300 K) around the edge of the reflector. This added noise is called spillover noise, and is a very important contribution to the system temperature at a telescope.

- T_{loss} Any lossy element in the feed path will also contribute noise.

- T_{rec} - receiver noise.

Radio Frequency Interference

- An important (but unwanted) contributor to the system temperature is terrestrial interference (RFI).
- If the bandwidth of the interference is large compared to the spectral resolution, the interference is called broad band. Steady, broad band interference increases the system temperature, and provided this increase is small its effects are relatively benign.
- RFI varies on a very rapid time scale, causing a rapid fluctuation in the system temperature. This is considerably more harmful, since such fluctuations could have harmonics which are mistaken for pulsars etc.
- In aperture synthesis telescopes such time varying effects will also produce artifacts in the resulting image

System temperature for GMRT antennas

- System temperatures T_{sys} for GMRT antennas
- 150 MHz: 615 K
- 235 MHz: 237 K
- 325 MHz: 106 K
- 610 MHz: 102 K
- 1280 MHz: 73 K

$$\sigma = \frac{T_{\rm sys}}{\sqrt{\Delta\nu\tau}}$$ Radiometer Equation

	Frequency (MHz)				
	151	235	325	610	1420
System temperature ($T_{\rm system}$, K)					
Receiver temperature ($T_{\rm R}$)	295[†]	106[†]	53	60	45
Typical $T_{\rm sky}$ (off Galactic plane)	308	99	40	10	4
Typical $T_{\rm ground}$	12	32	13	32	24
$T_{\rm system}$ ($= T_{\rm R} + T_{\rm sky} + T_{\rm ground}$)	615	237	106	102	73

At low frequencies

- Below 300 MHz, Galactic noise can be a significant or dominant contribution to the total noise. This, combined with mutual coupling between antennas, makes it difficult to predict the sensitivity of these instruments (Ellingson et al. 2010).

- The noise temperatures that are much less than the antenna temperatures associated with the background Galactic synchrotron radiation, such that the resulting total system noise temperature is dominated by Galactic noise.

- This is quite different than usually assumed case that internal noise associated with the receivers dominates and does not scatter into the array, so that the noise associated with different antennas is uncorrelated.

Antenna temperature

- The term *antenna temperature* to refer to the component of the power received by the antenna that results from a cosmic source under study. Antenna temperature temp of an ideal resistor that would produce the same Rayleigh-Jeans power per unit freq as the antenna output

- $T_A = P/k$

Single antenna performance

- Power to temperature approximation. Works because in Rayleigh-Jeans approximation of the Planck's Radiation Law valid for long wavelengths.

- Power given out by the source $P = k_{\rm B} T \Delta\nu$

- Power received at the antenna (power entering a feed is amplified by a factor g^2 – g is the voltage gain $P_{\rm a} = g^2 k_{\rm B} T_{\rm a} \Delta\nu$

- There is also a system noise $P_{\rm N} = g^2 k_{\rm B} T_{\rm sys} \Delta\nu$

k_B=1.38x10^{-16} erg/K; 1 Watt=10^7 erg/s

Single Antenna performance

- Antenna power related to flux density *S*, $P_A = \frac{1}{2} S_\nu A \Delta\nu$
- Area of the antenna *A*. *But not whole antennas will collect radiation.*
- Antenna efficiency. η_{a} ($A_{\mathrm{eff}} = \eta_{\mathrm{a}} A$)

$$P_{\mathrm{a}} = \frac{1}{2} g^2 \eta_a A S \Delta\nu \equiv g^2 k_{\mathrm{B}} K S \Delta\nu \qquad K = \frac{\eta_{\mathrm{a}} A}{2 k_{\mathrm{B}}}$$

- The factor 1/2 takes account of the fact that the antenna responds to only one-half the power in the randomly polarized wave. K measure of antenna performance or 'sensitivity'.

Antenna collecting area

- The GMRT antennas are parabolic reflector antennas.
- The collecting area *A* of a reflector antenna, for radiation incident in the center of the main beam, is equal to the geometrical area multiplied by an aperture efficiency factor, which is typically within the range 0.3–0.8.
- Collecting area = geometric area A x Aperture efficiency η_a
- E.g., GMRT antennas (A=45m), η_a ~60% - 40% from lowest frequency 150 MHz to highest frequency 1450 MHz

Antenna performance

- Specify the performance of an antenna in terms of Jy/K, that is, the **flux density (Jy), of a point source that increases T_A by one kelvin**. Sensitivity (or "gain") of an telescope is often measured in K/Jy, and usually changes with observing frequency. This is also called the telescope DPFU (Degree per Flux Unit).

