Calibration II

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with due thanks to several friends / collaborators at

UCT, IDIA (SA), NCRA-TIFR (India) and NRAO (USA)

TELESCOPE SENSITIVITY

- Noise limit for imaging with interferometric radio telescopes

$$\sigma = \frac{T_{\rm sys}}{A_{\rm eff} \times \sqrt{(\Delta\nu \times \Delta t)}}$$

- Sensitivity improvements achieved by
  - wide band receivers,
  - long integration times
  - more antennas
  - long baselines $\sigma_{\rm confusion} \propto (\nu^{-2.7}/B_{\rm max}^2)$
    - B_{max} ~100 km @200 MHz, the confusion noise is ~1 μJy beam^{-1}.

IMAGING CHALLENGES AT LOW FREQ.

- Wide-field imaging
  - account for direction dependent (DD) effects
  - PB: time, frequency and polarisation dependence
  - *w*-term
- Wide-band imaging
  - … plus frequency dependence of the sky brightness
- Data volume $\propto N_{\rm ant}^2 \times N_{\rm channel} \times t$
  - Sky brightness $\Longrightarrow$ multi-scale deconvolution
  - Ionospheric effects $\Longrightarrow$ need for DD solvers

IMPLICATIONS FOR IMAGING

- Long baselines $B_{\max} > 2\ \mathrm{km} \implies \mathrm{DR} > 10^4$
- Wide-field effects:
  - *w*-term, PB effects and ionosphere effects
- Larger data volume

- Wide-field, wide-band, high resolution, high dynamic range imaging using large data sizes
  - a natural consequence of low frequency and high sensitivity imaging.

CALIBRATION AND IMAGING

- Standard calibration and imaging
    - (DI instrumental effects)
- w/ DD instrumental + propagation effects
    - correction for *w*-term and for PB
        - image plane correction
        - Fourier plane correction
        - pointing self-calibration
    - Mosaicing
- w/ advanced image parameterisation
    - multi-scale CLEAN (deconvolution)
    - multi-frequency synthesis (imaging)
    - full polarisation (Stokes) calibration and imaging

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

$$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i(ul + vm + w(n-1))}\ dl\ dm$$

MEASUREMENT EQUATION

$$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

MEASUREMENT EQUATION

$$\Gamma(\vec{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \vec{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \vec{b} \cdot \vec{s}/\lambda} d\Omega$$

$\Gamma(\vec{b})$: mutual coherence function

$E(\vec{s}, t)$: complex amplitude of the radiation emanating from the source in the direction $\vec{s}$

$\vec{s} = \vec{s}_0 + \vec{\sigma}$: point near the phase centre

$\vec{b} \cdot \vec{s}/c$: time difference between the incoming radiation collected at two antennas separated by $\vec{b}$

$$d\Omega = \frac{d\vec{s}}{R^2}$$

MEASUREMENT EQUATION

- $$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$$

- $$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i(ul + vm + w(n-1))}\, dl\ dm$$

  - (for $w \simeq 0, n \simeq 1$)

    - $$V(u, v) = \int I(l, m)\ e^{-2\pi i(ul + vm)}\, dl\ dm$$

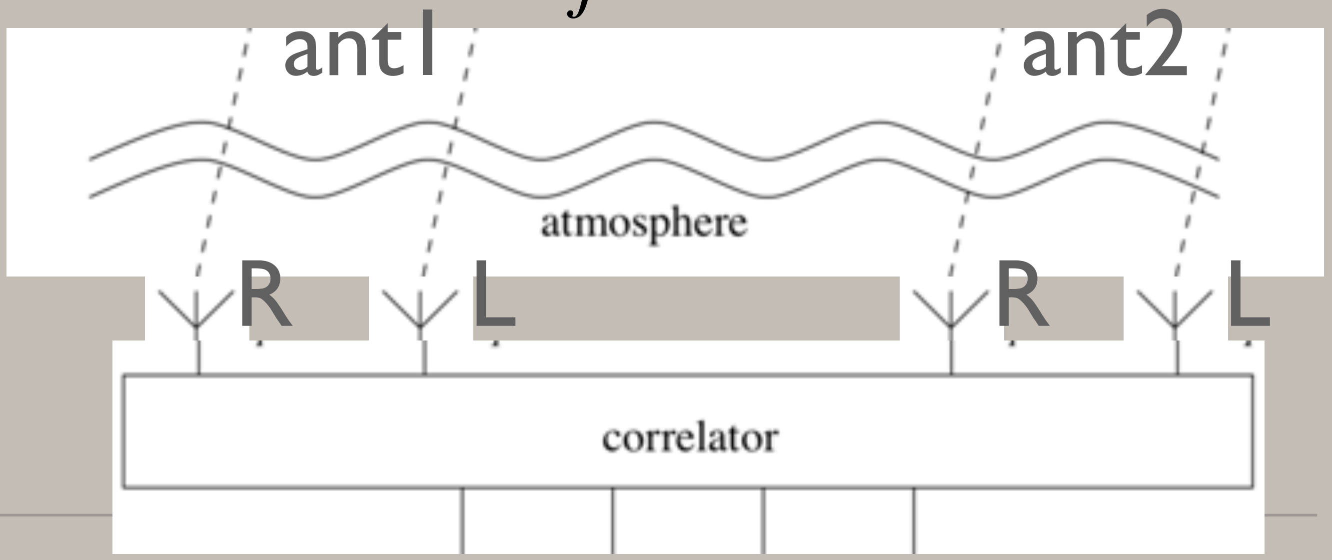
    - (this is van-Cittert Zernike theorem)

MEASUREMENT EQUATION

- $$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s},t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$$
- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \; dm$$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \;\; E^l]_i^T$
    - (two nominal orthogonal components of incident electric field are measured at each antenna *i*)

MEASUREMENT EQUATION

- $$\Gamma(\overrightarrow{b}) = \int \left\langle E(\vec{s}, t) \cdot E^*(\vec{s}, t - \overrightarrow{b} \cdot \vec{s}/c) \right\rangle e^{-2\pi i \overrightarrow{b} \cdot \vec{s}/\lambda} d\Omega$$
- $$V(u, v, w) = \int \frac{I(l, m, n)}{n} e^{-2\pi i (ul + vm + w(n-1))} \, dl \, dm$$
- Polarised radiation:
  - $\overrightarrow{E_i} = [E^r \; E^l]_i^T$
  - (four cross-correlation products, $\langle \overrightarrow{E_i} \otimes \overrightarrow{E_j^*} \rangle$ per baseline)
  - $\overrightarrow{V_{ij}} = [V^{rr} \; V^{rl} \; V^{lr} \; V^{ll}]_{ij}^T$
  - $\vec{I} = [I^{rr} \; I^{rl} \; I^{lr} \; I^{ll}]^T$



MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
    - (a 2×2 matrix product)
      - complex gains, G,
      - polar'n leakage, D and
      - feed config'n, C.
  - DD: $J_i^{sky} = [EPF]$
    - (a 2×2 matrix product)
      - AIPs, E,
      - PA effects, P and
      - tropospheric / ionospheric effects, and Faraday R'n, F.

