



# Receivers and Correlators

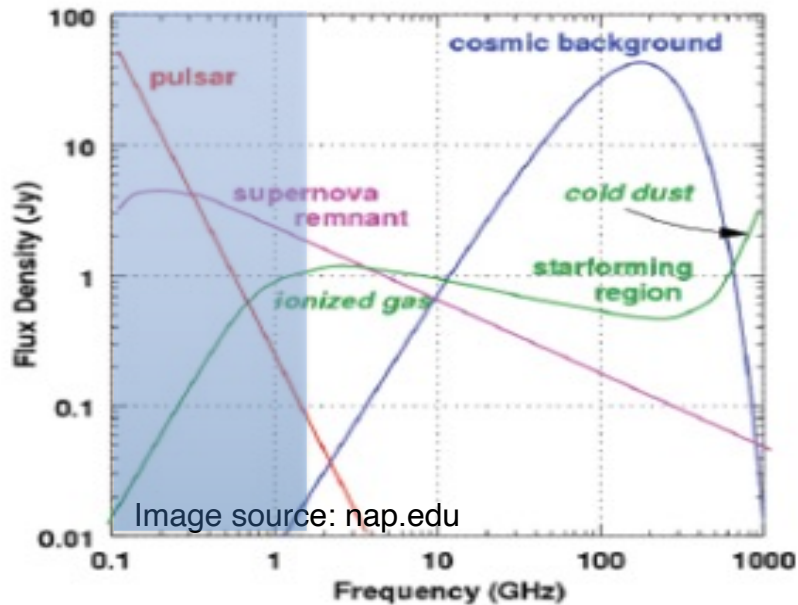
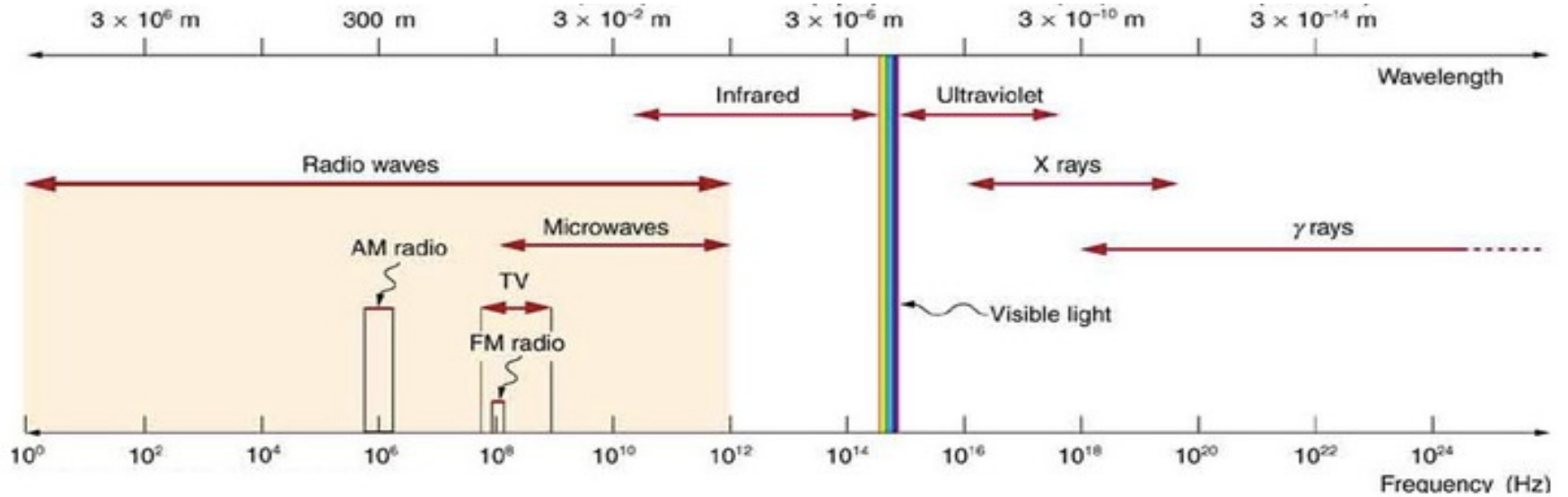
Jayanta Roy

NCRA-TIFR  
Pune

RAS 2023 on 14<sup>th</sup> March

# Radio Astronomy

## EM spectrum



Objects in radio spectrum

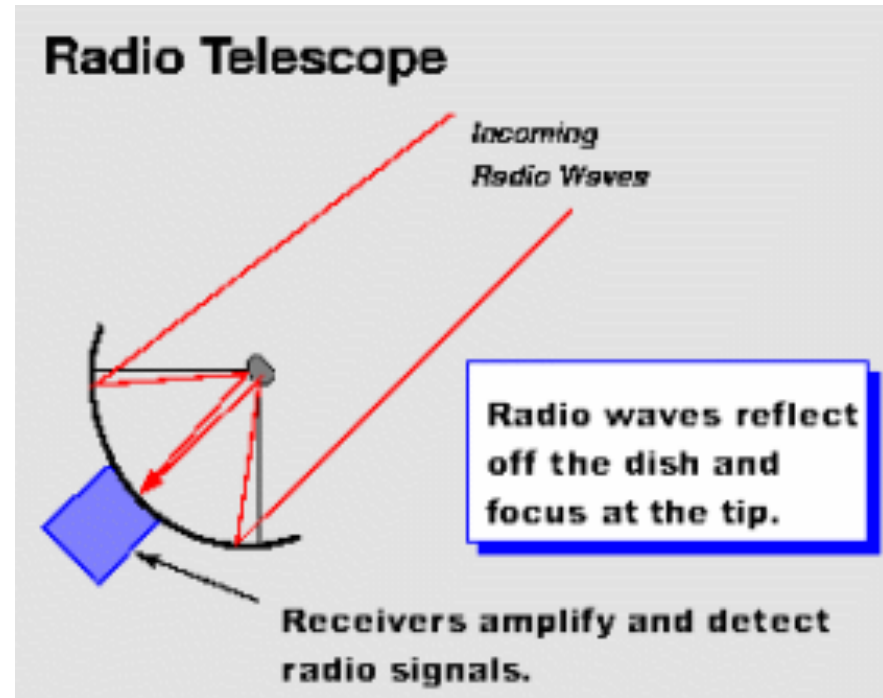
# A Basic Radio Telescope

- Collects radio waves from the celestial sky (from a narrow range of angles), over an effective aperture area
- Focuses the radiation to a feed antenna that converts the signal to an electrical voltage – in 2 orthogonal polarisations
- Converts the voltage signal to power  $\propto$  strength of source signal + receiver noise
- For high sensitivity (to see faint sources out to the distant part of the universe)

Large collecting area  $\Rightarrow$  Large dishes  
High quality, low noise electronics in the receivers

Large bandwidth of observations

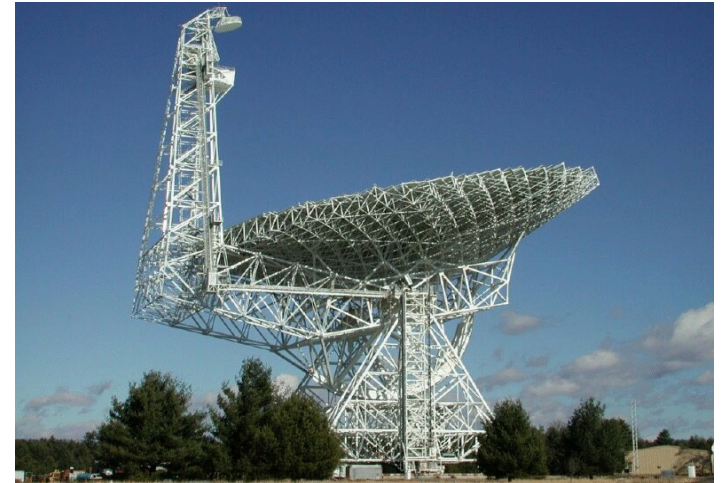
Long integration time to achieve the desired signal-to-noise level



- Celestial radio signals are VERY weak ; unit of flux used is :  
 $1 \text{ Jy} = 10^{-26} \text{ W / m}^2 / \text{Hz}$
- Input radio power into a typical telescope is  $\sim -100 \text{ dBm}$  !

# Single Dish versus Array Telescopes

- Resolution and sensitivity depend on the physical size (aperture) of the radio telescope
- Due to practical limits, fully steerable single dishes of more than  $\sim 100$  m diameter are very difficult to build  
resolution  $(\lambda / D) \sim 0.5$  degree at 1 metre (very poor)
- To synthesize telescopes of larger size, many individual dishes spread out over a large area on the Earth are used
- Signals from such array telescopes are combined and processed in a particular fashion to generate a map of the source structure
- resolution  $(\lambda / D_s)$  ,  $D_s =$  largest separation



The new 100-m Greenbank Telescope



The Very Large Array Telescope

# Introducing a modern radio telescope

## The GMRT

- The Giant Metre-wave Radio Telescope (GMRT) is a new, world class instrument for studying astrophysical phenomena at low radio frequencies (150 to 1450 MHz)
- Designed and built primarily by NCRA, a national centre of TIFR.
- Array telescope consisting of 30 antennae of 45 metres diameter, operating at metre wavelengths -- the largest in the world at these frequencies!

