Nonlinear electrodynamics: a model of inflationary universe

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Presentation Outline

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Motivation

- Accelerated expansion of the universe can be observed in the absence of dark energy and modified gravity.
- Nonlinear electrodynamics coupled to general relativity is investigated.

Presentation Outline

Few basics of Cosmology

The homogeneous and isotropic universe is described by Friedmann- Robertson -Walker(FRW) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

Where $a(t)$ is the scale factor, $k = 0, -1, +1$ for flat, open and closed universe. The Friedmann and Raychaudhury equations are,

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} \quad \text{and} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (3p + \rho)$$

Here $H \left(= \frac{\dot{a}}{a}\right)$ is the hubble constant, G the Newton's constant, ρ and p being the density and pressure of the matter-energy.
Assume that the universe is filled up with perfect fluid of density ρ and pressure p, its energy-momentum tensor $T^{\mu\nu}$ can be written as

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu}$$

where $u^\mu (= \frac{dx^\mu}{d\tau})$ is the fluid world velocity, τ being the proper time.

Few basics of Cosmology

The covariant conservation of $T^{\mu\nu}$ gives the fluid equation of continuity

$$\nabla_\mu T^{\mu\nu} = 0 \rightarrow \frac{\partial \rho}{\partial t} + 3H(\rho + p) = 0$$

Using $p = \omega\rho$ (fluid equation of state) and solving the equation of continuity, we find the scale factor $a(t)$

$$a(t) \propto \begin{cases} t^{\frac{2}{3(1+\omega)}} & , \omega \neq -1 \quad \text{(radiation(RD) and matter(MD))} \\ e^{Ht} = e^{\sqrt{\frac{\Lambda}{3}}t} & , \omega = -1 \quad \text{(cosmological constant}(\Lambda)) \end{cases}$$

Table: FRW solutions for the radiation and matter dominated universe($k = 0$)

	ω	$\rho(a)$	$a(t)$	$H(t)$	$H(a)$	$\rho(t)$	$p(t)$
MD	0	a^{-3}	$t^{2/3}$	$\frac{2}{3t}$	$\frac{2}{3a^{3/2}}$	t^{-2}	0
RD	$\frac{1}{3}$	a^{-4}	$t^{1/2}$	$\frac{1}{2t}$	$\frac{1}{2a^2}$	t^{-2}	t^{-2}

Inflationary Universe: Λ as the cosmological fluid

The universe, isotropic and homogeneous at large scale, requires exponential accelerated expansion (de-Sitter expansion) $\rightarrow$ Inflation.
The Raychaudhuri eqn:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(3p + \rho)$$

Condition for Inflation $\ddot{a}(t) > 0$ which requires $\rho + 3p < 0$ (violation of energy condition).
The Friedmann equation with the cosmological constant(Λ) is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad \rightarrow \quad H^2 = \left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{\Lambda}{3}$$

as the first and second terms fall rapidly as $\frac{1}{a^4}(\frac{1}{a^3})$ (for radiation(matter) dominated universe) and $1/a^2$ at late time. Solving we find

$$a(t) = \exp\left(\sqrt{\frac{\Lambda}{3}}\, t\right) \rightarrow \mathrm{de-Sitter\ expansion}$$

Inflation and the solution of horizon and flatness problem

Say, the cosmological constant(Λ) as a kind of cosmological fluid of energy density ρ_Λ and pressure p_Λ

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \text{constant} \to \dot{\rho}_\Lambda = 0$$

The fluid equation of continuity for Λ is,

$$\dot{\rho_\Lambda} + 3H(\rho_\Lambda + p_\Lambda) = 0 \quad \to \quad p_\Lambda = -\rho_\Lambda < 0$$

$\to$ So, Λ gives rise negative pressure $\to$ Λ a kind of dark energy !