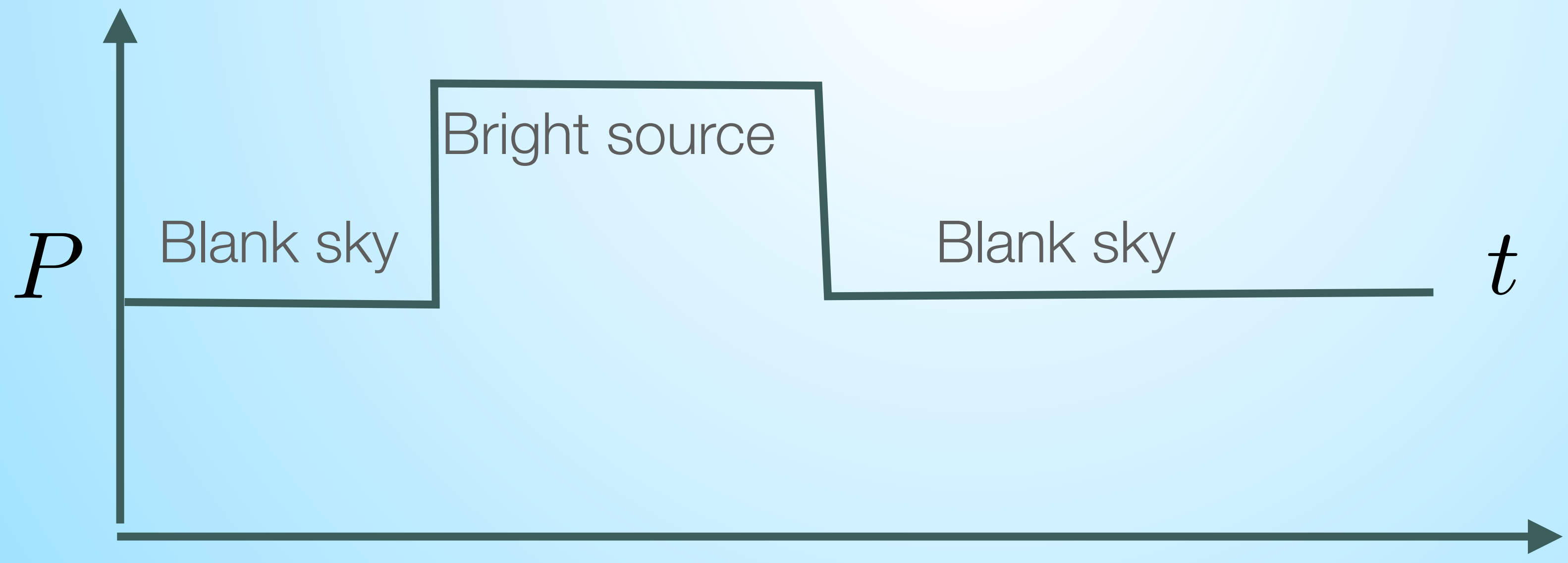
- Antenna gains for GMRT:
- 150 MHz: 0.33 K Jy^{-1} $Antenna^{-1}$
- 235 MHz: 0.33 K Jy^{-1} $Antenna^{-1}$
- 325 MHz: 0.32 K Jy^{-1} $Antenna^{-1}$
- 610 MHz: 0.32 K Jy^{-1} $Antenna^{-1}$
- 1280 MHz: 0.22 K Jy^{-1} $Antenna^{-1}$

System equivalent flux density (SEFD)

- System Equivalent Flux Density (SEFD): T_{sys} in terms of *SEFD*
- $SEFD = T_{sys}/K$
- SEFD takes into account the efficiency, collecting area of the antenna and system noise
- SEFD: Useful measure of the system performance. Ratio of the T_{SYS} to the DPFU with units Jy: $SEFD = T_{SYS}/DPFU$
- *Tsys* and antenna gains are calibration dependent but cancels out for SEFD.
- SEFD measured by determining fractional increase in power by going on and off source of a known flux density.

Measuring the SEFD of a real telescope

- The SEFD of a radio telescope can be measured by switching the telescope between a very bright source (of known flux density, S) and an empty patch of blank sky.



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System equivalent flux density (SEFD)

- It is convenient to express the rms uncertainty in terms of the system equivalent flux density (SEFD; units of Jy).

$$\mathrm{SEFD} = \frac{2kT_{\mathrm{sys}}}{A_e}$$

- What is the SEFD of a 25-m VLA antenna assuming a system temperature of 55 K and an effective area of 365 m^2?

$$\mathrm{SEFD} = \frac{2kT_{\mathrm{sys}}}{A_e} = \frac{2 * 1 \times 10^{-23} * 55}{365} = 300\,\mathrm{Jy}$$

- The SEFD is a good way to compare the sensitivity of telescopes because it takes the receiver system (T_{sys}) and the effective area (A_e) into account.

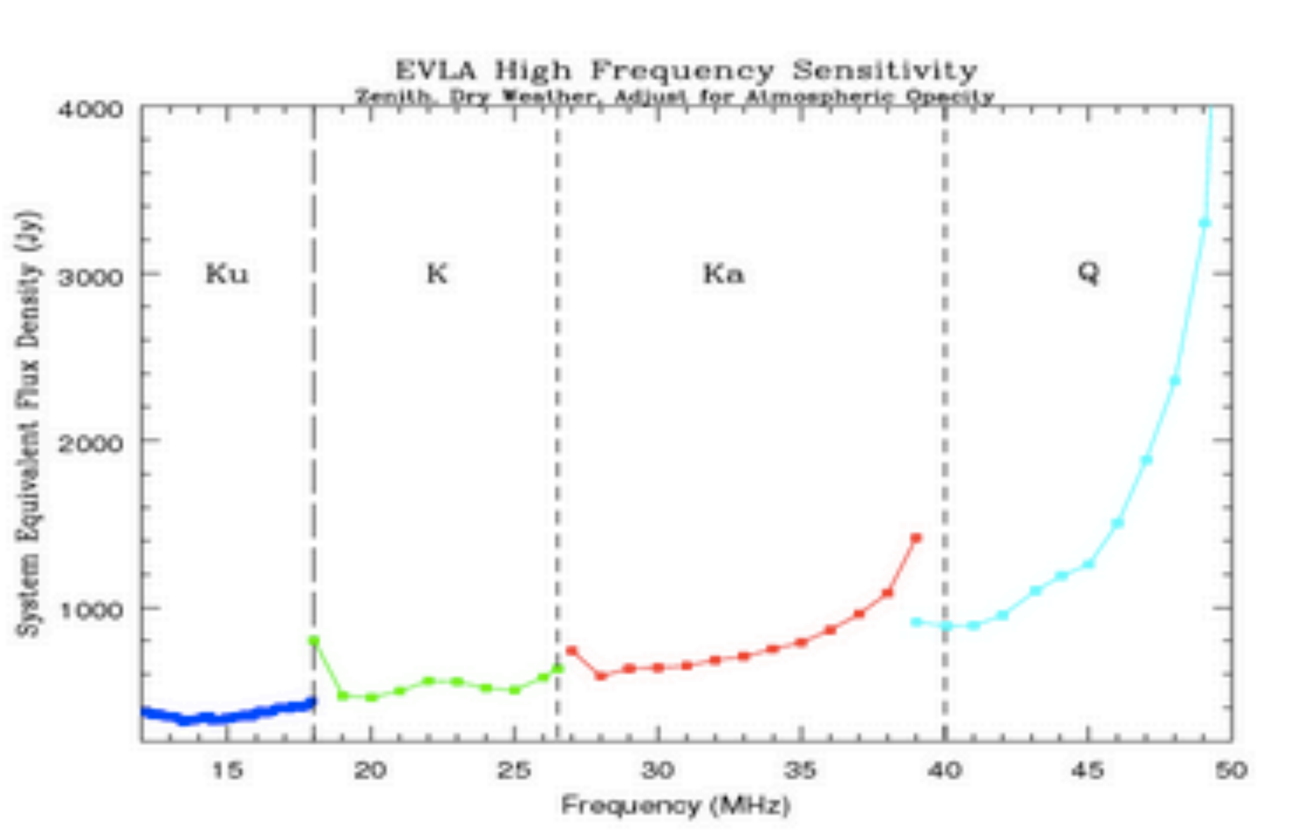
$$\sigma_{S_\nu} = \frac{\mathrm{SEFD}}{\sqrt{\Delta\nu\tau}}$$