MEASUREMENT EQUATION

- $\overrightarrow{E_i} = [E^r \ E^l]_i^T$
  - (suffers from propagate effects and receiver electronics)
- (Jones matrices describe this modulation for the electric field incident at each orthogonal pair of feeds $\overrightarrow{E_i} = [E^r \ E^l]_i^T$)
  - DI: $J_i^{vis} = [GDC]$
  - DD: $J_i^{sky} = [EPF]$
    - $K_{ij}^{\{vis,\ sky\}} = [J_i \otimes J_j^{\dagger}]^{\{vis,\ sky\}}$
    - (effect on each baseline ij is described by the outer-product of these antenna-based Jones matrices, a 4×4 matrix!)
- $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma}/\lambda} d\Omega$

MEASUREMENT EQUATION

- $$\vec{V}_{ij}^{Obs}(\nu,t) = G_{ij}\; W_{ij}(\nu,t) \int P_{ij}\; M_{ij}(s,\nu,t)\; \vec{I}(s,\nu)\; e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\; d\vec{s}$$

DATA observed visibility

DI gains

Weights

DD & Ionospheric Corrections

Antenna PB

MT-MFS

non-coplanar baselines *w*-projection

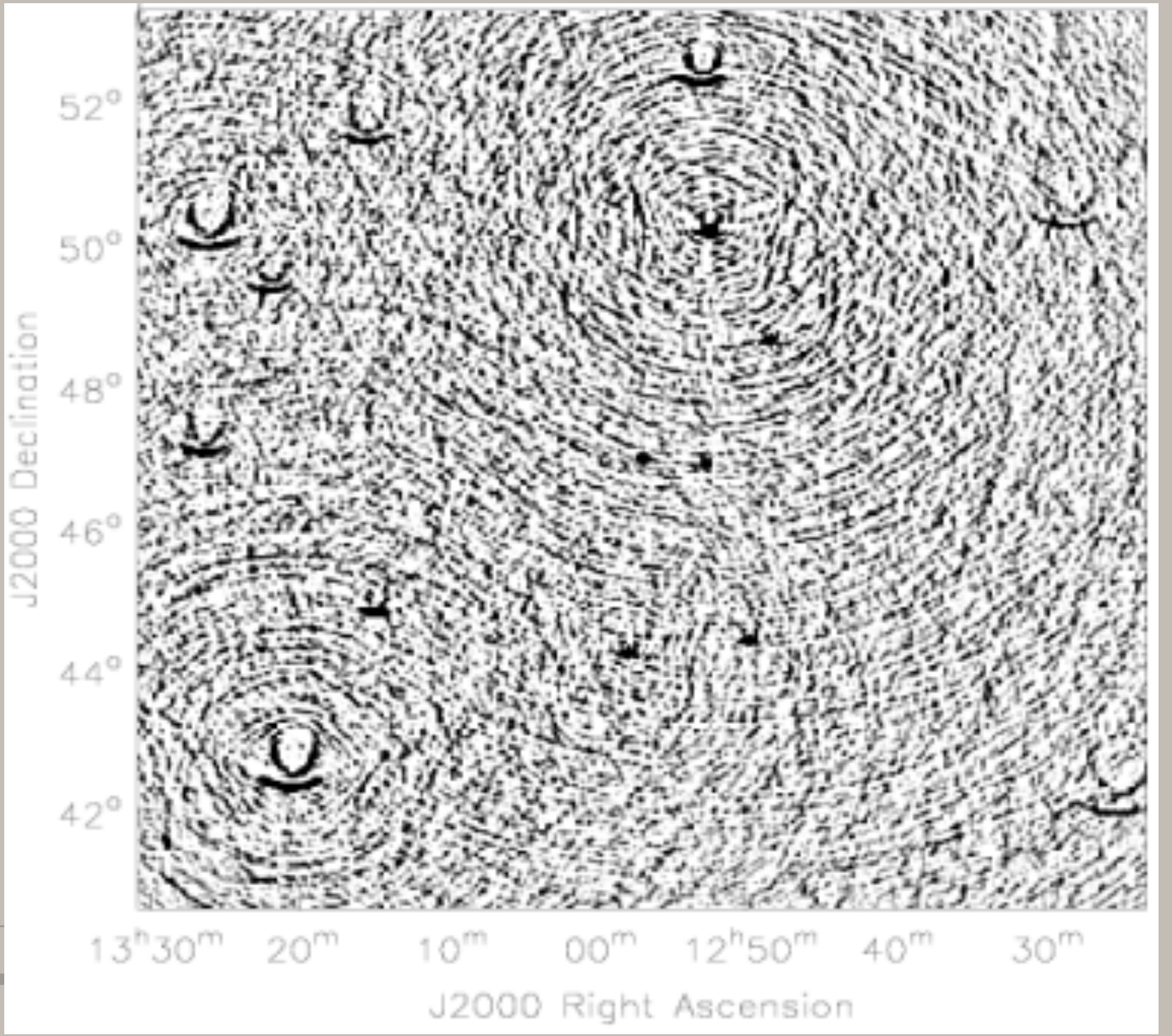
- $$\vec{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \vec{b} \cdot \vec{\sigma}/\lambda} d\Omega$$

CALIBRATION AND IMAGING

- Standard calibration and imaging
    - (DI instrumental effects)
- w/ DD instrumental + propagation effects
    - correction for ***w*-term** and **for PB**
        - **image plane** correction
        - **Fourier plane** correction
        - pointing self-calibration
    - Mosaicing
- w/ advanced image parameterisation
    - **multi-scale CLEAN** (deconvolution)
    - **multi-frequency synthesis** (imaging)
    - **full polarisation** (Stokes) calibration and imaging

W-TERM

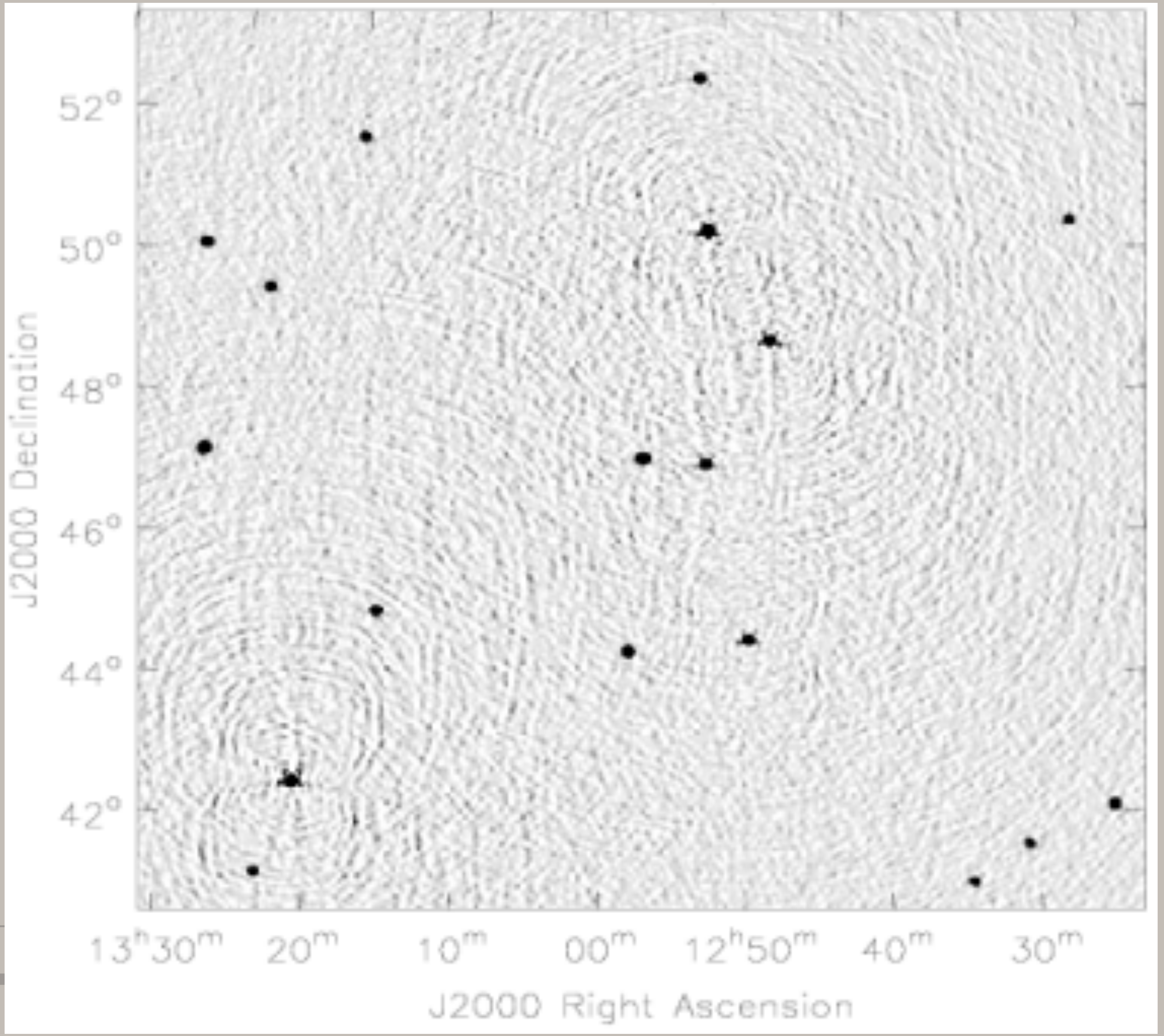
- $V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$
- $e^{\iota w\sqrt{1-l^2-m^2}}$