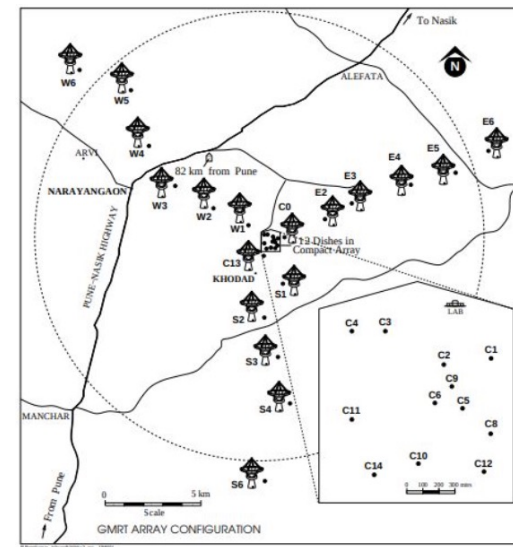




# GMRT with upgrades

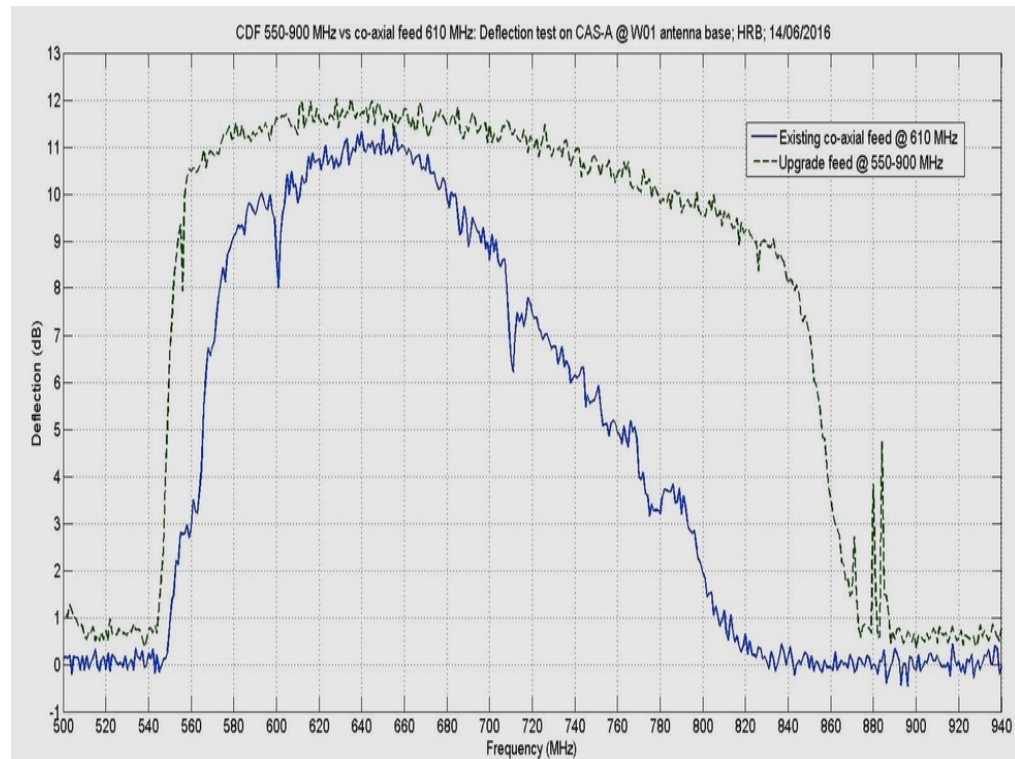


Array located at 80 km north to Pune consisting of 30 antennas over 25 km maximum baseline



A radio interferometer with fully steerable dishes of 45 metres diameter, operating over 120-250, 250-500, 550-850 and 1060-1460 MHz bands having good  $G/T_{\text{sys}}$

An increase of instantaneous bandwidth from 32 to 200/400 MHz makes GMRT an excellent instrument for imaging and time-domain studies



# Sub-systems of the GMRT

Mechanical sub-system

Servo sub-system

Antennas (feed and RF)

Analog Receiver sub-system

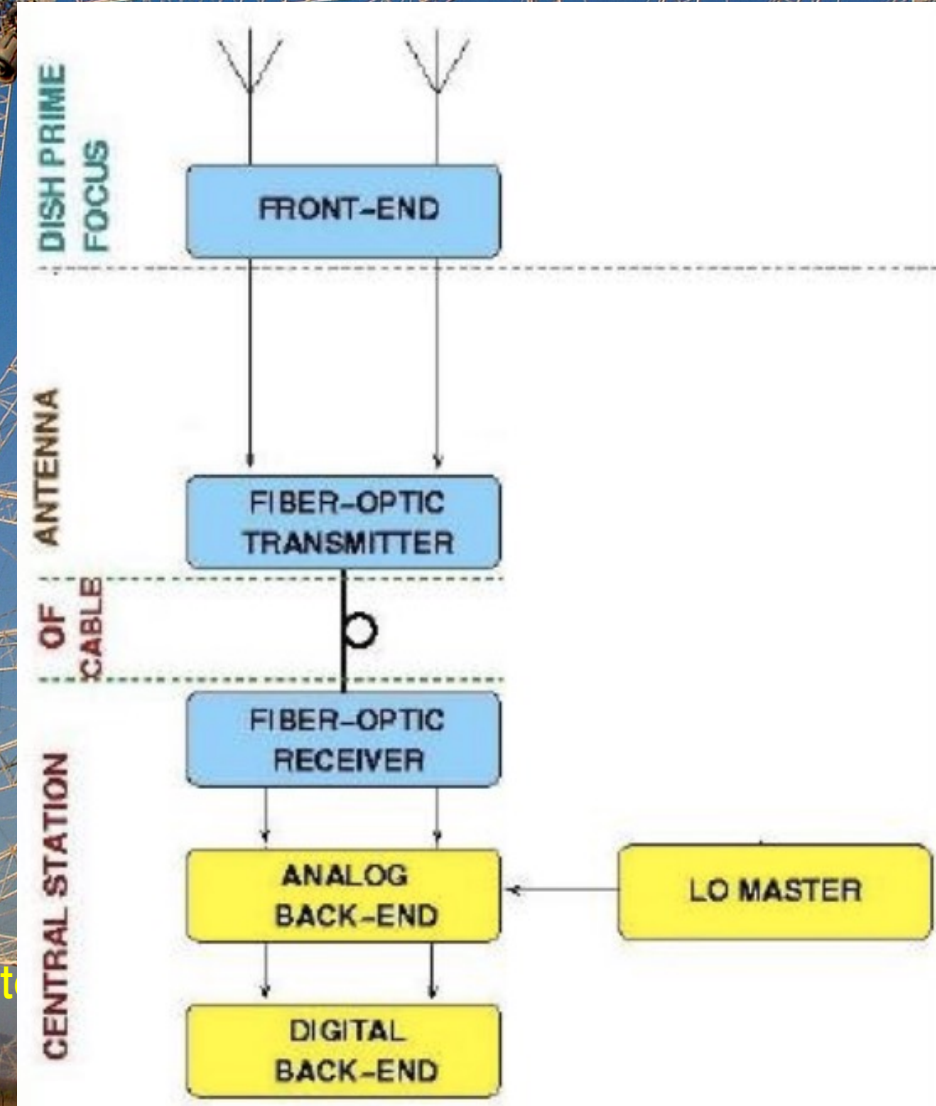
Optical fibre sub-system

Digital Receiver sub-system -- correlator

Telemetry sub-system

“On-line” Control and Monitor sub-system

Off-line data processing chain(s)