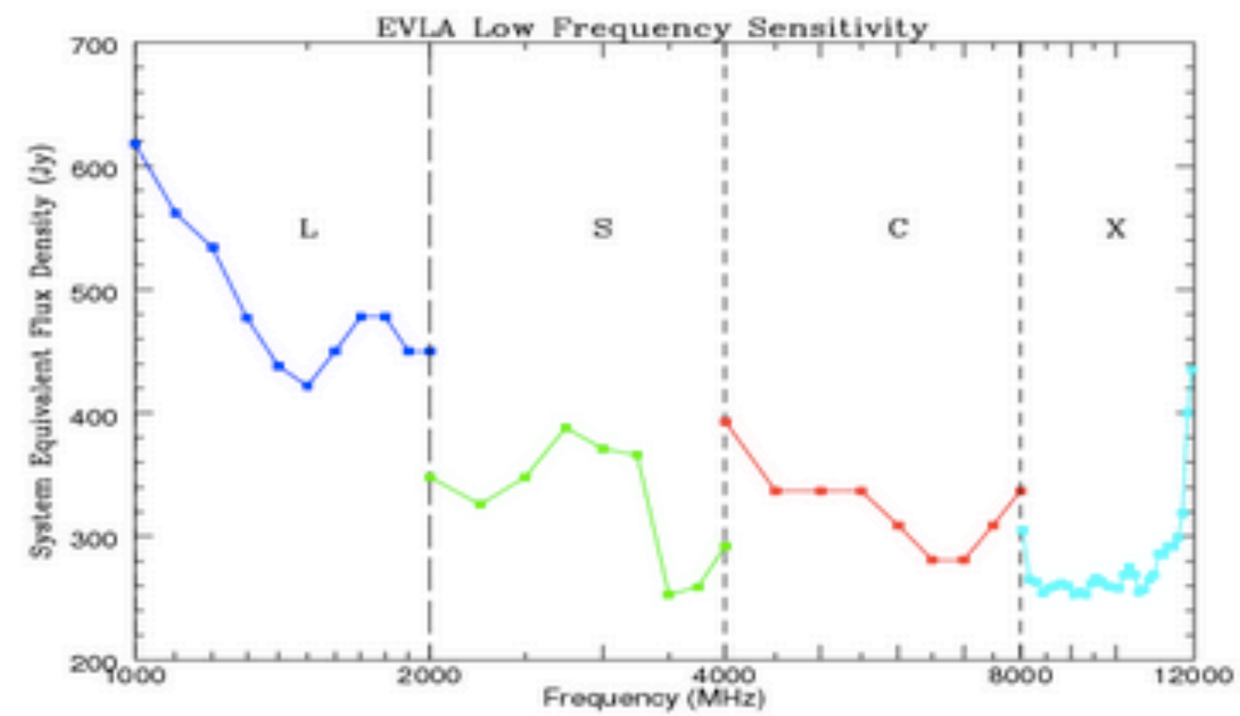
SEFD for GMRT antennas

- SEFD for GMRT:
- 150 MHz: 1864 Jy
- 235 MHz: 718 Jy
- 325 MHz: 331 Jy
- 610 MHz: 319 Jy
- 1280 MHz: 332 Jy