Credits: S. Bhatnagar, synthesis imaging NRAO workshop

W-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $e^{\iota w\sqrt{1-l^2-m^2}}$
- divide the FoV into a no. of FACETS



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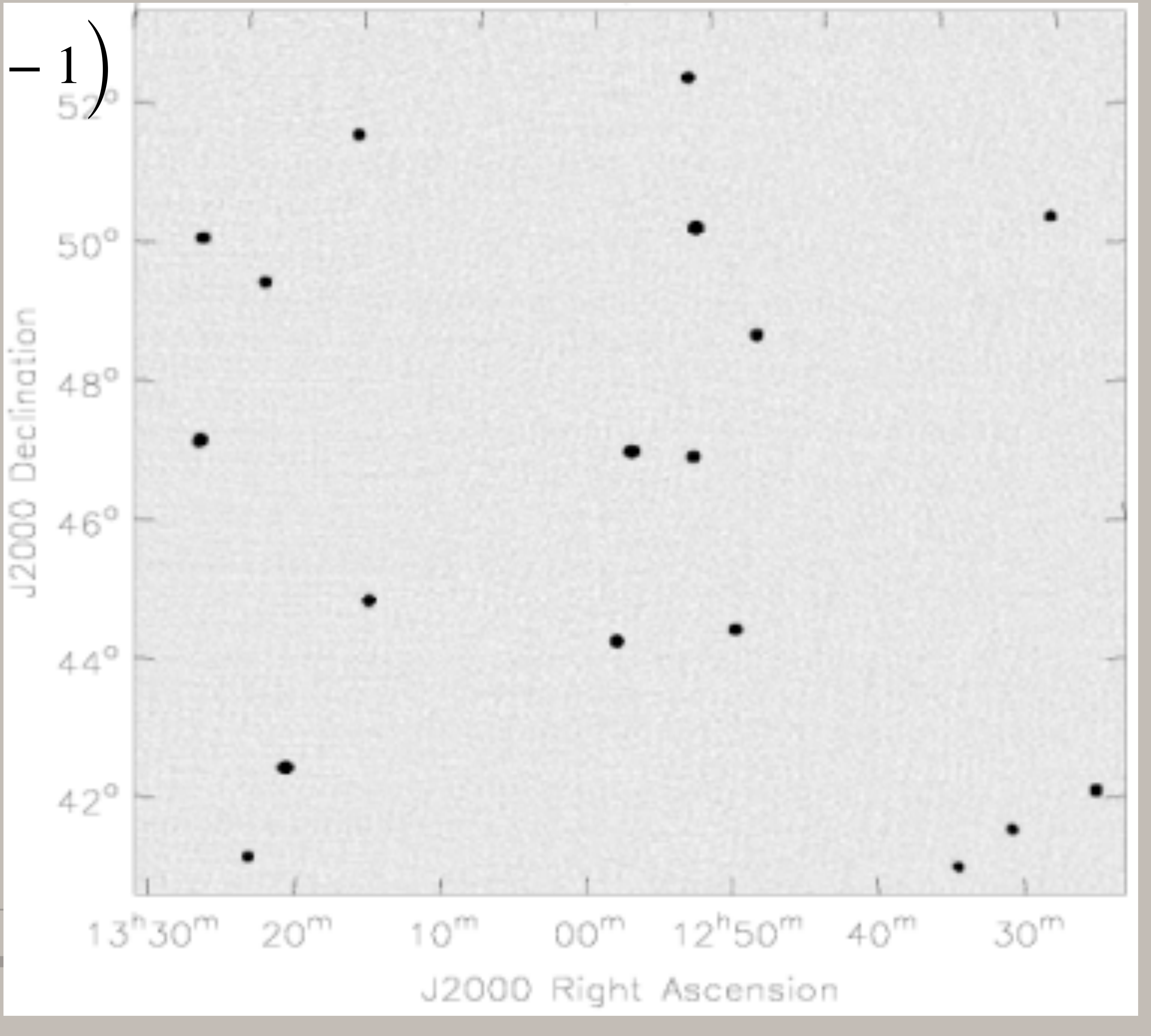
W-TERM

- $$V(u,v,w) = \int \frac{I(l,m,n)}{n} e^{-2\pi i(ul+vm+w(n-1))}\, dl\, dm$$
- $$K_{ij}^{Sky} = e^{w_{ij}\left(\sqrt{1-l^2-m^2}-1\right)}$$
- An order-of magnitude faster than FACETing, and
- for the same amount of computing time provides higher DR images.



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CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma}/\lambda} d\Omega$$

is different for each baseline

- Assumption:
  - sky is (not) variable, and
  - Antenna power pattern is (not) changing!

CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}^{obs}_{ij} = [K^{vis}_{ij}] \int [K^{sky}_{ij}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma}/\lambda} d\Omega$$
  - $$\overrightarrow{V}^{obs}_{cn\times 1} = [K^{vis}_{cn\times cn}][S_{cn\times cm}][F_{cm\times cm}][K^{sky}_{cm\times cm}]\vec{I}^{sky}_{cm\times 1}$$

vector of n visibilities

projection operator describing the uv-coverage

Fourier transfer operator

Pixelated image of sky

CORRECTION FOR **PB**

- **A**-projection
  - $\overrightarrow{V}_{ij}^{obs} = [K_{ij}^{vis}] \int [K_{ij}^{sky}] \vec{I}^{sky}(\vec{s}) e^{-2\pi i \overrightarrow{b} \cdot \overrightarrow{\sigma}/\lambda} d\Omega$
  - $\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s, \nu, t)\ I(s, \nu)\ e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\ ds$

- Visibility depends on time and frequency!