Solving Flatness Problem

For the exponential expansion $a(t) = \exp\left(\sqrt{\frac{\Lambda}{3}}\ t\right)$ we find

$$\mid \Omega - 1 \mid = \frac{k}{a^2 H^2} = \frac{3k}{\Lambda} exp\left(-\sqrt{\frac{4\Lambda}{3}}\ t\right)$$

where $\Omega = \frac{\rho}{\rho_c}$ and $\rho_c = \frac{3H^2}{8\pi G}$ (the critical density). This makes $\Omega \to 1$ irrespective of all subsequent expansion of the universe $\to$ thus the inflation solves the flatness problem !

Presentation Outline

Nonlinear Electrodynamics coupled to gravity

• In early universe, the strong and highly nonlinear electromagnetic field acts as the main source of gravity!
• NLE field can mimic DE near the Planck scale which can drive inflation in early universe.
The (non-minimal) action of nonlinear electromagnetic field coupled with gravity is given by

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L} + \frac{1}{\kappa^2}R\right)$$

where $\kappa^2 = 8\pi G$ and the lagrangian density ($\mathcal{L}$) of nonlinear electrodynamics (NLED) (Kruglov 2017)

$$\mathcal{L} = -\frac{\mathcal{F}}{(\beta\mathcal{F}+1)^2} \simeq -\mathcal{F} + 2\beta\mathcal{F}^2 + \mathcal{O}(\beta^2) + ...$$

Where $\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B^2 - E^2)$ and $\beta\mathcal{F}$ is dimensionless.
The Einstein's equation and electromagnetic field equatios

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}, \quad \partial_\mu\left(\frac{\sqrt{-g}F^{\mu\nu}(\beta\mathcal{F}-1)}{(\beta\mathcal{F}+1)^3}\right) = 0$$

$T_{\mu\nu}$ in NLED theory coupled to gravity

The energy-momentum tensor $T_{\mu\nu}$ can be obtained as

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} = -g_{\mu\nu}\mathcal{L} + \frac{(\beta\mathcal{F} - 1)}{(1 + \beta\mathcal{F})^3} \; F_{\mu\alpha}F_{\nu}{}^{\alpha}$$

The energy-momentum conservation gives

$$\nabla^{\mu} T_{\mu\nu} = 0 \to \dot{\rho} + 3H(\rho + 3p) = 0$$

The energy density(ρ) and pressure(p) of the electromagnetic field is obtained as

$$\rho = \frac{(1 - \beta\mathcal{F})E^2}{(1 + \beta\mathcal{F})} + \frac{\mathcal{F}}{(1 + \beta\mathcal{F})^2}, \quad p = -\frac{\mathcal{F}}{(\beta\mathcal{F} + 1)^2} + \frac{(E^2 - 2B^2)(\beta\mathcal{F} - 1)}{3(\beta\mathcal{F} + 1)^3}$$

For the magnetic universe(where $B = B(t)$ and $\langle B \rangle = 0$ (the universe is isotropic) and $\vec{E} = 0$ as the average electric field E is screened by the charged primordial plasma) consisting of fluid of density $\rho_B(t)$ and pressure $p_B(t)$ are given by

$$\rho_B(t) = \frac{2B^2}{(2 + \beta B^2)^2}, \quad p_B(t) = -\frac{2B^2}{(2 + \beta B^2)^2} - \frac{8}{3}\frac{B^2(\beta B^2 - 2)}{(2 + \beta B^2)^3}$$

Einstein equations in NLED theory

The Friedmann equation and Raychaudhuri equation for the universe filled up with magnetic fluid are given by

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_B = \frac{8\pi G}{3}\frac{2B^2}{(2+\beta B^2)^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(3p_B + \rho_B) = -\frac{4\pi G}{3}\frac{4B^2(2-3\beta B^2)}{(2+\beta B^2)^3}$$

Solving the equation of continuity

$$\frac{d\rho_B}{dt} + 3\frac{\dot{a}}{a}\left(\rho_B + p_B\right) = 0$$

we find

$$\frac{dB(t)}{dt} = -2B(t)\frac{1}{a(t)}\frac{da(t)}{dt}$$

Integrating this equation between $t = t_0$ to $t = t$, we find

$$B(t) = B(t_0)\frac{a^2(t_0)}{a^2(t)} \rightarrow B(t)a^2(t) = B(t_0)a^2(t_0) \text{ (Magnetic flux conservation)}$$