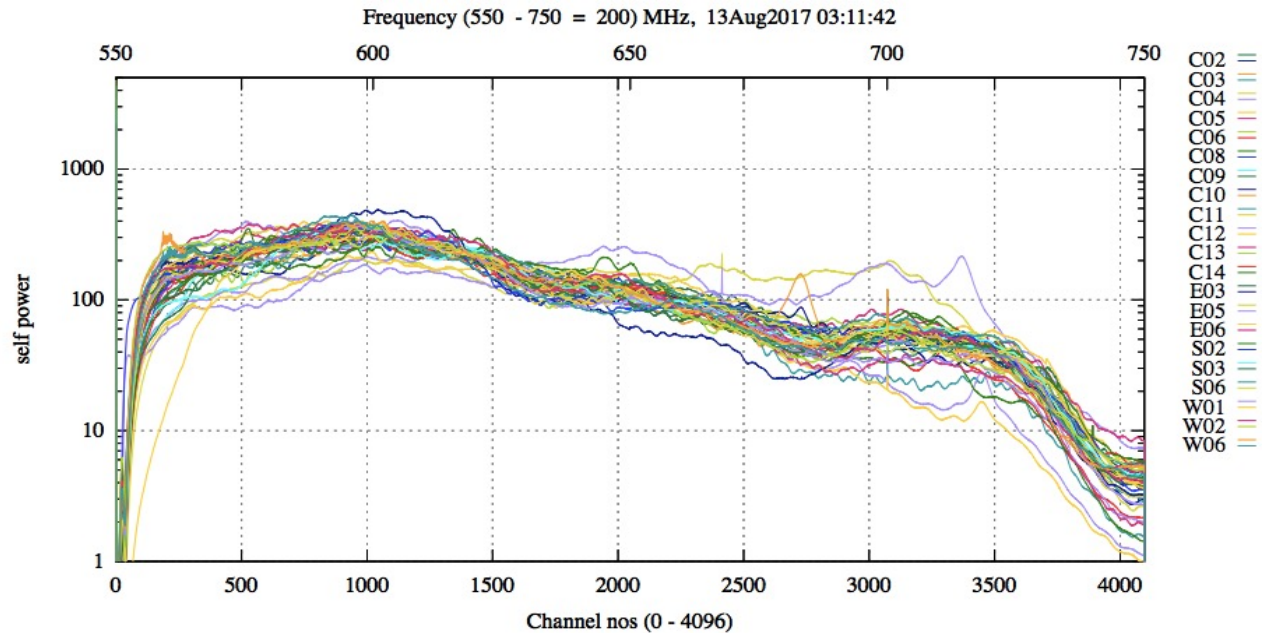
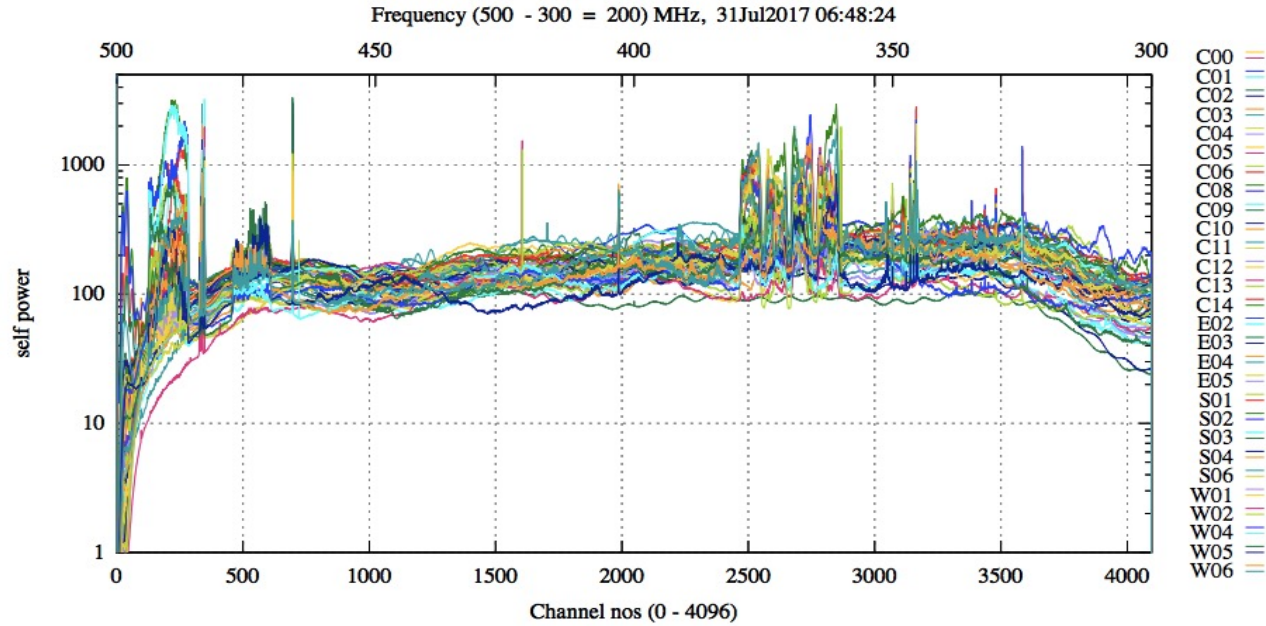
# Feed/Dipole

Antenna primary feeds are placed on a rotating turret near the focus of the 45-m dishes.

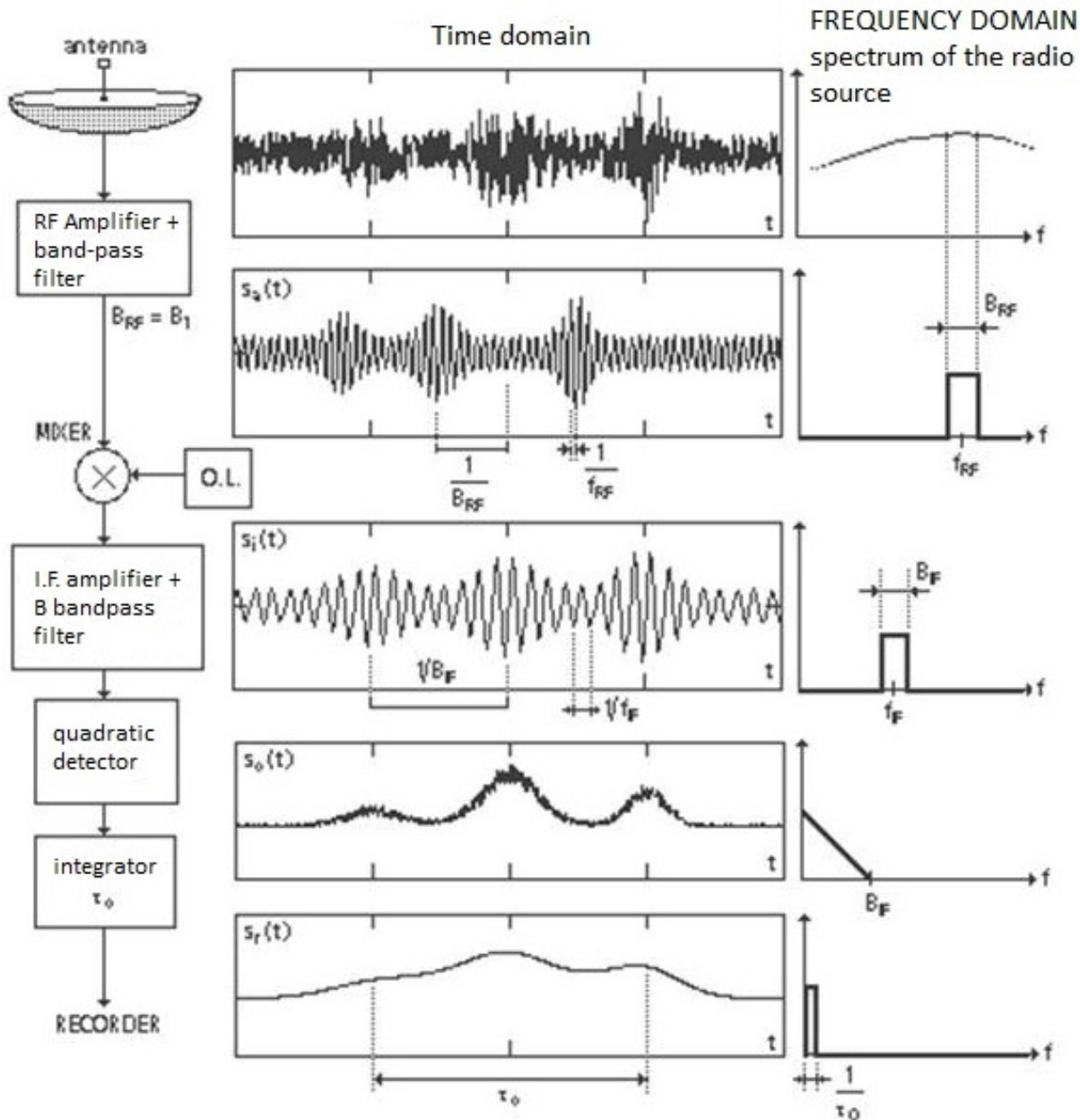




# Band of signals



# Radio Receivers



Sensitivity or radiometre Equation:

$$\Delta T = \frac{T_{sys}}{\sqrt{B_F \tau_0}}$$

$$P_{radio\_source} = S A_{eff} = k T \text{ (watts/Hz)}$$

$$\text{Gain (G) of radio system} = T / S \text{ (Kelvins/Jy)}$$

$$G_{Arecibo} = 11 \text{ K/Jy}$$

$$G_{GMRT} = 9 \text{ K/Jy or } 1.8 \text{ K/Jy}$$

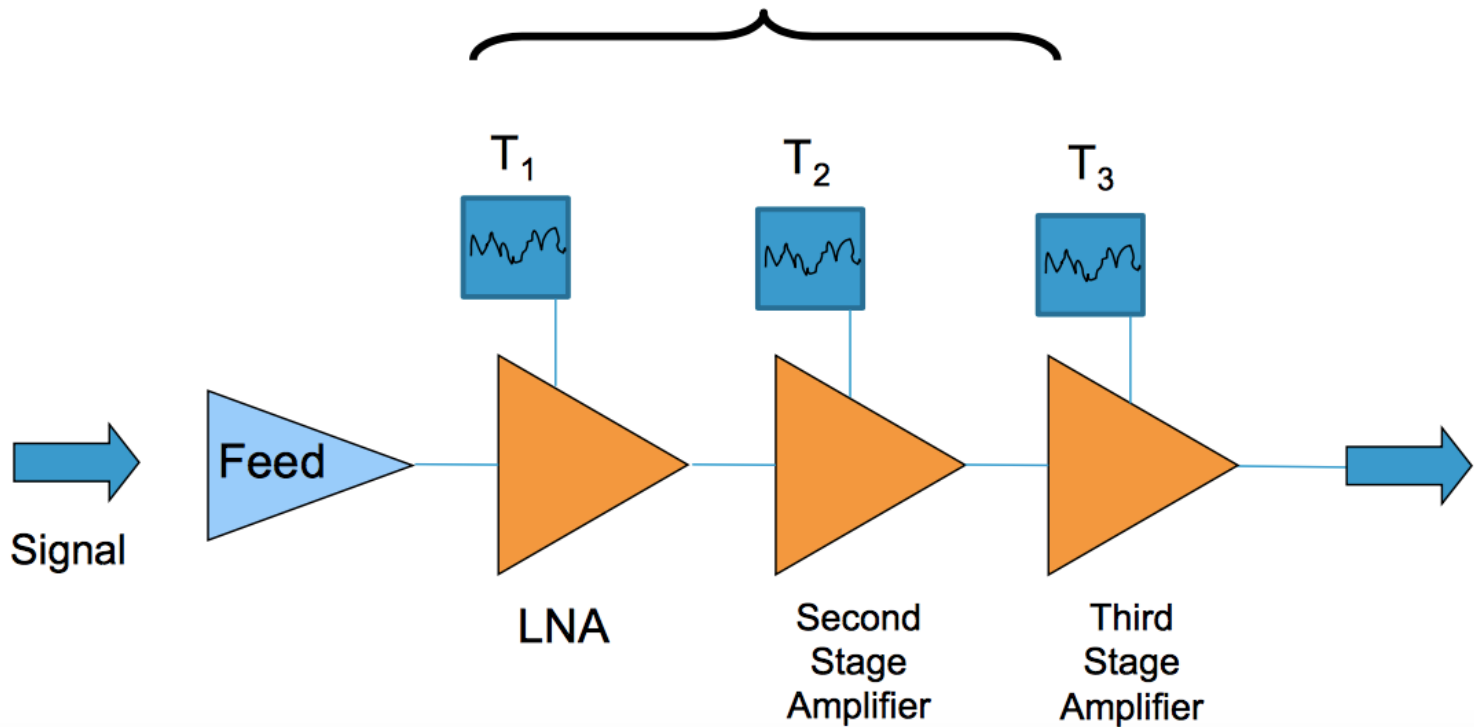
$$G_{GBT} = 2 \text{ K/Jy}$$

$$G_{Effelsburg} = 1.5 \text{ K/Jy}$$

$$G_{Parkes} = 0.74 \text{ K/Jy}$$

# Receiver noise temperature

$$T_{system} = T_1 + \frac{T_2}{Gain_{LNA}} + \frac{T_3}{Gain_{LNA} \times G_2} + \frac{T_4}{Gain_{LNA} \times G_2 \times G_3} \dots\dots$$