CALIBRATION AND IMAGING

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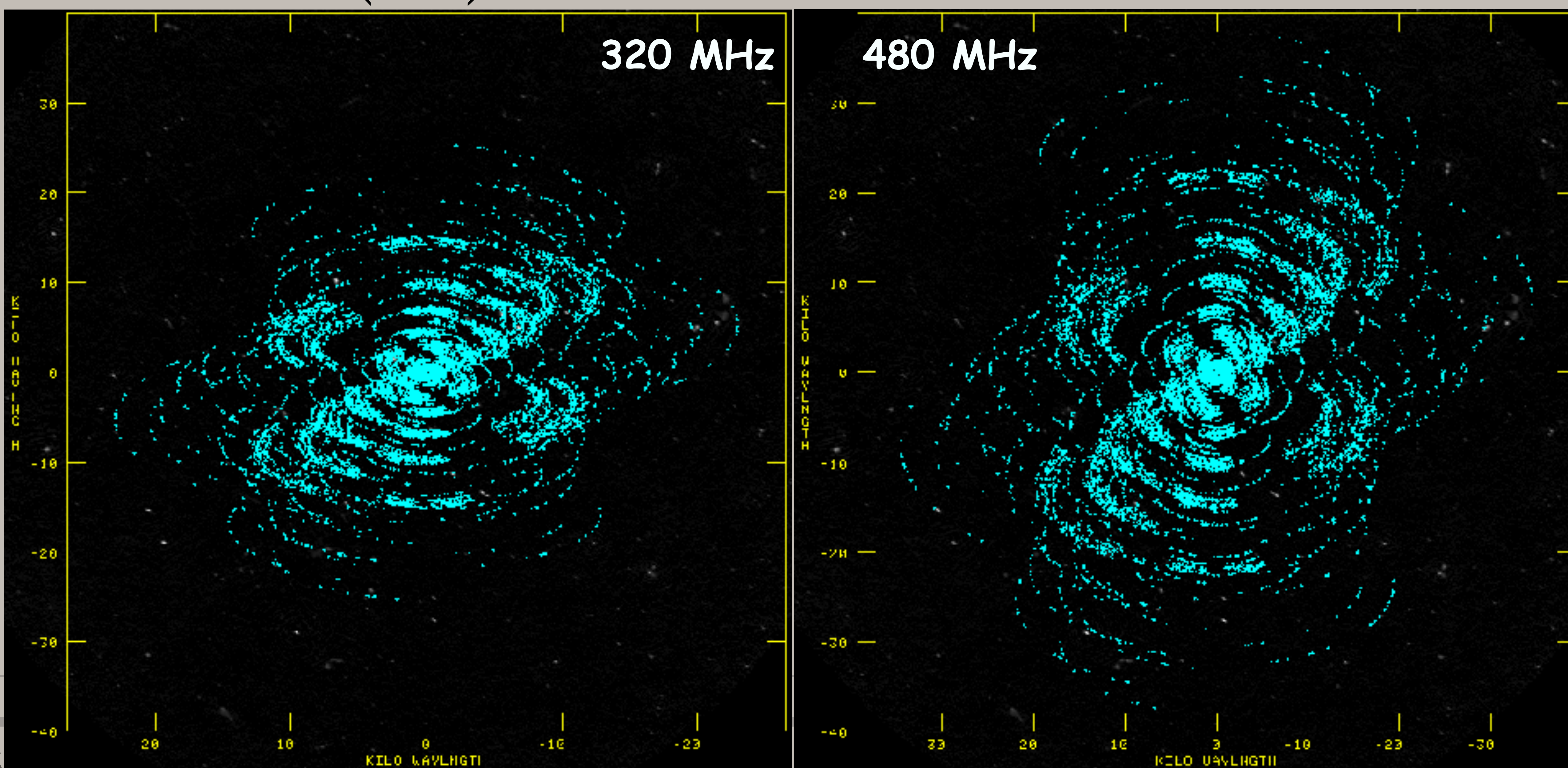
        - pointing self-calibration
    - Mosaicing
- w/ advanced image parameterisation
    - multi-scale CLEAN (deconvolution)
    - **multi-frequency synthesis** (imaging)
    - **full polarisation** (Stokes) calibration and imaging

CORRECTION FOR **PB**

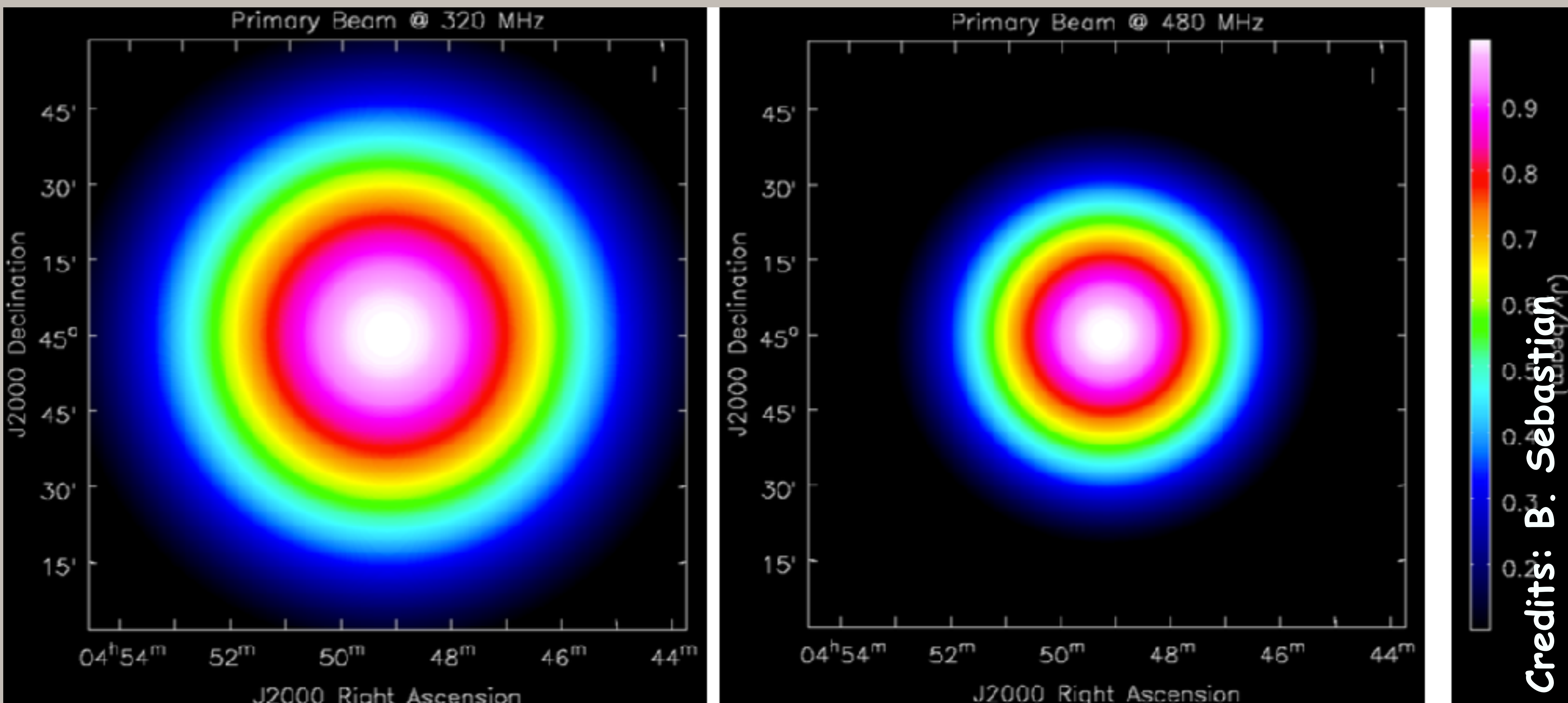
- **multi-frequency** synthesis

$$I_{\nu}^{sky} = I_{\nu 0}^{sky} \left(\frac{\nu}{\nu 0} \right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$