Inflation in NLED theory coupled to gravity

The deceleration parameter q can be defined as (using the Friedmann and Raychaudhuri equations)

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\rho_B + 3p_B}{2\rho_B} = \frac{2 - 3\beta B^2}{2 + \beta B^2} \Longrightarrow B^2 = \frac{1}{\beta}\frac{2 - 2q}{3 + q}$$

- We see $B = 0$ for $q = 1 \rightarrow$ which corresponds to no inflation as $\ddot{a} < 0$.
- B is singular for $q = -3$. This tells q lies in the range $-3 < q < 1$
- Note that for $q = -1$, one finds

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = 0 \rightarrow \frac{d}{dt}\left(\frac{\dot{a}}{a}\right) = 0 \rightarrow \frac{\dot{a}}{a} = H = \mathrm{const} \quad \rightarrow a(t) = a_0 exp(Ht)$$

which corresponds to de-Sitter(inflationary) expansion. Here a_0 is a constant.

Inflation in Nonlinear electrodynamics

From the Raychaudhuri equation we see that the acceleration of the magnetic universe $\ddot{a}(t) > 0$ requires $\rho_B + 3p_B < 0$ i.e.

$$2 - 3\beta B^2 < 0 \Longrightarrow B > \sqrt{\frac{2}{3\beta}}$$

is required to drive inflation in the magnetic universe.

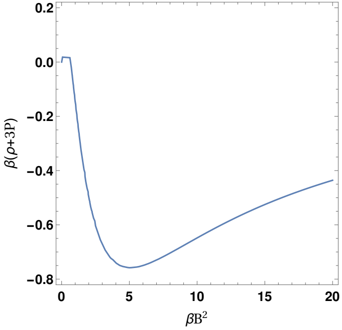

Figure: $\beta(\rho + 3p)$ is plotted against βB^2.

1. $\beta(\rho + 3p)$ remains **negative** for a wide range of βB^2: it first decreases and becomes minimum and then rises again with βB^2.
2. $\beta(\rho + 3p)$ is found to be **minimum** at $\beta B^2 = 5.184$.
3. The magnetic field plays a crucial role in Inflation (as $\ddot{a} > 0$ for $\beta(\rho + 3p) < 0$).

Below βp (here $p = p_B$) is plotted against $\beta\rho$ (here $\rho = \rho_B$).

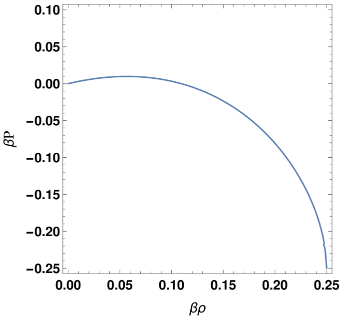

Figure: βp is plotted against $\beta\rho$.

1. The magnetic energy density ρ_B is found to be maxmium at $\rho_B^{max} = 0.25/\beta$.
2. We also see that as $\beta\rho_B \to 0.25$ and $\beta P_B \to -0.25$, $\beta(\rho_B + p_B) \to 0$.
3. As $\beta(\rho_B + p_B) \to 0$, from the equation of continuity with $\dot{\rho_B} = 0$ we find $B = \sqrt{\frac{2}{\beta}} = \mathrm{constant}$ during inflation.

Inflation in Nonlinear electrodynamics

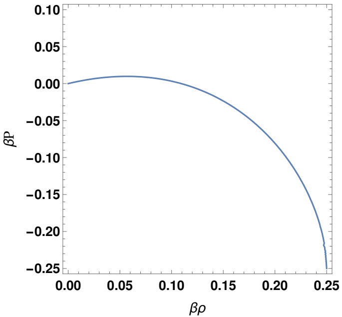

Figure: βp is plotted against $\beta\rho$.