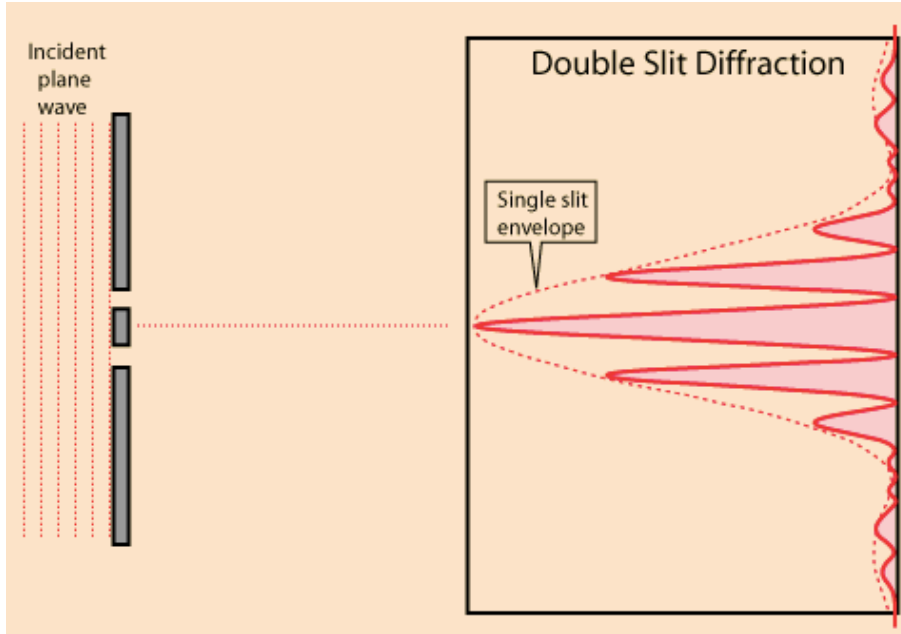
Noise in the data

$$\sigma_S = \frac{T_{sys}}{G \sqrt{2 \Delta \nu t_{samp}}}$$



# Radio Interferometers

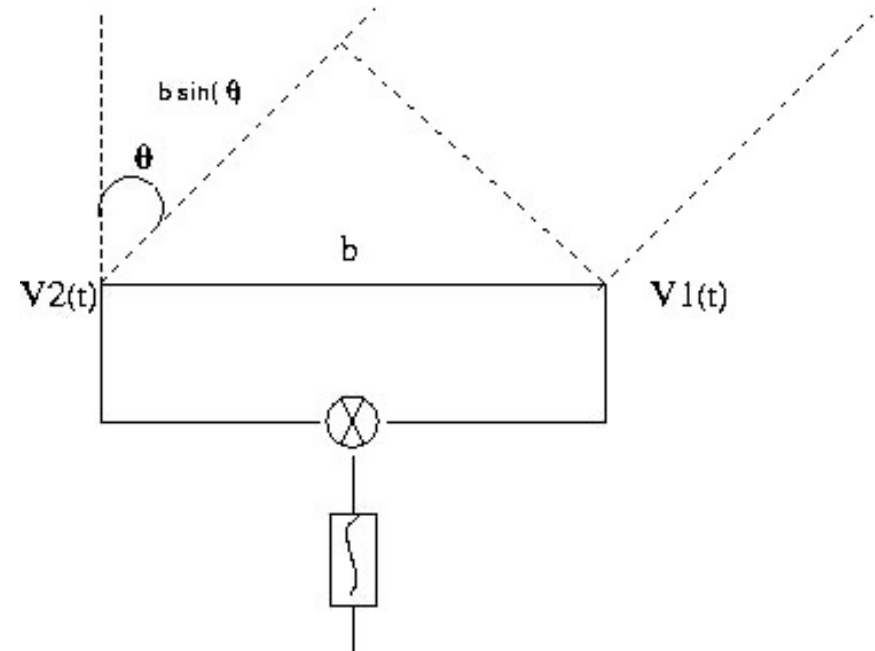
## Interference fringe patterns



Diffraction limit of a telescope =  $1.22 \lambda/D$

Interferometer measures the spatial coherence function of the incident electric field

Signals arrive at Correlator from different Antennas have different *propagation* and *instrumental* delay.



$$\tau = b/C \sin(\theta)$$

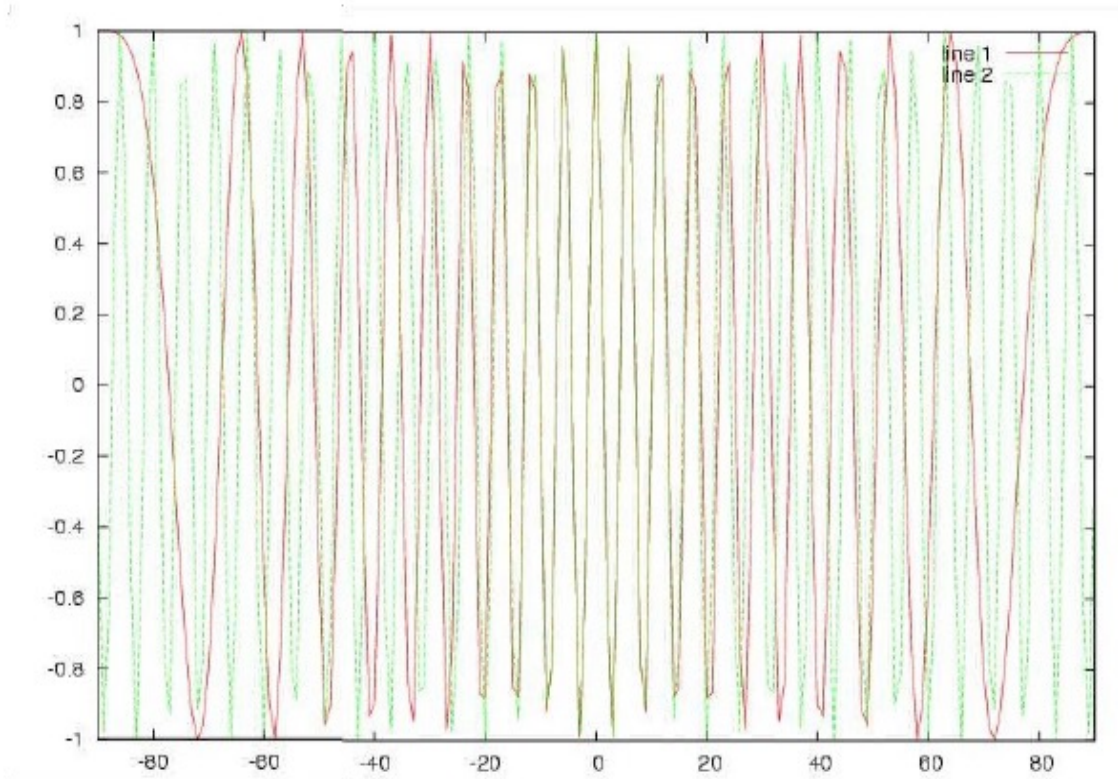
$$d\tau/dt = b/C \cos(\theta) d\theta/dt$$