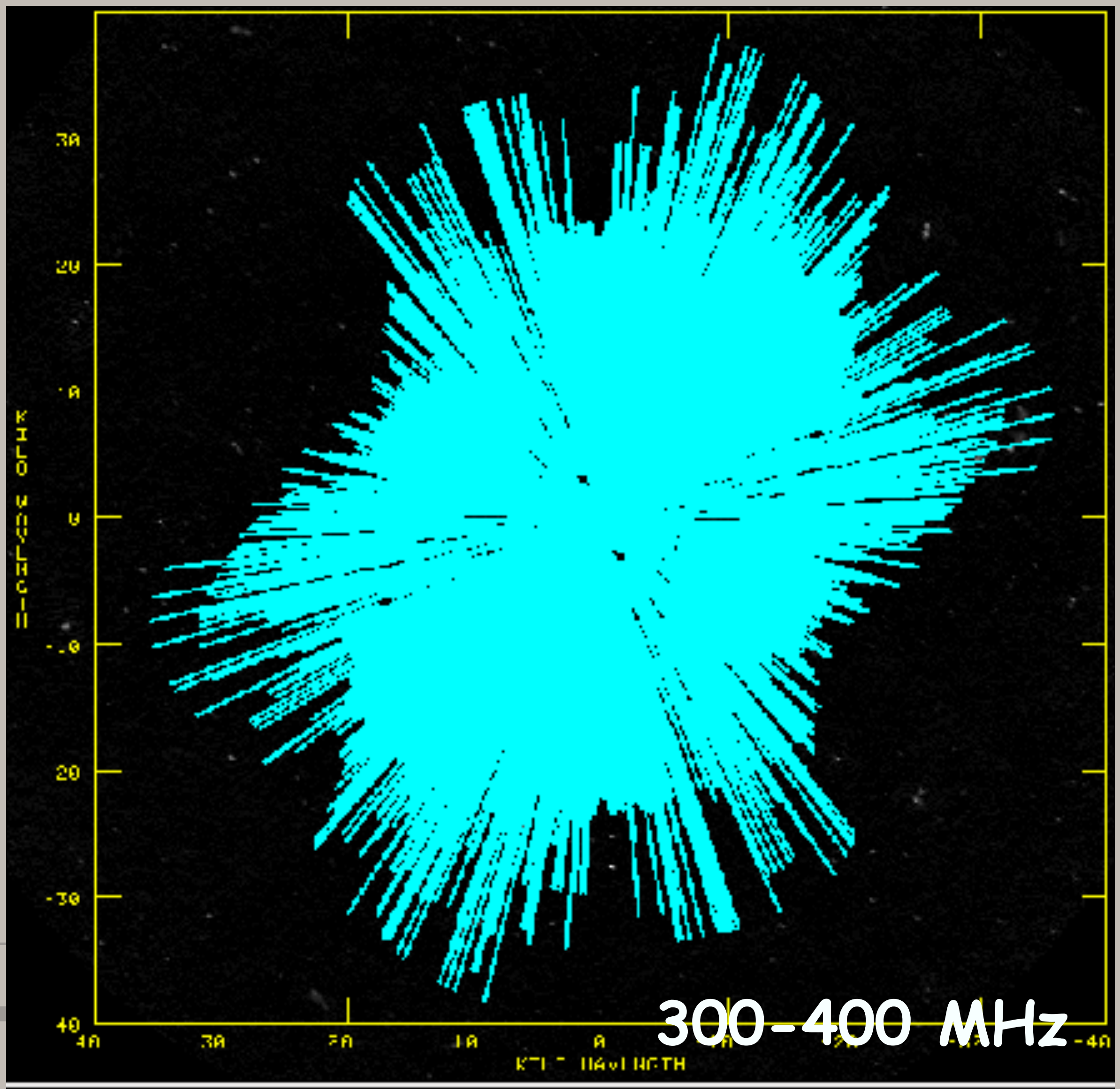
CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{i(ul+vm+w(\sqrt{(}1-l^2-m^2)-1))}\ ds$$

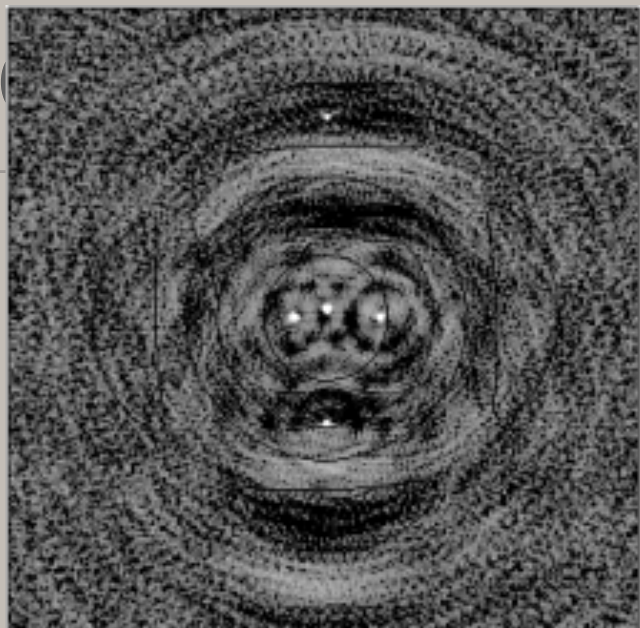


CORRECTION FOR **PB**

- **A**-projection
  - $$\overrightarrow{V}_{ij}^{obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{i(ul+vm+w(\sqrt(1-l^2-m^2)-1))}\ ds$$
- **multi-frequency** synthesis
  - $$I_{\nu}^{sky} = I_{\nu 0}^{sky}\left(\frac{\nu}{\nu 0}\right)^{I_{\alpha}^{sky} + I_{\beta}^{sky} log(\frac{\nu}{\nu 0})}$$
  - $$I_0 = I_{\nu_0}$$
  - $$I_1 = I_{\alpha} \times I_{\nu_0}$$
  - $$I_2 = (I_{\alpha}(I_{\alpha} - 1)/2 + I_{\beta}) \times I_{\nu_0}$$

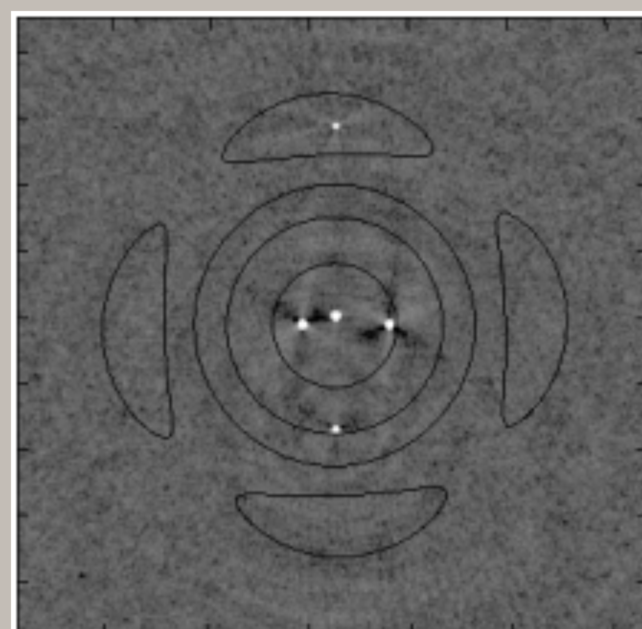


ON FOR **PB**



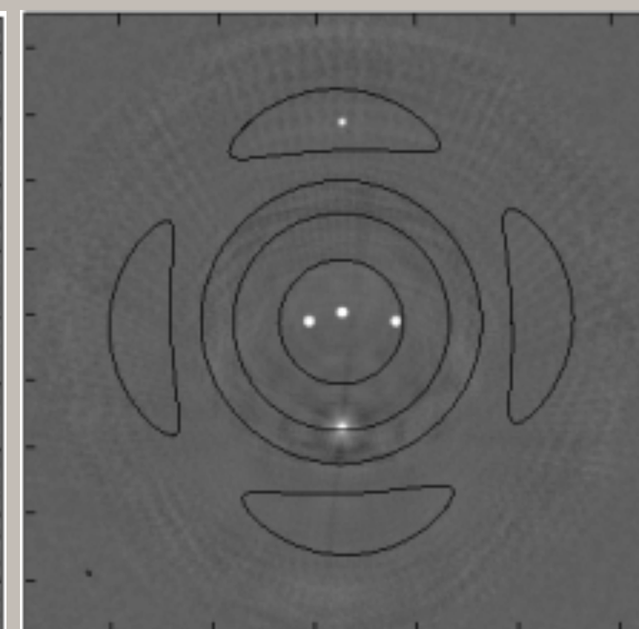
FT

(standard imaging)



FT

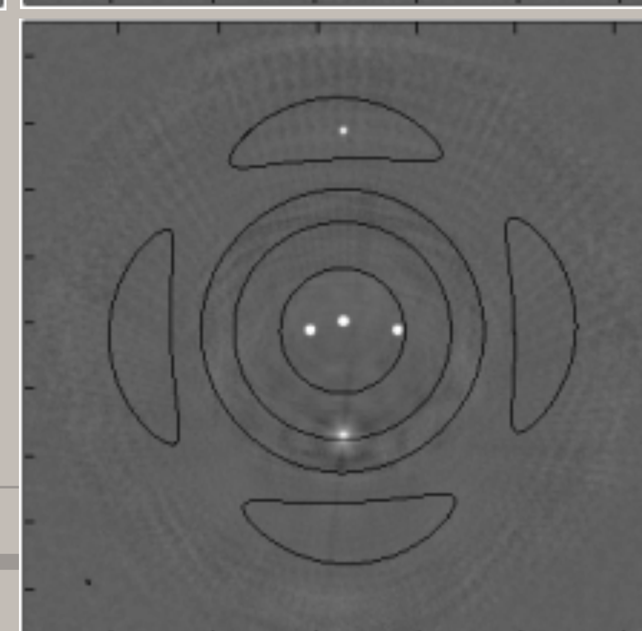
+ MT-MFS



FT

+ MT-MFS

+ A-projection



FT

+ MT-MFS

+ WB A-projection

Credits: S. Bhatnagar (NRAO, USA)