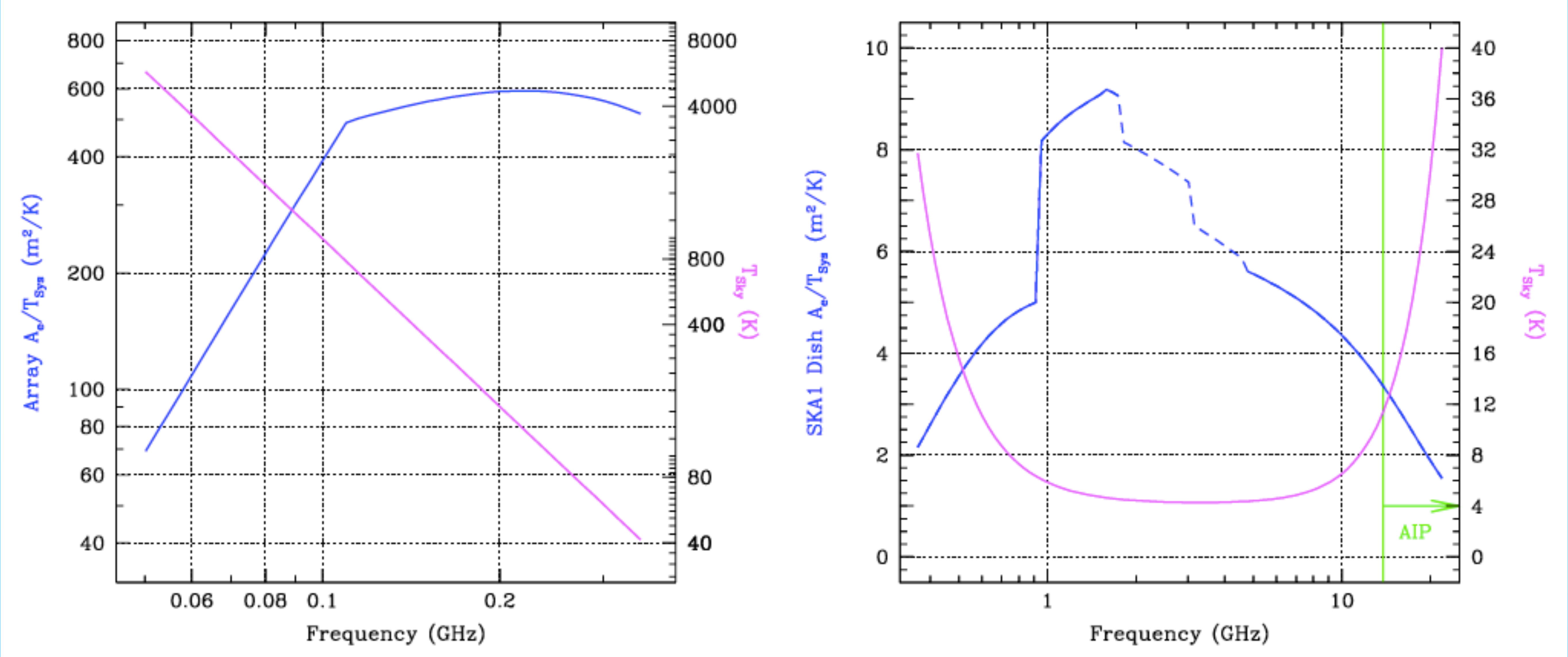
SEFD for VLA antennas

Band	Code	Effective BW	SEFD	σ(cont)	σ(line)
GHz		GHz	Jy	μJy	mJy
1 – 2	L	0.75	400	5.5	2.2
2 – 4	S	1.75	350	3.9	1.7
4 – 8	C	3.5	300	2.4	1.0
8 – 12	X	4	250	1.8	0.65
12 – 18	Ku	6	280	1.7	0.61
18 – 27	K	8	450	2.3	0.77
27 – 40	Ka	8	620	3.2	0.90
40 -- 50	Q	8	1100	5.6	1.4





SKA1 Low and Mid sensitivities



Signal-to-noise

- For **signal**, bigger must be better, because we are collecting radio waves and/or photons.
- For our telescopes, that is the effective area, A_{eff}
- For **noise**, we must want as little as possible We describe noise in radio astronomy in terms of temperture, and in this case, the system temp, T_{sys}
- Radio sensitivity $\propto A_{eff} / T_{sys}$

Signal to noise

- The ratio of the signal power from a source to the noise power in the receiving amplifier is T_A/T_{sys}. Because of the random nature of the signal and noise, measurements of the power levels made at time intervals separated by $(2\Delta\nu)^{-1}$ can be considered independent.

- A measurement in which the signal level is averaged for a time τ contains approximately $2\Delta\nu\tau$ independent samples.

- The signal-to-noise ratio at the output of a power-measuring device attached to the receiver is increased in proportion to the square root of the number of independent samples and is of the form

- Typical values of $\Delta\nu$ and τ are of order 1GHz and 6h, which result in a value of 4×10^6 or the factor $(\Delta\nu\tau)^{1/2}$. As a result, it is possible to detect a signal for which the power level is less than 10^{-6} times the system noise.

Sensitivity of an antenna

- The signal from an antenna is first passed through an amplifier. The amplifier is characterized by its power gain factor, *G*; receiver temperature; and the bandwidth $\Delta\nu$. The gain factor is assumed to be constant. If the gain is sufficiently high, this amplifier sets the noise performance of the entire system, which we denote as T_{sys} to include the contributions from atmosphere, ground pickup, and ohmic losses.

- We assume that the passband has a rectangular shape that is flat between a lower cutoff frequency ν_0 and upper cut off frequency $\nu_0+\Delta\nu$.

- The signal is converted to a digital data stream sampled at the Nyquist rate. According to the Nyquist sampling theorem, a band limited signal can be represented by samples taken at intervals of $(2\Delta\nu)^{-1}$.

- We assume there is no quantization error in this sampling process. In this case, the original signal can be exactly reconstructed from the sampled sequence by convolution with a sinc function. The sampled signal has the same statistical properties as the corresponding analog signal.

Sensitivity equation

$$\mathrm{snr} = \frac{\mathrm{GS}\sqrt{\Delta\nu\tau}}{\mathrm{T_{sys}}}$$

- The output of a total power detector attached to a radio telescope too will show random fluctuations. Supposing a telescope with system temperature T_{sys}, power gain G, and bandwidth Δv is used. For source S, increase in system temperature GS

- Ideally look at a 'blank' part of the sky, and then switch to a region containing the source. Clearly if the received power increases, then one has detected radio waves from this source .