## *Monochromatic radiation*

$$V_1(t) = \cos(2\pi\nu t)$$

$$V_2(t) = \cos(2\pi\nu(t - \tau_g))$$

Output of the interferometer :  $\cos(2\pi\nu\tau_g)$



Try with different b  
and  $\nu$  combinations

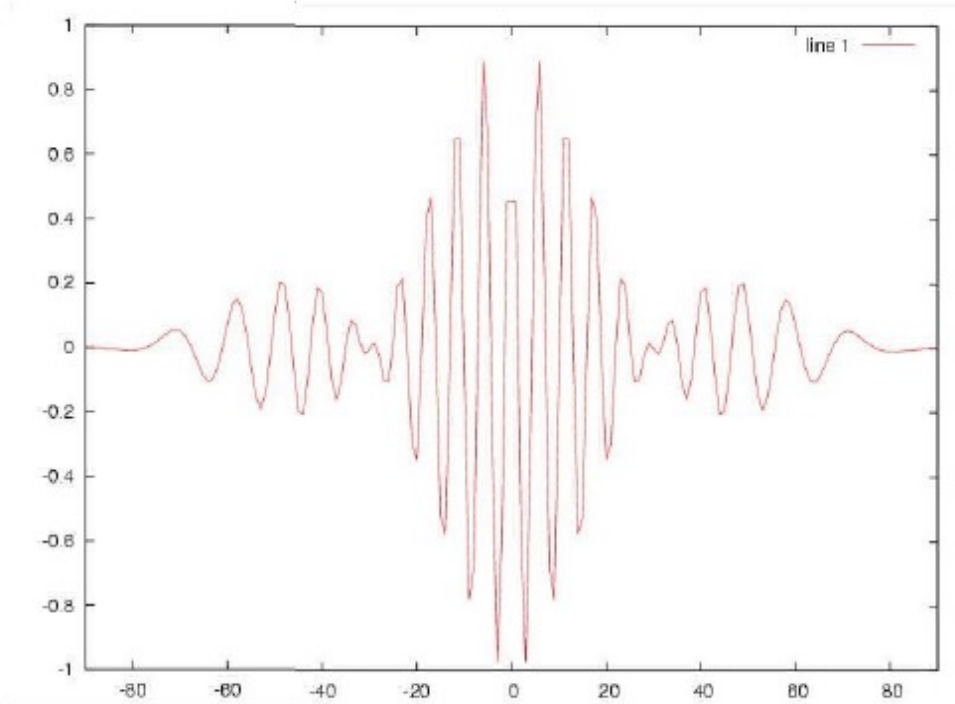
Fringe rate is maximum at zenith and minimum when source is rising or setting

## *Quasi-monochromatic radiation*

Radiation spectrum contains all frequencies in a band  $\Delta\nu$  around  $\nu$

Averaging over the all  $\nu$  reduce the amplitude of the fringe

Output of the interferometer :  $\cos(2\pi\nu\tau_g) \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g}$



Increase  $\Delta\nu$  to see how fringe amplitude decreases

**Increase  $\Delta\nu$  without losing fringe amplitude !!**

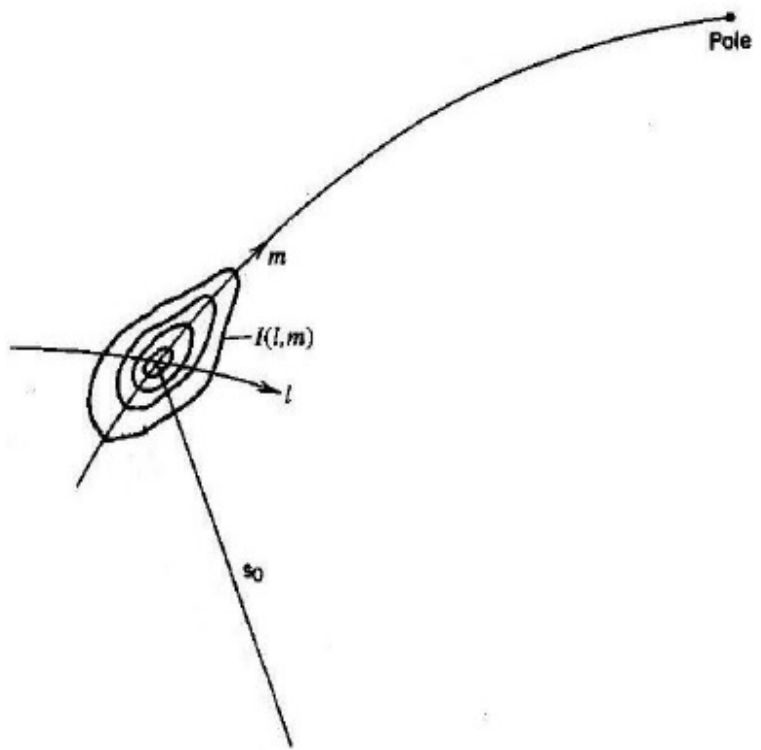


# Mapping from Antenna spacing co-ordinates (X, Y, Z) to Projected baseline co-ordinates (u, v, w)

$$u_{\lambda} = X_{\lambda} \sin(H) + Y_{\lambda} \cos(H)$$

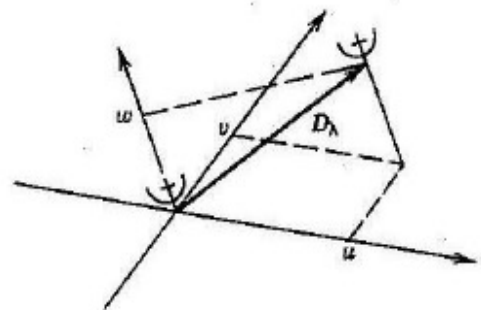
$$v_{\lambda} = -X_{\lambda} \sin(\delta) \cos(H) + Y_{\lambda} \sin(\delta) \sin(H) + Z_{\lambda} \cos(\delta)$$

$$w_{\lambda} = X_{\lambda} \cos(\delta) \cos(H) - Y_{\lambda} \cos(\delta) \sin(H) + Z_{\lambda} \sin(\delta)$$



All fringe and delay corrections apply for a specific point on the sky  $S_0$  :  
Phase tracking center

w-term for a baseline is giving path length difference between two antennas



## Required parameters :

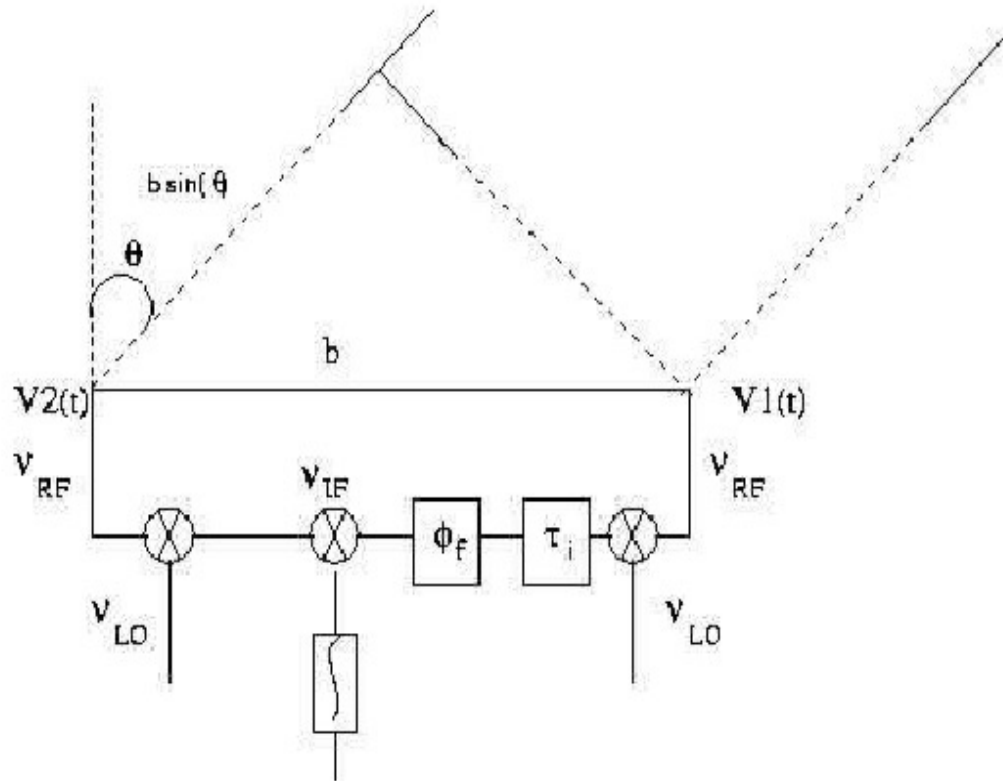
Delay  $\tau_g = \frac{w}{c}$

Fringe phase  $\phi = w_\lambda$

Fringe frequency  $\frac{dw_\lambda}{dt} = \frac{dw_\lambda}{dH} \frac{dH}{dt} = (\nu_{\text{observation}}) \frac{d\tau_g}{dt}$

$$= -\omega_e [X_\lambda \cos(\delta) \sin(H) + Y_\lambda \cos(\delta) \cos(H)]$$

## What is fringe stopping and delay tracking



**Delay suffered at RF frequency**

**Correction applies at IF frequency**

$$\begin{aligned} & \langle \cos(\phi_v + 2\pi\nu_{IF}t - 2\pi\nu_{RF}\tau_g) \cos(2\pi\nu_{IF}(t - \tau_i) + \phi_f) \rangle \\ & = \cos(\phi_v + 2\pi\nu_{LO}\tau_g - \phi_f) \end{aligned}$$

Applying this time varying phase  $\phi_f$  is called : fringe stopping

Applying this additional delay  $\tau_i$  is called delay tracking



# Logical flow of the fringe stopping and delay correction :

- Get antenna co-ordinates (x,y,z)
- Get source co-ordinates (RA,DEC)
- Read the time-stamp value
- Calculate the HA(t) of the source
- Estimate the projected baseline co-ordinate (u,v,w)
- delay  $\tau = \frac{W(t)}{C} + \tau_{Fix}$  ; phase  $\Phi = 2\pi\tau(v_{RF} + v_i)$
- New  $\tau = \tau + \frac{d\tau}{dt} \Delta(t)$
- Linear interpolation goes on till re-calculation of  $(\tau, \dot{\tau})$