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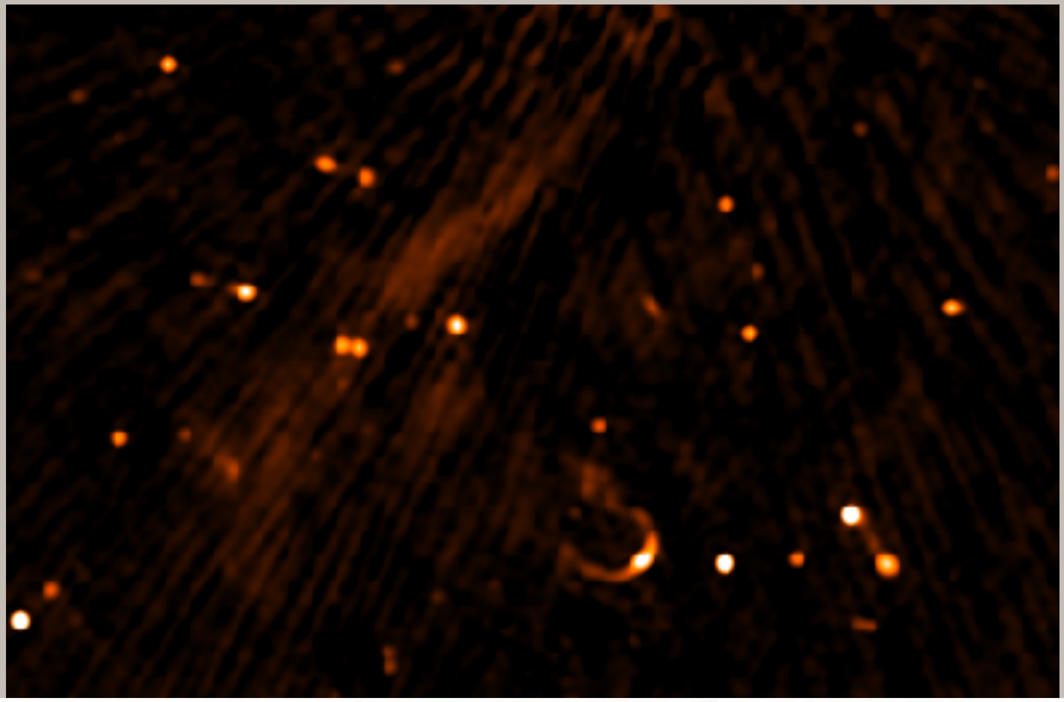
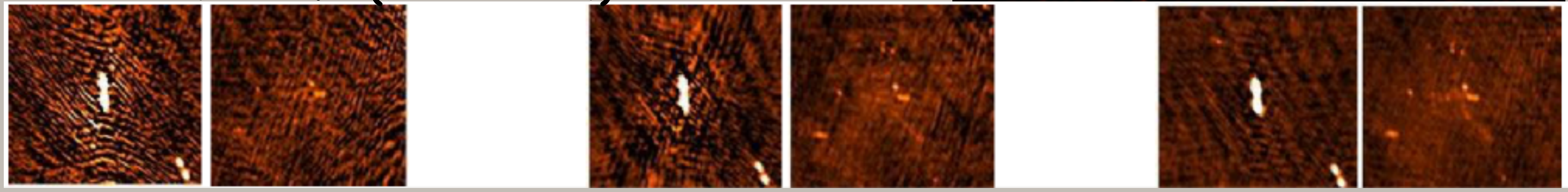
PEELING: DD CALIBRATION

- antenna based gains are determined in the direction of each compact source.
    - subtract these gains (contribution of compact sources from the observed data using a DFT) and the residual visibilities are imaged again.
- drawbacks of peeling…



Credits: H. Intema (Leiden Obs.)



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STOKES PARAMETERS

Sir George Gabriel Stokes

- Defined by George in 1852
  - Adopted for astronomy by Chandrasehkar in 1947.
  - Not a vector quantity! Deals with power instead of electric field amplitudes.
    - Can be used for partially polarised radiation.
    - The correlator can produce ALL Stokes parameters simultaneously (not so easy in optical astronomy!)

HOW DO WE MEASURE STOKES ….?

Sir George Gabriel Stokes

- Stokes parameters are the auto-correlation & cross-correlation products returned from the correlator, but input to the correlator can come from different feed types.
- Feeds normally designed to approximate pure linear or circular.
  - Circular feeds – frequency dependent response
  - adds 90^{o} phase to *R* for *L*, so:
    - *I* from *RR* + *LL*
    - *V* from *RR* - *LL*

STOKES PARAMETERS

Sir George Gabriel Stokes

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
  - $RR = \mathscr{A}(RR)e^{i\psi RR} = I + V$
  - $LL = \mathscr{A}(LL)e^{i\psi LL} = I - V$
  - $RL = \mathscr{A}(RL)e^{i\psi RL} = Q + iU$
  - $LR = \mathscr{A}(LR)e^{i\psi LR} = Q - iU$

Sir George Gabriel Stokes

STOKES PARAMETERS

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
  - $$I = \frac{(RR + LL)}{2}$$
  - $$\frac{Q}{I} = \frac{\mathrm{Re}(RL + LR)}{RR + LL}$$
  - $$\frac{V}{I} = \frac{RR - LL}{RR + LL}$$
  - $$\frac{U}{I} = \frac{\mathrm{Im}(RL - LR)}{RR + LL}$$

Sir George Gabriel Stokes

STOKES PARAMETERS

- I – total intensity and sum of any two orthogonal polarisations
- Q & U – completely specify linear polarisation
- V – completely specifies circular polarisation
- Stokes parameters (as percentages of I)
- Is it really that simple?
  - No, there are leakages…
    - The total intensity can leak into the polarised components (I into $\{Q,U,V\}$).

MUELLER MATRIX

Hans Mueller

- The leakage of each polarisation into the other can be measured and quantified in a 4×4 matrix (Mueller 1943).

$$M = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix}$$

$$\begin{bmatrix} RR + LL \\ RL + LR \\ RL - LR \\ RR - LL \end{bmatrix} = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix} \cdot \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
- $I \Leftrightarrow V$ leakage
- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- RL phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
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- Alignment => PA calibration
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Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

POLARISATION CALIBRATION

- Flux density scale
- $I \Leftrightarrow Q$ leakage
- $I \Leftrightarrow U$ leakage
- $I \Leftrightarrow V$ leakage
- Alignment => PA calibration
- Ellipticity, $Q \Leftrightarrow V$
- RL phase, $U \Leftrightarrow V$