- Even on a blank region of sky is fluctuating, one needs to be sure that the increase in antenna temperature is not a random fluctuation but is indeed due to the astrophysical source? In order to make this decision, one needs to know what the rms is in the fluctuations.

- For a total power detector with instantaneous rms T_{sys}, the rms after integrating a signal of bandwidth Δv Hz for τ seconds is $T_{sys}/\sqrt{\Delta\nu\tau}$. The signal to noise ratio is **snr=(GS √ Δvτ)/ $\mathbf{T_{sys}}$** - the fundamental equation for the sensitivity of a single dish telescope (thermal noise).

Radio Telescope Sensitivity in T_{sys}

- Defined as flux density of a source whose output increment equals the noise.

$$\sigma_p = K \frac{2kT_{\rm sys}}{\eta_a A \sqrt{\Delta\nu\tau}}$$

- K is a system efficiency factor, defined w.r.t. a perfect analog system.
- k is Boltzmann's constant (1.38×10^{-23} Watt/Hz/K)
- T_{sys} is a measure of the noise produced by the system (K)
- A is the physical collecting area of the 'sensor' (antenna).
- η_a is the aperture efficiency of the antenna.
- $\Delta\nu$ is the bandwidth (Hz)
- Δt is the integration time (sec).

‘Sensitivity’ for a two-element interferometer

2-element interferometer

- The ***point-source sensitivity*** of a two-element interferometer can be derived from the radiometer equation for a total-power receiver on a single antenna for two identical elements.

- Thus the effective collecting area A of the two-element interferometer equals the effective collecting area of each element. The noise voltages from the two interferometer elements are almost completely uncorrelated, while the noise voltages going into the square-law detectors are completely correlated (identical).

- In the limit where the antenna temperature ΔT contributed by the point source is much smaller than the system noise T_{sys}, the correlator output noise is $\sqrt{2}$ lower than the square-law detector noise from each antenna.

2-element interferometer

- An aperture synthesis telescope can be regarded as a collection of two element interferometers. An interferometer composed of two antennas i, j, looking at a point source of flux density S (the point source is at the phase center and hence that in the absence of noise the visibility phase is zero)

- Antenna g^2 and system temperatures be $T_{sys,i}$ and $T_{sys,j}$. If $n_i(t)$ and $n_j(t)$ are the noise voltages, then $\sigma_i^2 = <n_i^2(t)> = T_{sys,i}$, and $\sigma_j^2 = <n_j^2(t)> = T_{sys,j}$.

- If $v_i(t)$ and v_j (t) are the voltages induced by the incoming radiation from the point source, $< v_i^2(t)>=< v_2^2(t)>=GS$

2-element interferometer

- An instantaneous correlator output is given by
- $r_{ij}(t)=(v_i(t)+n_i(t))(v_j(t)+n_j(t))$
- The mean of the correlator output is hence
- $<r_{ij}(t)>=<(v_i(t)+n_i(t))(v_j(t)+n_j(t))>=<v_i(t)(v_j(t)>=GS$
- Here we have assumed that the noise voltages of the two antennas are not correlated, and also of course that the signal voltages are not correlated with the noise voltages.
- To determine the noise in the correlator output, we would need to compute the rms of $r_{ij}(t)$,

2-element interferometer

- $<r_{ij}(t)r_{ij}(t)>=<(v_i(t)+n_i(t))(v_j(t)+n_j(t))(v_i(t)+n_i(t))(v_j(t)+n_j(t))>$
- If we assume that all the random processes involved are Gaussian processes the complexity is considerably reduced because for Gaussian random variables
- $<r_{ij}(t)r_{ij}(t)>=3(GS)^2+(\sigma_i^2+ \sigma_j^2)GS+\sigma_i^2\sigma_j^2 = (GS)^2+(GS+T_{sys,i})(GS+T_{sys,j})$
- The variance of $r_{ij}(t)$ is $<\sigma_{ij}^2>= <r_{ij}(t)r_{ij}(t)>-<r_{ij}(t)>^2=(GS)^2+(GS+T_{sys,i})(GS+T_{sys,j})$

2-element interferometer

- Hence for $\Delta\nu$ and τ, average variance $\sigma_{ij}^2=\{(GS)^2+(GS+T_{sys,i})(GS+T_{sys,j})\}/2\Delta\nu\,\tau$

$$snr = \frac{(\sqrt{2T\Delta\nu}GS)}{\sqrt{T_{s_i}T_{s_j}}}$$

- Or $SNR=(\sqrt{2\Delta\nu\,\tau})GS/\sqrt{\{(GS)^2+(GS+T_{sys,i})(GS+T_{sys,j})\}}$
- For weak source Tsys>>TA; $SNR=(\sqrt{2\Delta\nu\,\tau})GS/\sqrt{T_{sys,i}T_{sys,j}}$
- For strong source Tsys<<TA; $SNR=(\sqrt{2\Delta\nu\,\tau})GS/\sqrt{2(GS)^2}=\sqrt{\Delta\nu\,\tau}$
- It can be shown that for an n-element array with same Tsys;
- $SNR=\sqrt{(N(N-1)\Delta\nu\,\tau)}GS/T_{sys}$

$$snr = \frac{\sqrt{N(N-1)T\Delta\nu}\ GS}{T_s}$$

2-element interferometer in T_{sys}

- For an unpolarized point source of flux-density S, $k\Delta T = SA_{eff}/2$

- So for a single antenna

$$\sigma_s = \frac{2kT_{\rm sys}}{A_{\rm eff}(\Delta\nu_{\rm RF}\tau)^{1/2}}$$

- For a two element interferometer

$$\sigma_s = \frac{2^{1/2}kT_{\rm sys}}{A_{\rm eff}(\Delta\nu_{\rm RF}\tau)^{1/2}}$$

- The point-source sensitivity of a two-element interferometer is therefore $2^{1/2}$ better than the sensitivity of each antenna, but $2^{1/2}$ worse than that of a single dish whose area is that of two antennas.