Total phase 
$$\Phi = 2\pi(v_{RF} + v_i) \left( \frac{W(t)}{C} + \tau_{fix} \right) = 2\pi v_{RF} \left( \frac{W(t)}{C} + \tau_{fix} \right) + 2\pi v_i \left( \frac{W(t)}{C} + \tau_{fix} \right)$$

$$\Phi_{fmg}(t) = 2\pi v_{RF} \left( \frac{W(t)}{C} + \tau_{fix} \right)$$

$$\Phi_{fstc}(v, t) = 2\pi v_i \tau_{frac}$$

$$\dot{(\phi_{fmg})}_{max} = 5 \text{cycles} / \text{sec}$$

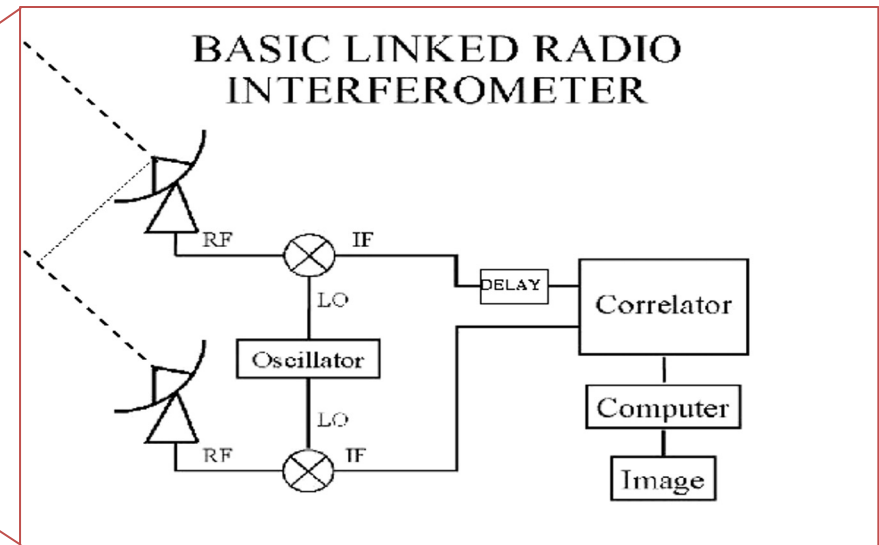
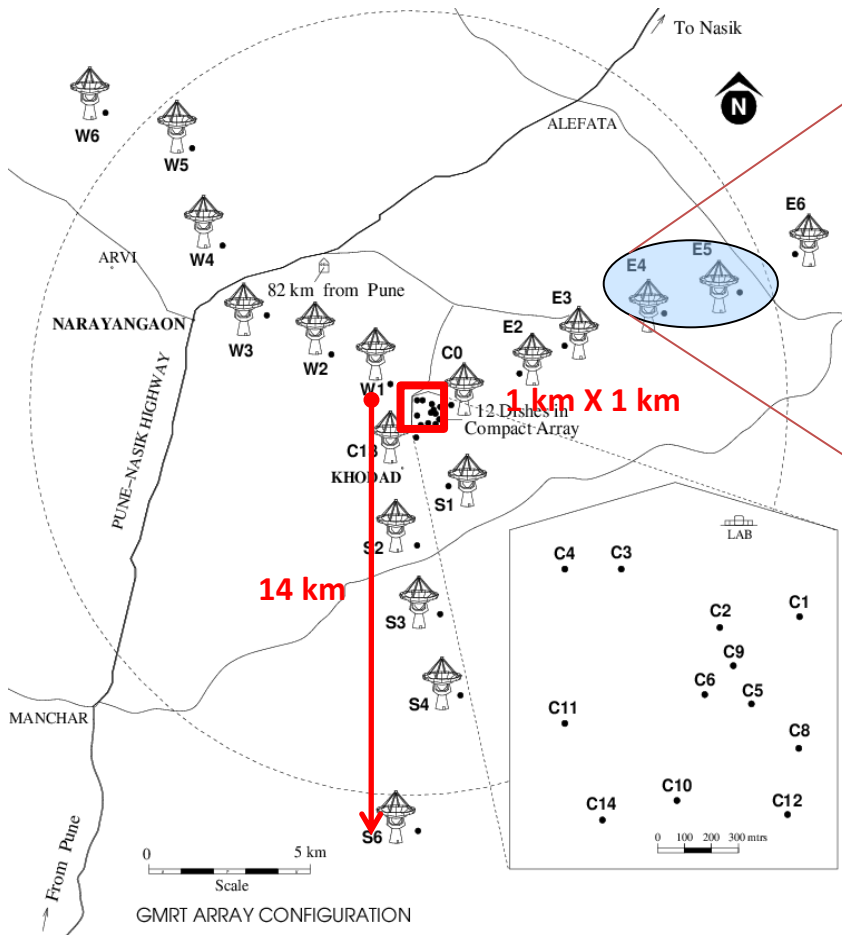
$$(\tau)_{max} = 150 \mu\text{s}$$

$$\dot{(\tau)}_{max} = 3 \text{ns} / \text{sec}$$

$$\Delta \tau = 2 \text{ns}$$

# The GMRT array distribution :

## Concept of Radio Interferometry and Aperture Synthesis



➤ Signals from pair of antennae are cross-correlated (cross-spectrum is obtained)

➤ Product of Interferometer :  
Visibility Function :  $V(r_1, r_2)$

$$V(r_1, r_2) = \langle E(r_1) E^*(r_2) \rangle$$

➤  $\sim N(N-1)$  such instantaneous measurements (Fourier components of the image)

➤ Reconstruction of Source Brightness Distribution :  $I \xleftrightarrow{\text{FFT}} V$  (Aperture Synthesis)

# Design consideration of a back-end for an array telescope

- Digitisations of the analog signals : more bits per sample ➡ better dynamic range
- Ability to correct for variable time delays between pair of antennae ➡ delay and fringe correction
- Extract the spectral information about the celestial source ➡ realization of FFT
- Variable spectral resolutions ➡ ranges from studying continuum sky to finer emission/absorption features of the HI cloud
- Complex correlator ➡ in order to get  $N^2$  instantaneous measurement of the Fourier components of the sky brightness distribution
- Variable time resolution ➡ snapshot imaging to study the dynamic sky
- Ability to observe the Polarized sky
- A high time resolution total power receiver ➡ to study the time domain features of the periodic signal from Pulsars
- Ability to add sophisticated algorithms to detect and filter out RFI signals at various stages in processing pipeline (wish-list)

# Digitization of signals

## Sampling

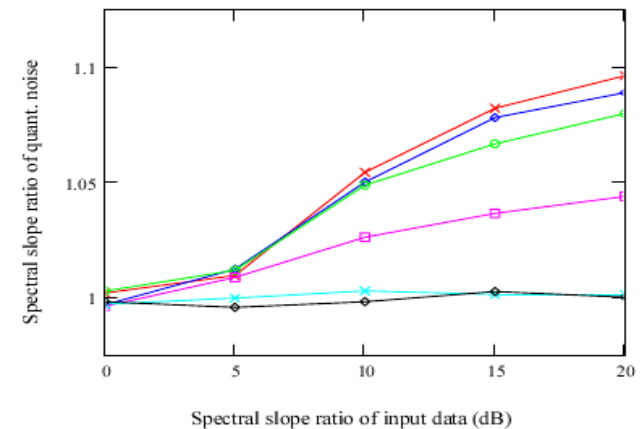
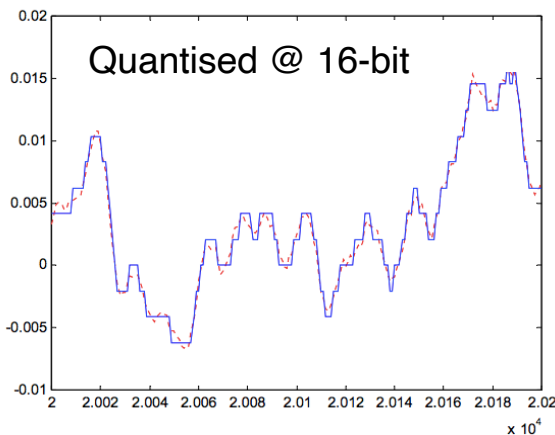
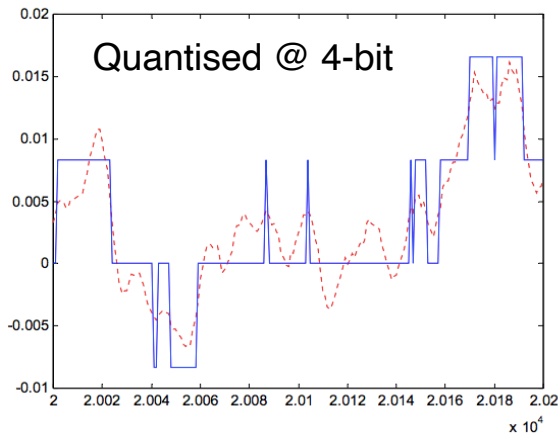
Band-limited signal down-converted to baseband sampled at Nyquist rate with 8 bits per sample  
Input power level adjustment for  $10 \times$  sigma range

## Quantization

Discretization add quantization noise, more severe for fewer levels system.