1. From the Friedmann equation, it follows $H^2 = \frac{8\pi G}{3}\frac{2B^2}{(2+\beta B^2)} = \frac{8\pi G}{3\beta} = \text{constant}$ during inflation.
2. Combining the Friedmann and Raychaudhuri equations, one finds $\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -4\pi G(\rho_B + p_B)$. During inflation $\rho_B + p_B = 0$, $\rightarrow$ one finds

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = 0 \;\rightarrow a(t) = c_0 e^{Ht}$$

the exponential acceleration(de-Sitter) expansion! Here c_0 is a constant and $H = \sqrt{\frac{8\pi G}{3\beta}}$.

Inflation in Nonlinear electrodynamics

1. During the inflationary phase, we find $\rho = \rho_{max} = \mathrm{constant}$ and this gives the magnetic field $B = \sqrt{\frac{2}{\beta}} = 2\sqrt{2}\,(\rho_{max})^{1/2}$.
2. In a typical inflationary model, where the reheating temperature $T_{reh} \sim \rho_{inf}^{1/4} = 10^{16}$ GeV (the grand unified scale), the energy density at the time of inflation is found to be $\rho_{inf} = 10^{64}\ \mathrm{GeV}^4$.
3. The magnetic field required to trigger the inflation is found to be

$$B_{start} = \sqrt{\frac{2}{\beta}} = 2\sqrt{2}(\rho_{max})^{1/2} = 2\sqrt{2}\times 10^{32} \times \frac{1}{2\times 10^{-20}}\ \mathrm{Gauss} \sim 10^{52}\ \mathrm{Gauss}$$

   where, $1\ \mathrm{Gauss} = 2\times 10^{-20}\ \mathrm{GeV}^2$.
4. The e-fold number N required for inflation

$$N = ln\frac{a_{end}}{a_{start}} = ln\sqrt{\frac{B_{start}}{B_{end}}}$$

   For $N = 70$ (say), one finds $e^{2\times 70} = 10^{61} = \frac{10^{52}}{B_{end}}$ from which gives $B_{end} = 10^{-9}$ Gauss. at the end of inflation.

e-fold number(N) from horizon problem solution

- To explain the horizon problem, the largest scale observed today $\lambda(t_0) = 1/H_0$ should be within the horizon at the beginning of inflation

$$\frac{1}{a_0 H_0} < \frac{1}{a_i H_i} \rightarrow \frac{1}{H_0}\frac{a_f}{a_0}\frac{a_i}{a_f} < \frac{1}{H_i}$$

- Noting $a_i/a_f = e^{-N}$ and the photon temperature(T) drops as $T \propto 1/a$, one can write $a_f/a_0 = T_0/T_f$ where T_0 is the CMB temp. today and T_f is the temp. after reheating, we find

$$\frac{1}{H_0}\frac{T_0}{T_f}e^{-N} < \frac{1}{H_i}$$

- The e-fold number N can be calculated as

$$N > ln\left(\frac{T_0}{H_0}\right) + ln\left(\frac{H_i}{T_f}\right)$$

e-fold number(N) from horizon problem solution

- Since $B \propto 1/a^2$ and $T \propto 1/a$, $B \propto T^2$, we find N as

$$N > ln\left(\frac{\sqrt{B_0}}{H_0}\right) + ln\left(\frac{H_i}{\sqrt{B_f}}\right)$$

- At present $B_0 = 10^{-10}$ G ($\sim 10^{-30}\ \mathrm{GeV}^2$) and the present Hubble parameter $H_0 = 10^{-42}$ GeV. This gives

$$N > ln\left(\frac{10^{-15}}{10^{-42}}\right) + ln\left(\frac{H_i}{\sqrt{B_f}}\right) \simeq 62 + ln\left(\frac{H_i}{\sqrt{B_f}}\right)$$

- A knowledge of the magnetic field B and the Hubble parameter H allows us to estimate N, the e-fold number.
- CMB observation suggests $N \simeq 60 - 70$ for inflation.

Exit from eternal inflation

- The deceleration parameter q lies in the range $-3 < q < 1$. B is singular at $q = -3$ and $q = -1$ corresponds to de-Sitter or exponential acceleration. As q moves from $q = -1$ to $q = 1$ region in the course of universe expansion, B varies from $B = \sqrt{2/\beta}$ to $B = 0$.
- As $B \to 0$ (for $q \to 1$) at the end of inflation, $\ddot{a} < 0$, the acceleration of the universe (i.e. inflation) stops $\to$ graceful exit from inflation !
- Hence, in NLED theory there is no exit problem for inflation!