For multiple array

- An interferometer with N dishes contains N(N-1) independent two-element interferometers, so its point-source sensitivity is

$$\sigma_s = \frac{2kT_{\rm sys}}{A_{\rm eff}[N(N-1)\Delta\nu_{\rm RF}\tau]^{1/2}}$$

- In the limit of large N, N(N-1)—>N^2, and the point-source sensitivity of an interferometer approaches that of a single antenna whose area equals the total effective area NA_{eff} of N interferometer antennas.

- Practical interferometers are slightly less sensitive than this because their correlators use digital multipliers that sample and quantize the input voltage, not perfect analog multipliers. For example, a multiplier that samples at twice the Nyquist rate with three quantization levels (-1,0,+1) is only 0.89 times as sensitive as a perfect multiplier.

Sensitivity of two element interferometer in SEFD

- Presume two identical antennas, whose signals are correlated (multiplied).

- The rms fluctuations of the product (single correlation):

$$\sigma_p = \frac{SEFD}{\eta_c \sqrt{2\Delta\nu\tau}} \, \mathrm{Jy}$$

- Here, SEFD is for each antenna, and η_c is the 'correlator efficiency', again defined w.r.t. the perfect analog multiplier. For the JVLA, η_c~0.95.

- If we have N antennas, we have N(N-1)/2 unique baselines, so we get (N >> 1, assuming all antennas are identical)

- $$\sigma_p = \frac{SEFD}{\eta_c N \sqrt{\Delta\nu\tau}} \, \mathrm{Jy}$$

Sensitivity of an image

Dynamic Range vs Image Fidelity

- Dynamic Range is usually defined as: DR = Image Peak/Image Noise
  - This makes sense for a field comprised of unresolved objects.
- For highly extended objects, we are more focused on the 'Fidelity'.
  - This a related, but different metric.
- Image Fidelity can be limited by a number of effects:
  - Gaps in the spatial frequency ('u-v') coverage
  - Errors or insufficiency in the deconvolution/selfcalibration process
  - Errors in the instrumental response and in calibration
- Whereas, Dynamic Range is much less strongly affected by the first two of these.
- Dynamic Range is primarily a diagnostic of instrumental performance and calibration methodology

Image sensitivity

- uGMRT, ALMA and JVLA are extremely sensitive but sensitivity may not be the only thing you need for your science!
- Image quality depends on UV coverage + density
- Image Fidelity is best when UV coverage is well matched to the source brightness distribution
- Poor UV coverage à reduced DYNAMIC RANGE and lower sensitivity
- Sensitivity varies with distance from the centre of the beam.

Image sensitivity

- In the image plane, i.e. after Fourier transformation of the visibilities.

- Most of the astronomical analysis and interpretation will be based on the image, it is the statistics in the image plane that is usually of interest. The intensity at some point (l, m) in the image plane is given by:

$$I(l, m) = \frac{1}{\mathrm{M}} \sum_p w_p \mathcal{V}_p e^{-i2\pi(lu_p + mv_p)}$$

- Here w_p is is the weight given to the pth visibility measurement V_p, and there are a total of M independent measurements.

Image sensitivity

$$I(l,m) = \frac{1}{\mathrm{M}} \sum_p w_p \mathcal{V}_p e^{-i2\pi(lu_p + mv_p)}$$

- The cross-correlation function in the image plane, <I(l, m)I(l',m')> is hence:

$$\left\langle I(l,m) I(l^{'},m^{'}) \right\rangle = \frac{1}{\mathrm{M}^2} \sum_p \sum_q w_p w_q \left\langle \mathcal{V}_p \mathcal{V}_q^* \right\rangle e^{-i2\pi(lu_p + mv_p)} e^{i2\pi(l^{'}u_q + m^{'}v_q)}$$

- In the absence of any sources, the visibilities are uncorrelated with one another, and hence, we have

$$\left\langle I(l,m) I(l^{'},m^{'}) \right\rangle = \frac{1}{\mathrm{M}^2} \sum_m w_p^2 \sigma_p^2 e^{-i2\pi((l-l^{'})u_p + (m-m^{'})v_p)}$$

Image sensitivity

$$I(l,m) = \frac{1}{\mathrm{M}} \sum_p w_p \mathcal{V}_p e^{-i2\pi(lu_p + mv_p)}$$

- In the case that all the noise on each measurement is the same, and that the weights given to each visibility point is also the same, (i.e. uniform tapering), the correlation in the map plane has exactly the same shape as the dirty beam. Further the variance in image plane would then be $\sigma_{\mathcal{V}}^2/\mathrm{M}$, where $\sigma_{\mathcal{V}}^2$ is the noise on a single visibility measurement.

Image sensitivity

- Sensitivity of a single polarization image formed by N identical antennas

$$\Delta I_{\rm m} = \frac{1}{\eta_{\rm s}} \frac{SEFD}{\sqrt{N\,(N-1)\,\Delta\nu\,t_{\rm int}}}.$$

- If simultaneous dual polarization observations, then sensitivity of an image of Stokes parameters I, Q, U and V will obey Gaussian statistics i.e.

$$\Delta I = \Delta Q = \Delta U = \Delta V = \frac{\Delta I_{\rm m}}{\sqrt{2}}.$$

Image Sensitivity

- The sensitivity and synthesized point spread function (PSF) quality at a specified angular resolution are determined
  - by the total system equivalent flux density (SEFD),
  - the array configuration,
  - the duration of source tracking,
  - the fractional bandwidth being sampled as well as the method of visibility data weighting employed in imaging.
- These quantities will vary with the central observing frequency.