Variation of gain with frequency makes the SNR of correlated signal varies across the band due to quantization noise

No. of levels	Quantize efficiency
3	80.9%
8	96.25%
16	98.84%
32	99.65%
256	99.99%

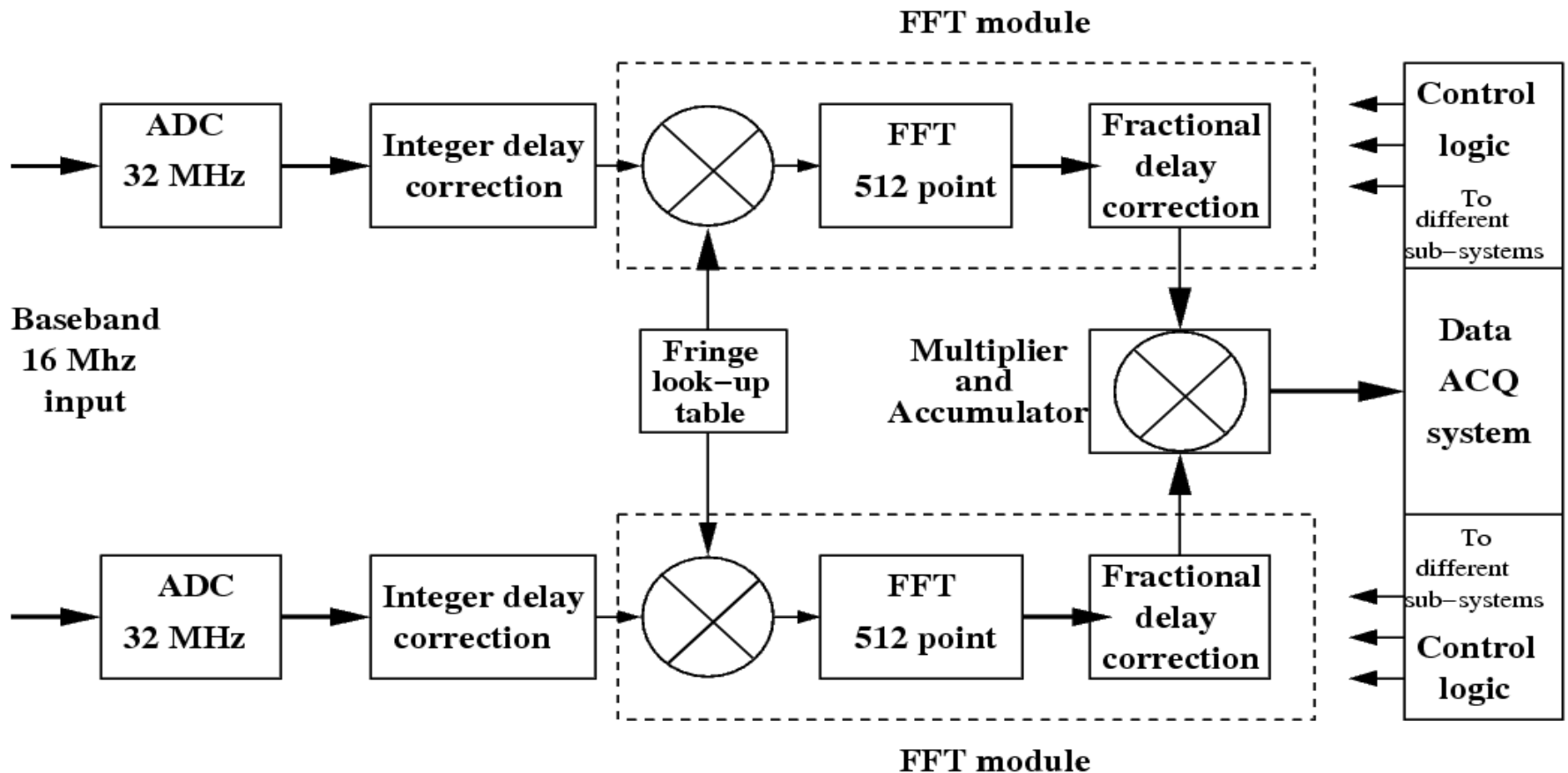




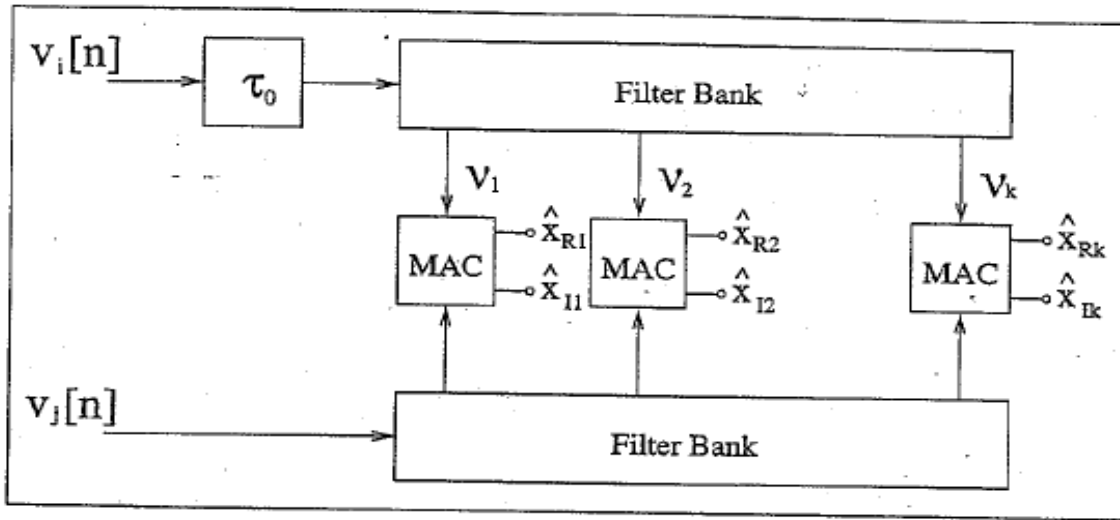
# Digital Backend of a radio telescope like the GMRT

➤ Simultaneous operation as

- FX correlator as an Imaging instrument
- Beamformer as a Pulsar receiver



# Spectral correlator : FX Vs XF



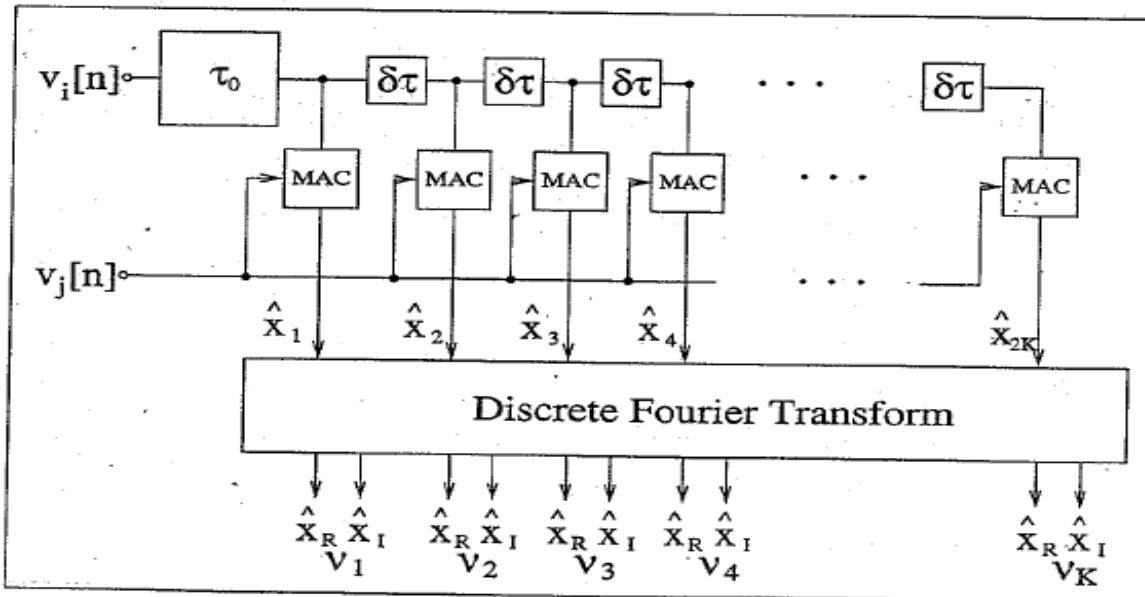
FX correlator

$$\sim 2 * Na * BW * \log(NFFT) +$$

$$BW * Na * (Na + 1)$$

$$= 2 * Na * BW [\log(NFFT) + (Na + 1) / 2]$$

Cops



XF correlator

$$\sim BW * Na * (Na + 1) *$$

$$[1 + \log(NFFT)]$$

Cops

For GMRT, Na=30, NFFT=1024

$$C_{XF} / C_{FX} > 3$$

# Spectral correlator : FX Vs XF

## Sensitivity

FX operates on block of data determined by the FFT algorithm. Cross-correlation is derived from fewer pair of samples than XF → loss of sensitivity in FX, requires overlapping adjacent blocks, net increase in computing load in FX

## Quantization

Correction for quantization efficiency before correlation possible for XF, but difficult for FX → XF is advantageous for small no. of bit correlator