Presentation Outline

Conclusion

- The Friedmann and Raychaudhuri equations in the early universe dominated by strong and highly nonlinear magnetic field coupled to gravity are obtained.
- The role of magnetic field in inflation is highlighted from the deceleration parameter q and the condition for acceleration $\ddot{a} > 0$ on is obtained by setting $\rho_B + 3p_B < 0$ (from Raychaudhuri equation).
- The fact that the magnetic energy density ρ_B remains constant during inflation gives the magnetic field $B = \sqrt{\frac{2}{\beta}} = 2\sqrt{2}(\rho_{max})^{1/2} \sim 10^{52}$ Gauss, where $\rho_{inf} = \rho_{max} = 10^{64}\ \mathrm{GeV}^4$ assuming the reheating takes place at $T_{reh} \sim \rho_{inf}^{1/4} \sim 10^{16}\ \mathrm{GeV}$, the GUT scale.
- Assuming that the magnetic field driven inflation will produce the e-fold number $N = 70$, the magnetic field at the end of the inflation is found to be $B_{end} = 10^{52} \times e^{-2N}\ (N = 70) = 10^{52} \times 10^{-61} = 10^{-9}$ Gauss.
- Horizon problem solution for $B_0 = 10^{-10}$ Gauss and $H_0 = 10^{-43}$ GeV, predicts the e-fold number N

$$N > ln\frac{\sqrt{B_0}}{H_0} + ln\frac{H_i}{\sqrt{B_f}} \sim 62 + ln\frac{H_i}{\sqrt{B_f}}$$

- There is no exit problem in magnetic field driven inflation.

Presentation Outline

Cosmic magnetism and Square Kilometer Array

- The estimated magnetic field 10^{-9} Gauss that we obtain at the end of magetic field driven inflation can compete with the field produced by Active galactic nuclei and violent star-formation activity or the field produced at the galaxy formation time at $z \sim 5$.
- The SKA can provide into the origin, evolu- tion and structure of cosmic magnetic fields.


Thank you
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Big bang cosmology - the Horizon Problem

1. The cosmic microwave background radiation(CMBR) spectrum suggests that the universe at the large scale is isotropic and homogeneous.
2. The inhomogeneities (gravitationally unstable which grows with time) of the CMBR spectrum were much smaller in the past (at the time of last scattering) than today.
3. The conventional big bang picture of the early universe (e.g. the CMB at last scattering) consists of a large number of causally-disconnected region (patches) of space of similar physical conditions.
   Q. Why the universe was so homogeneous at the times of last scattering than it is now? No dynamical reason.
4. The homogeneity problem is often referred to as the *horizon problem*.

Big bang cosmology: the Flatness Problem

The Friedmann equation(for $k \neq 0$) can be written as

$$\mid \Omega - 1 \mid = \frac{k}{a^2 H^2}$$

where $\Omega = \frac{\rho}{\rho_c}$ and $\rho_c = \frac{3H^2}{8\pi G}$ (the critical density). So, we find $\Omega = \Omega_0 = 1$ if $k = 0$ i.e. the universe is flat which agrees with the WMAP data $|\Omega - 1| < 0.02$ $\rightarrow$ the universe is flat.
In big bang cosmology (for $k \neq 0$) one finds

$$|\Omega - 1| \propto \begin{cases} t & \text{radiation dominated universe} \\ t^{2/3} & \text{matter dominated universe} \end{cases}$$

So any small departure of $k = 0$, will drive Ω away from $\Omega_0 = 1$ in big bang cosmology $\rightarrow$ requires extreme fine tuning at $k = 0$ for $\Omega = 1$. This is the *flatness problem*.