Constrained using calibrator with known Stokes parameters

Need calibrator with known PA

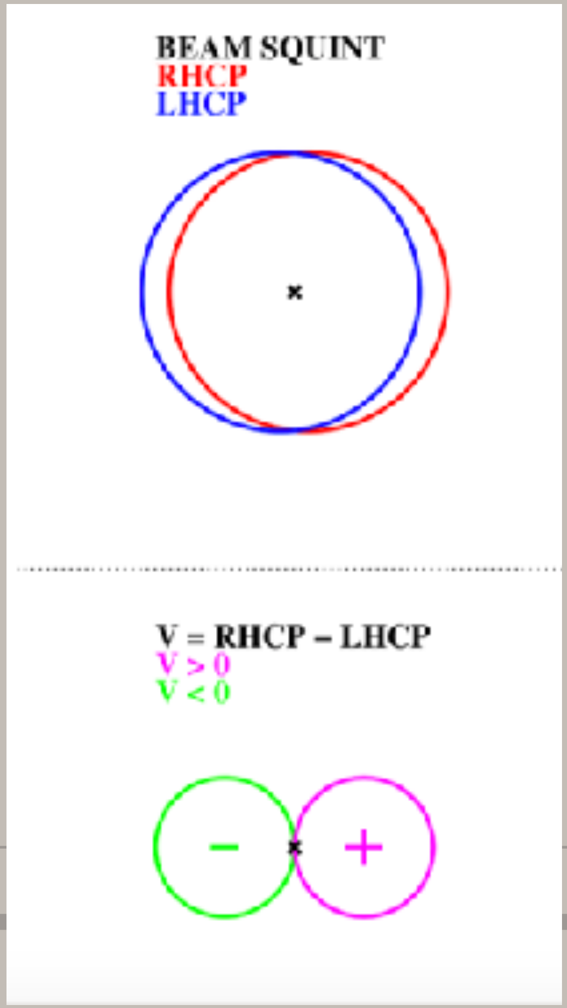
Stokes V ~ 0 for most calibrators so no need to worry too much unless you require very high precision

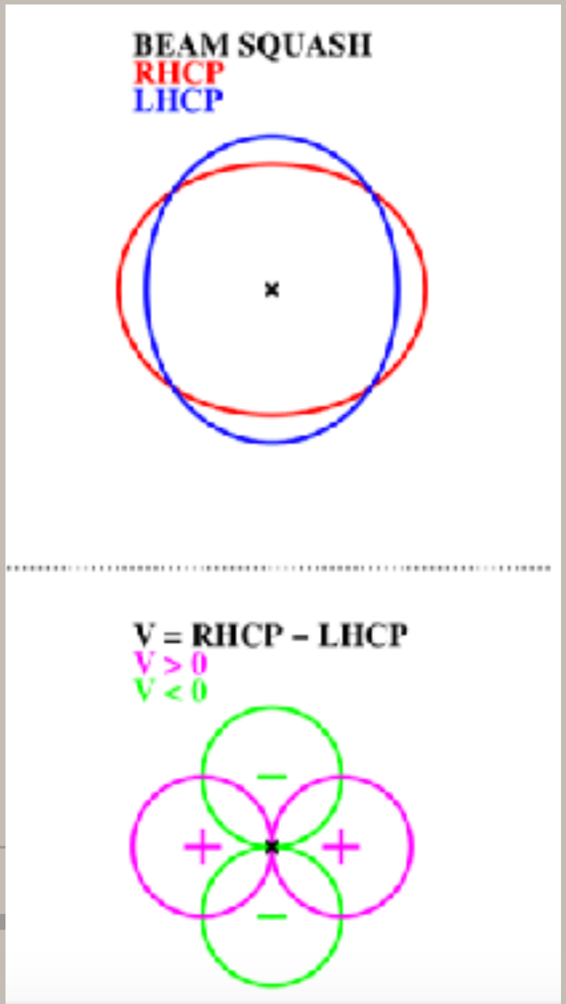
PUTTING THIS ALL TOGETHER

- In the end what we are trying to do is relate products from our correlator to the intrinsic polarised radiation from the source.
- So we need to correct the raw correlator outputs for
  - imperfections in the receiver (leakages).
    - The orientation of the receiver with respect to the telescope structure.
      - a.k.a. the changing parallactic angle.
  - Any measured propagation related polarisation effects (e.g. Faraday rotation).

BEAM EFFECTS

- For point sources, all of the previous is fine.
- What if the source you are looking at is extended compared to the telescope beam?
  - There are instrumental beam effects that can confuse the measurement of extended polarised signals. They are…
    - Squint
    - Squash





BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t) \int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu) e^{\imath \overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$

DATA

Weights

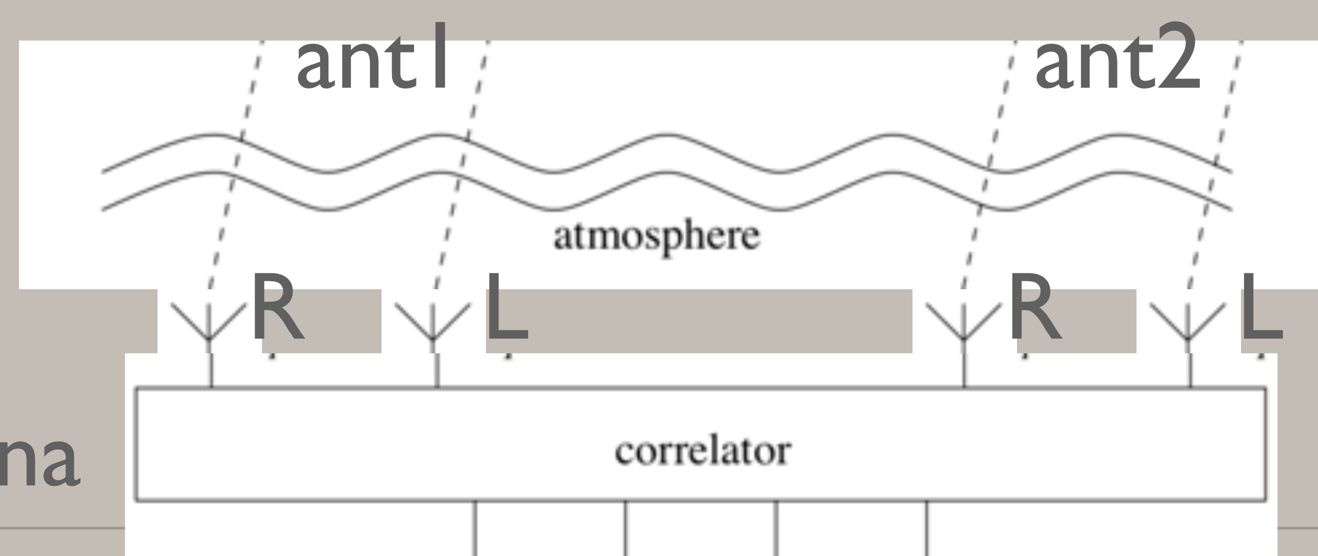
Mueller matrix

full-polarization vector of the sky brightness distribution

BEAM EFFECTS

- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t)\int M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)e^{\iota\overrightarrow{b_{ij}}.\vec{s}}\ d\vec{s}$$
- $$M_{ij}(\vec{s}, \nu, t) = E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)$$
- $$\overrightarrow{V}_{ij}^{Obs}(\nu, t) = W_{ij}(\nu, t)\mathscr{F}\left[\left(E_i(\vec{s}, \nu, t) \otimes E_j^*(\vec{s}, \nu, t)\right) \cdot \vec{I}(\vec{s}, \nu)\right]$$
  - $$= W_{ij}(\nu, t)\left[A_{ij} \star \overrightarrow{V}_{ij}\right]$$
  - $$\text{where}, A_{ij} = A_i \otimes A_j^*$$