Factors degrading image sensitivity

- Affected by fringe fitting and self calibration
- Errors in determining antenna calibration parameters will introduce errors in visibility data
- For an unresolved source, self-cal error $\sqrt{(N-1)/(N-3)}$
- Natural weighting and no tapering results in highest sensitivity but undesirable many times.

Factors degrading image sensitivity

- For the more commonly-used "robust" weighting scheme, intermediate between pure natural and pure uniform weightings, sensitivity a factor of about 1.2 worse.

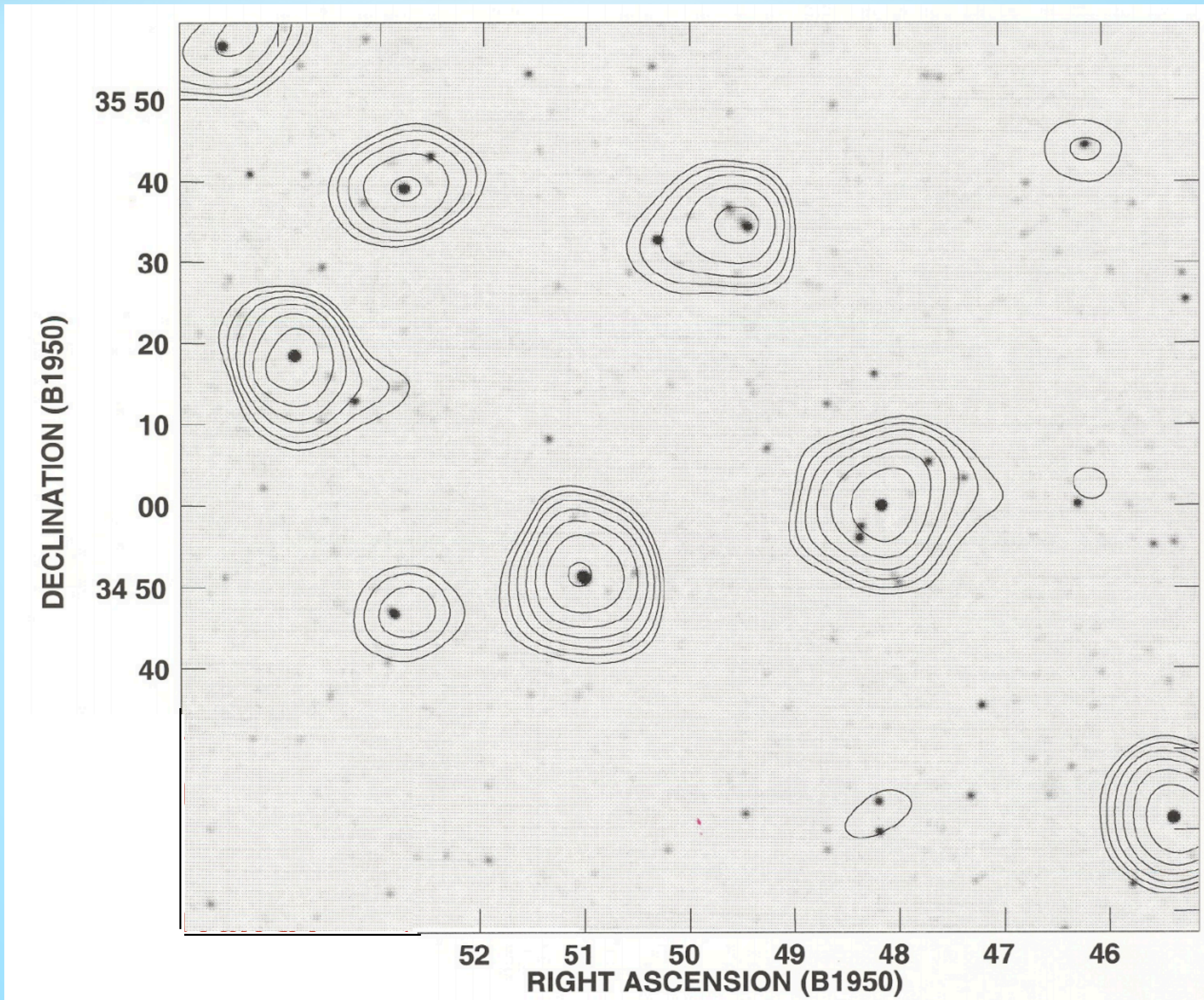
- Weather. The sky and ground temperature contributions to the total system temperature increase with decreasing elevation. This effect is very strong at high frequencies.

Confusion limit (affects sensitivity)

- There are two types of confusion:
- (i) Due to confusing sources within the synthesized beam, which affects low resolution observations the most. E.g. D configuration in VLA. Confusion noise should be added in quadrature to the thermal noise in estimating expected sensitivities.
- (ii) confusion from the sidelobes of uncleaned sources lying outside the image, often from sources in the sidelobes of the primary beam. This primarily affects low frequency observations.