Big bang cosmology - the Horizon Problem

The comoving horizon is defined in terms of casual horizon or maximum distance travelled by a light ray from time 0 to time t.

$$\tau = \int_0^\tau d\tau' = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da}{Ha^2}$$

where, $(aH)^{-1}$ is comoving Hubble radius which can be evaluated with ω as,

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2(1+3\omega)}}$$

The causal horizon for radiation dominated(RD) and matter dominated(MD) universe

$$\tau = \int_0^a \frac{da}{Ha^2} \propto \begin{cases} a & \text{RD} \\ a^{1/2} & \text{MD} \end{cases}$$

The comoving horizon grows monotonically with time $\rightarrow$ the comoving scale entering into the horizon today have been far outside the horizon at CMB decoupling.

Solving horizon and flatness problem

1. **Solving Horizon Problem**
   If the particles are separated by distance greater than Hubble radius $(aH)^{-1}$ then they can not communicate with each other now.
   Solving Flatness Problem
   For the exponential expansion $a(t) = \exp\left(\sqrt{\frac{\Lambda}{3}}\ t\right)$ we find

$$\mid \Omega - 1 \mid = \frac{k}{a^2 H^2} = \frac{3k}{\Lambda} exp\left(-\sqrt{\frac{4\Lambda}{3}}\ t\right)$$

   This makes $\Omega \to 1$ irrespective of all subsequent expansion of the universe. Thus the inflation solves the flatness problem.

Scalar Field Dynamics

The action of the inflaton scalar ϕ coupled to gravity is

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

For flat($k = 0$) FRW space time the energy density and pressure is given by,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

For the homogeneous scalar field $\phi(t)$, the acceleration $\ddot{a} > 0$, if $\rho_\phi + 3p_\phi < 0$ and $p_\phi < 0$, which will follow if

$$V(\phi) > \frac{1}{2}\dot{\phi}^2$$

Thus the potential is flat on which the inflaton rolls extremely slowly.
The dynamics of $\phi(t)$ and the Friedmann equation is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

Slow-roll parameters

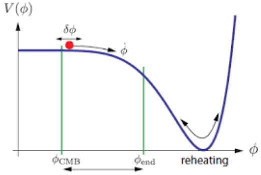
The slow roll parameter is defined as

$$\varepsilon \equiv \frac{3}{2}(\omega_\phi + 1) = \frac{1}{2}\frac{\dot{\phi}^2}{H^2}$$

where $\omega_\phi = p_\phi/\rho_\phi = -1$ (when $V(\phi) > \frac{1}{2}\dot{\phi}^2$) , ε is related to the evolution of Hubble parameter H

$$\varepsilon = -\frac{\dot{H}}{H^2} = 1 - \frac{a\ddot{a}}{\dot{a}^2} = -\frac{d\ln H}{dN}$$

Where $dN = Hdt$ (N, the e-fold number). The acceleration($\ddot{a} > 0$) of the universe occurs if $\varepsilon < 1$.



The second slow-roll parameter is defined as

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN} \rightarrow \mid \eta \mid < 1$$

$\mid \eta \mid < 1$ ensures change in ε per e-fold is small.

Slow-roll Inflation

In slow roll regime $\varepsilon, \mid \eta \mid \ll 1$ and the Hubble eqn and scalar field eqn. become

$$H^2 \approx \frac{8\pi G}{3} V(\phi) \approx \text{constant.} \quad \dot{\phi} \approx -\frac{1}{3H}\frac{dV}{d\phi}$$

Inflation ends when $\varepsilon(\phi_{end}) = 1$ and $\eta(\phi_{end}) = 1$.The number of e-fold(N) before the inflation ends is

$$N(\phi) = \ln\left(\frac{a_{end}}{a}\right) = \int_t^{t_{end}} Hdt = \int_\phi^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi \approx \int_{\phi_{end}}^{\phi} \frac{V}{V_{,\phi}} d\phi$$

olution of flatness (and horizon) problems require the total e-fold number(N_{tot}) exceeds about 60 i.e.

$$N_{tot} \equiv \ln \frac{a_{end}}{a_{start}} \geq 60$$

The fluctuations observed in CMB corresponds $N = 40 - 60$ number of e-folds before the end of Inflation.