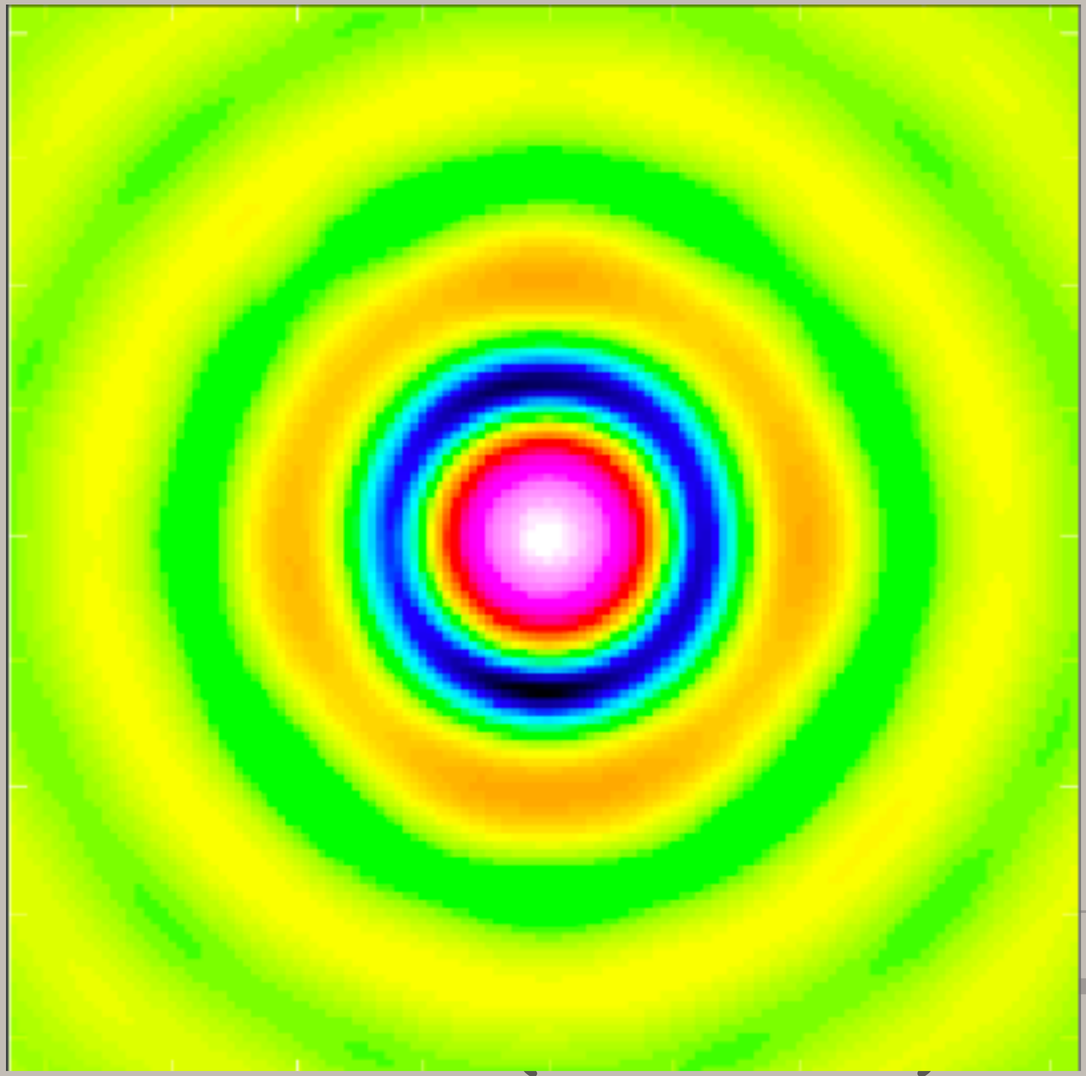
AIPs for two antenna



APERTURE ILLUMINATION P

- Holography data: MeerKAT
  - Obtained by Fourier Transforming the PB Holography measurements

Credits: S. Sekhar (UCT-IDIA)



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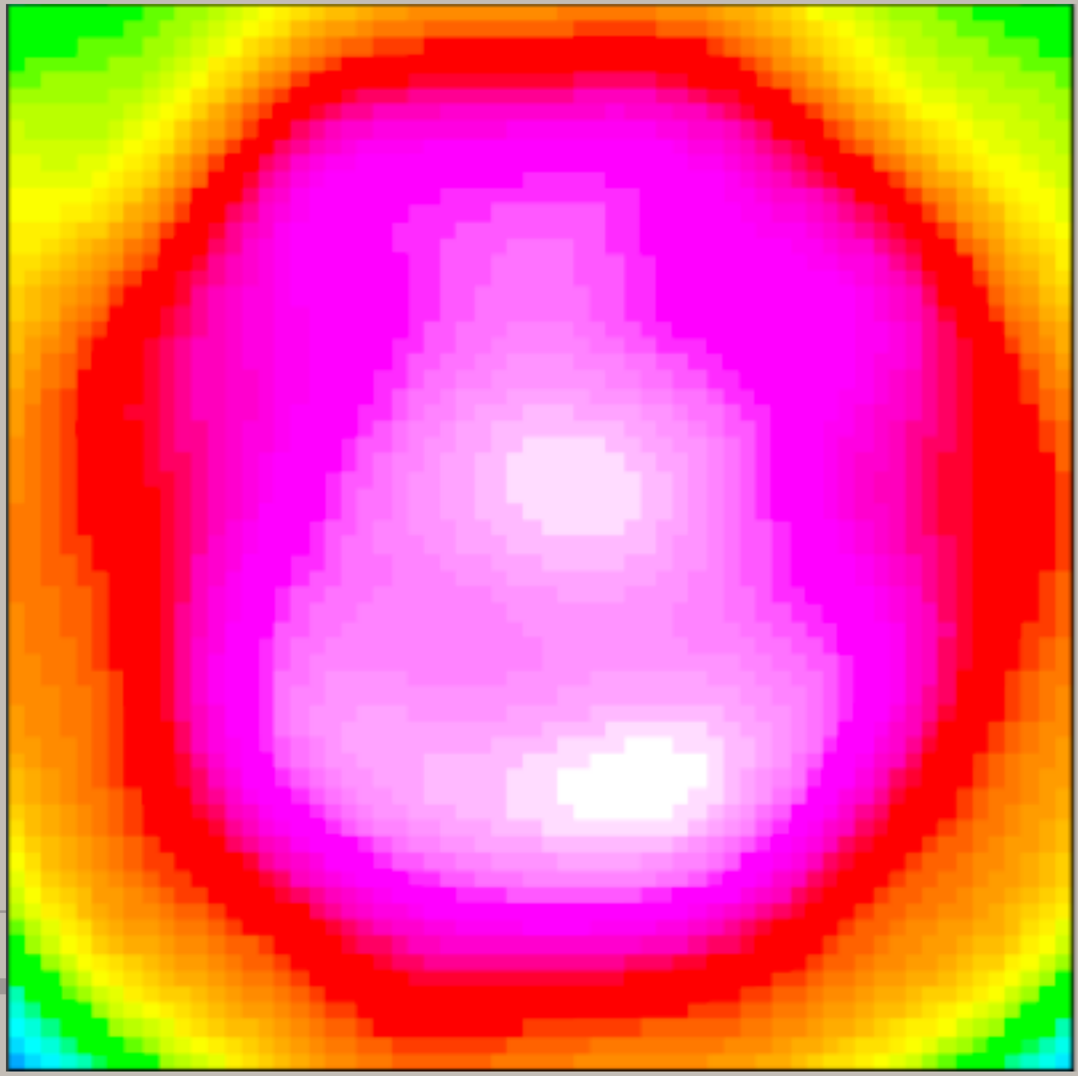

A-TO-Z SOLVER

- Use Zernike polynomials to directly model the complex aperture
  - it is a natural domain to model optical aberrations that cause PB weirdness
  - (Telescope agnostic - does not require ray traced model for different antennas/telescopes, only Holography)
  - Aperture size is fixed, independent of number of measured sidelobes.

MEASUREMENT EQUATION

$$\vec{V}_{ij}^{Obs}(\nu, t) = G_{ij}\ W_{ij}(\nu, t) \int P_{ij}\ M_{ij}(s, \nu, t)\ \vec{I}(s, \nu)\ e^{\imath \vec{b}_{ij}.\vec{s}}\ d\vec{s}$$

DATA
observed
visibility

DI gains

Weights

DD
&
Ionospheric
Corrections

Antenna PB
Bhatnagar+ 2008, 2013
Jagannathan+ 2019,…

MT-MFS
Rau+ 2011

non-coplanar
baselines
w-projection
Cornwell+ 2008

Ref.:
Rau+ 2009, Jagannathan+ 2019,
ASP Conf. Ser. Vol. 180 (1999)

Thank you all for your attention!