Confusion

NVSS (45") vs
GBT (12')
Credit: Scott Ransom



uGMRT sensitivity parameters

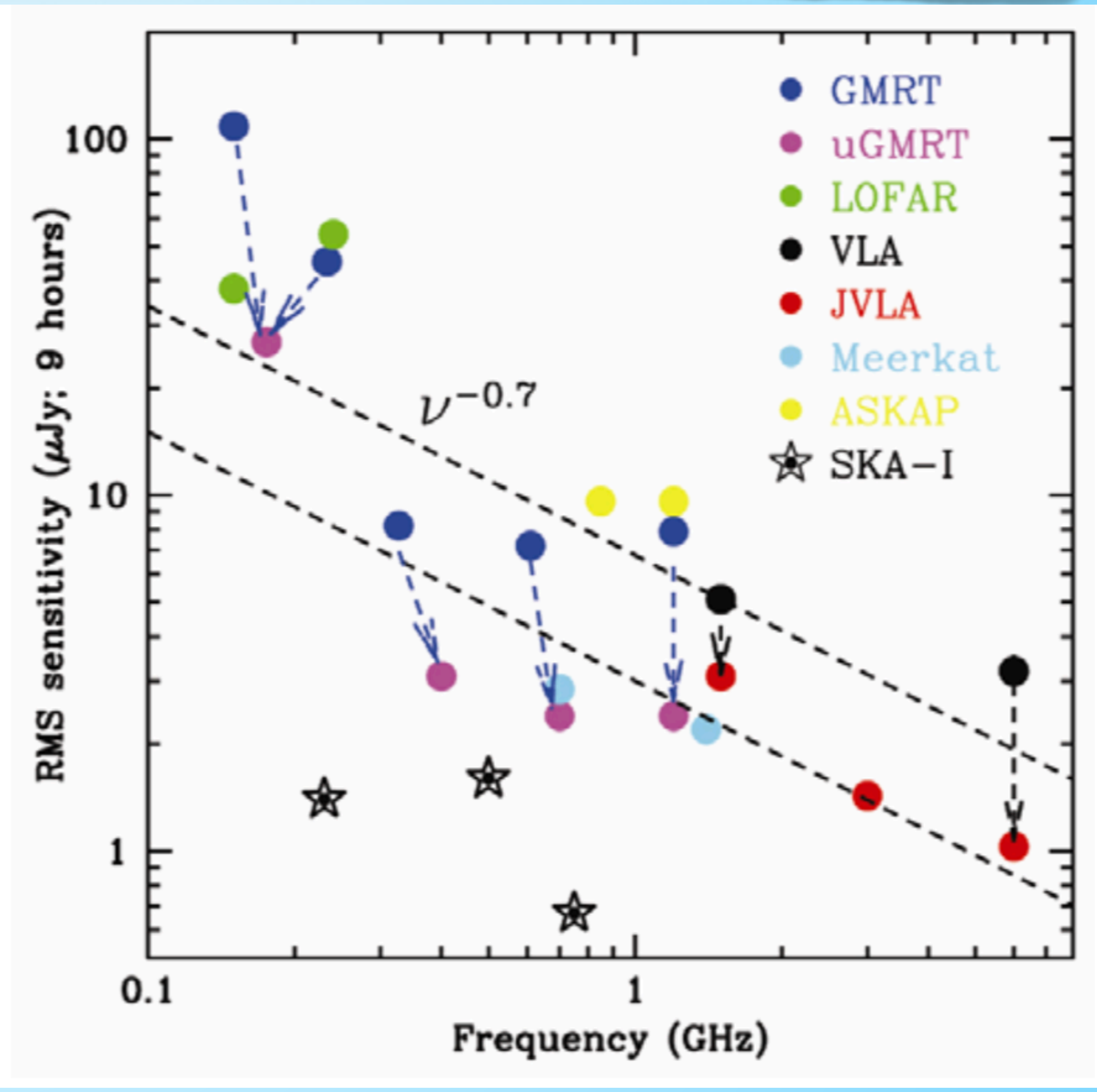
Table 1. System parameters of the uGMRT

	Frequency (f, MHz)			
	Band 2	Band 3	Band 4	Band 5
Frequency range (MHz)	120–250	250–500	550–850	1050–1450
Total system temperature (K)	760–200	165–100	105–100	80–75
Primary beam (arc min)	152*(185/f)	70*(375/f)	37*(700/f)	27*(1250/f)
Antenna gain (K/Jy/antenna)	0.33	0.38	0.35	0.28–0.22
Synthesized beam (arcsec)				
Whole array	17*(185/f)	8*(375/f)	4*(700/f)	2*(1250/f)
Central square	343*(185/f)	174*(375/f)	87*(700/f)	45*(1400/f)
Sensitivity				
RMS noise in image (μJy)*,**	190	50	40	45
Incoherent array sensitivity (μJy)*,***,****	1500	300	220	200

*For 30 antennas and average T_{sys} over the band.
**For 10 min integration and 100 MHz bandwidth.
***For 10 min integration and full bandwidth.
****Minimum detectable pulsar flux for long-period pulsars (5σ).

uGMRT sensitivity

- Comparison between the continuum sensitivities of existing and upcoming radio interferometers, for a 9 h on-source integration. The points show the sensitivities of GMRT, VLA, JVLA, uGMRT, LOFAR, MeerKAT, ASKAP and SKA-1-Mid . As can be seen, uGMRT will be the most sensitive interferometer in the world at frequencies 250–1500 MHz until the advent of Phase-1 of the SKA.



uGMRT sensitivity calculator

Users are advised to run the ETC on the Firefox or Chrome browsers.
Problems have been noticed in some versions of Safari.

1	Observation Type	?	Continuum
2	Observing Band	?	Band-2 (125-250 MHz)
3	Representative Frequency	?	200 MHz
4	Number of antennas	?	26
5	Bandwidth	?	200 MHz
6	Usable Bandwidth	?	50 MHz
7	Number of Polarizations	?	2
8	Image weighting	?	Natural
9	Source co-ordinates(J2000)	?	RA 00h 00m 00.(Dec 00d 00' 00.(
10	Sky temperature (T_sky, K)	?	0 auto calculate
11	Calculation Type	?	On-Source Time
12	RMS noise	?	100 μJy/Bm
13	On-source Time	?	00h 00m 00s
14	Fudge Factor	?	1
15	Overheads	?	00h 00m 00s auto calculate
16	Extra Bandpass/Polarization Time	?	00h 00m 00s
17	Total Time (13*14+15+16)	?	00h 00m 00s
18	Confusion Limit (σ_c*)	?	μJy/Bm

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