## Closure errors

FX correlator is less vulnerable to baseline dependent systematic effects

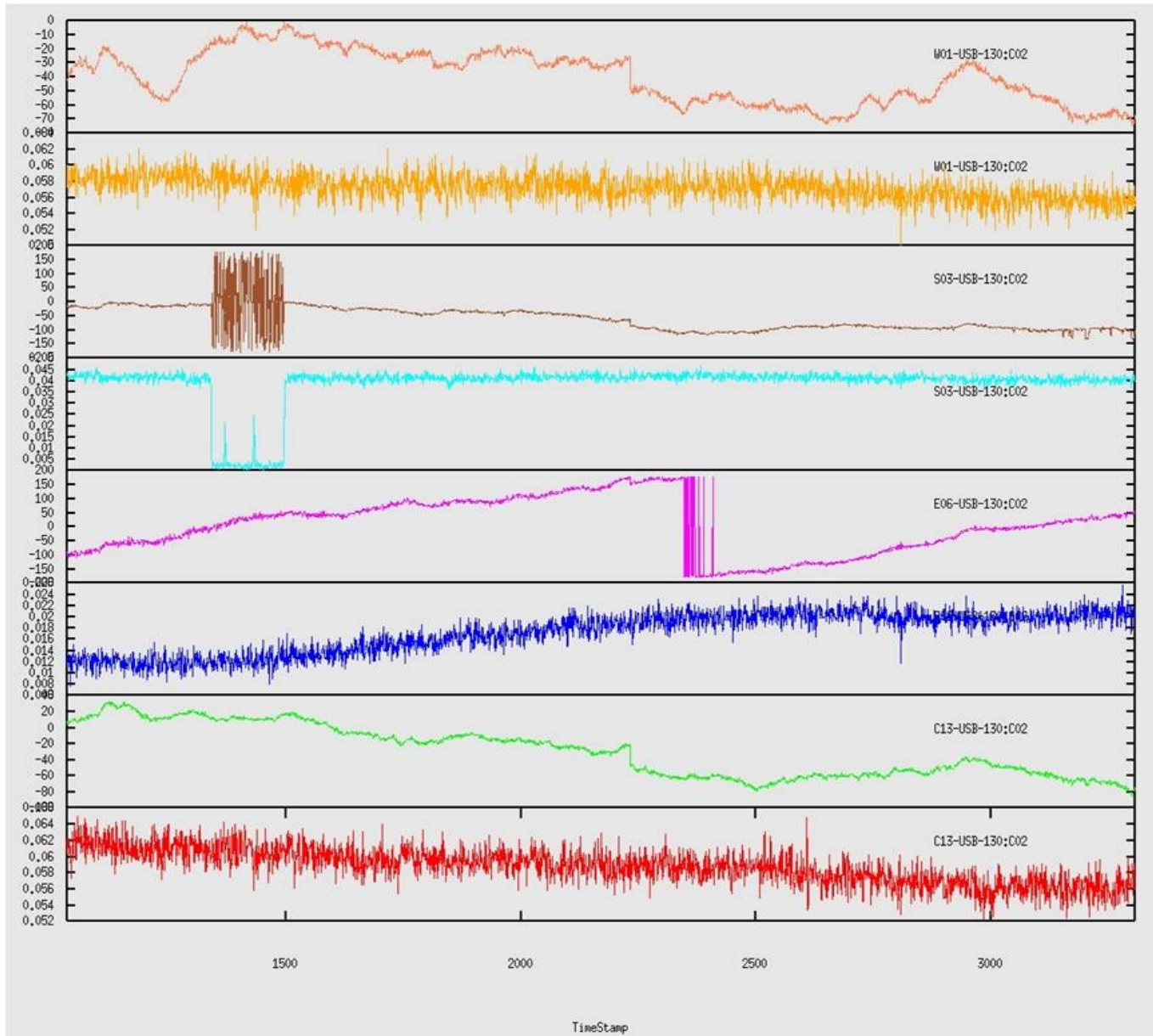
## Fractional sample correction

In XF correction can be done in base-line base after transform

## Improvement in the shape of channel bandpass

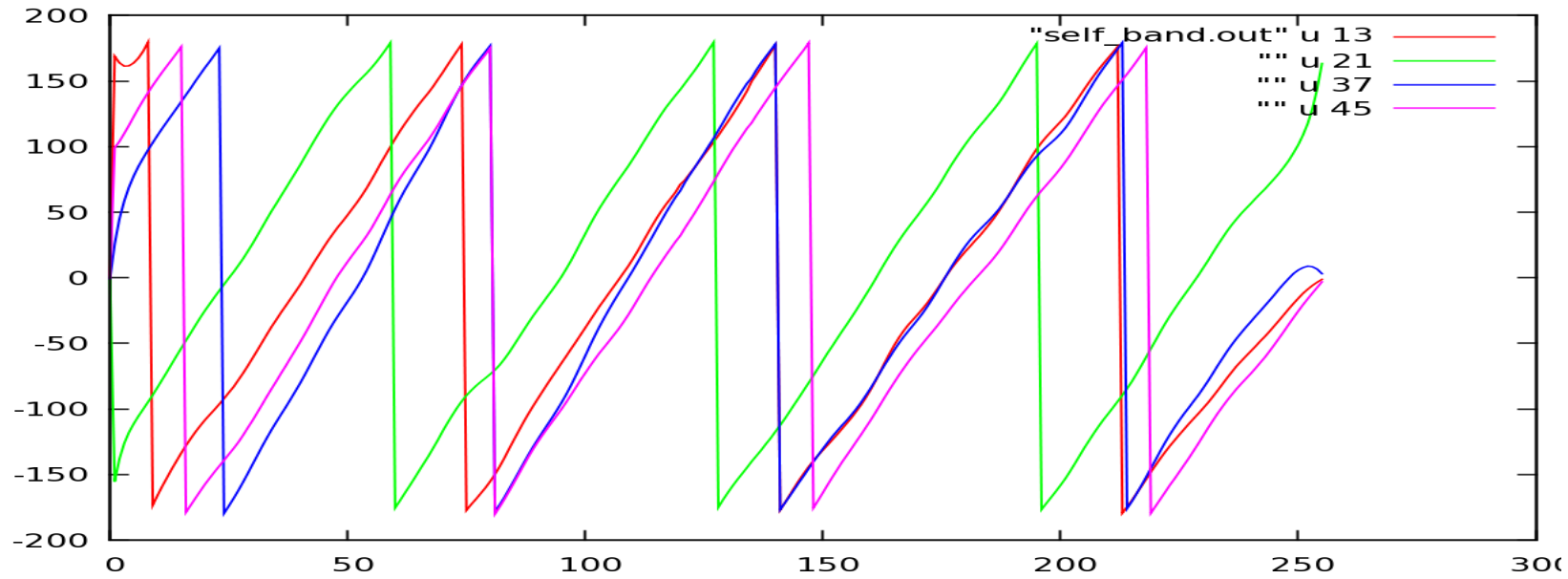
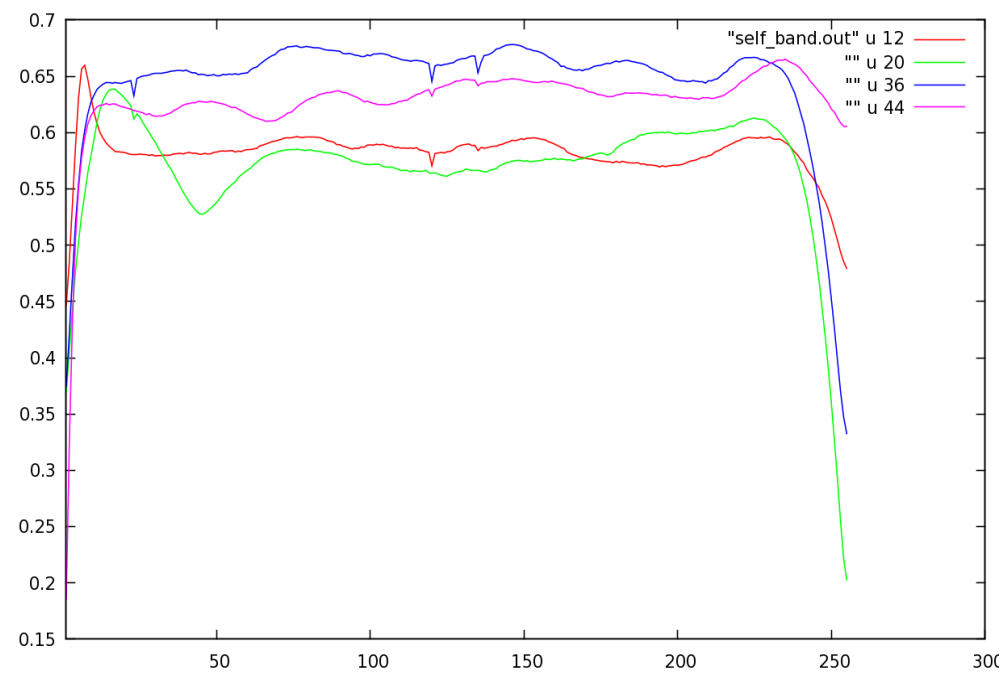
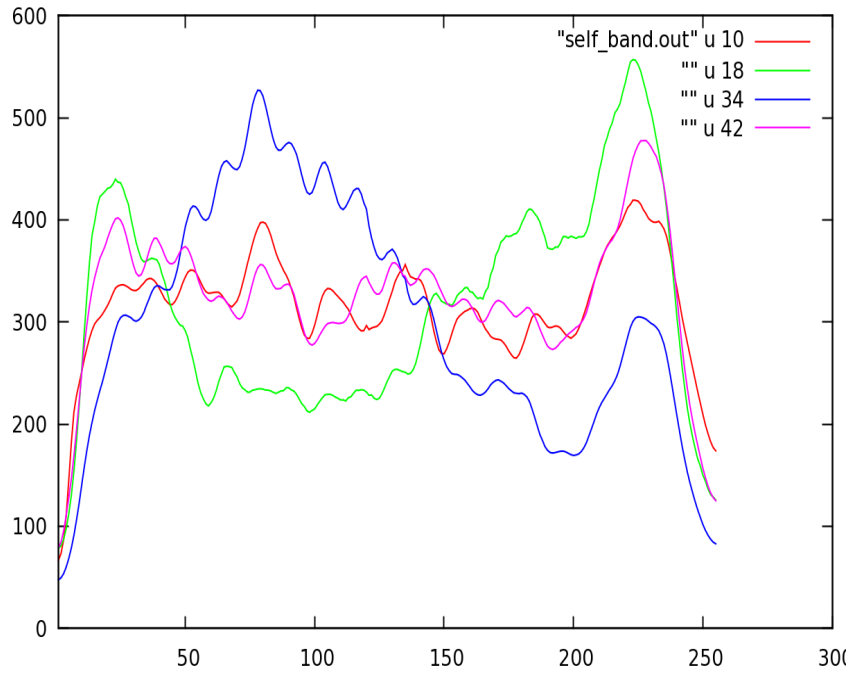
FX correlator bandpass function of each channel is  $\text{Sinc}^2$ , whereas for XF it is  $\text{Sinc}$

# Cross correlation output



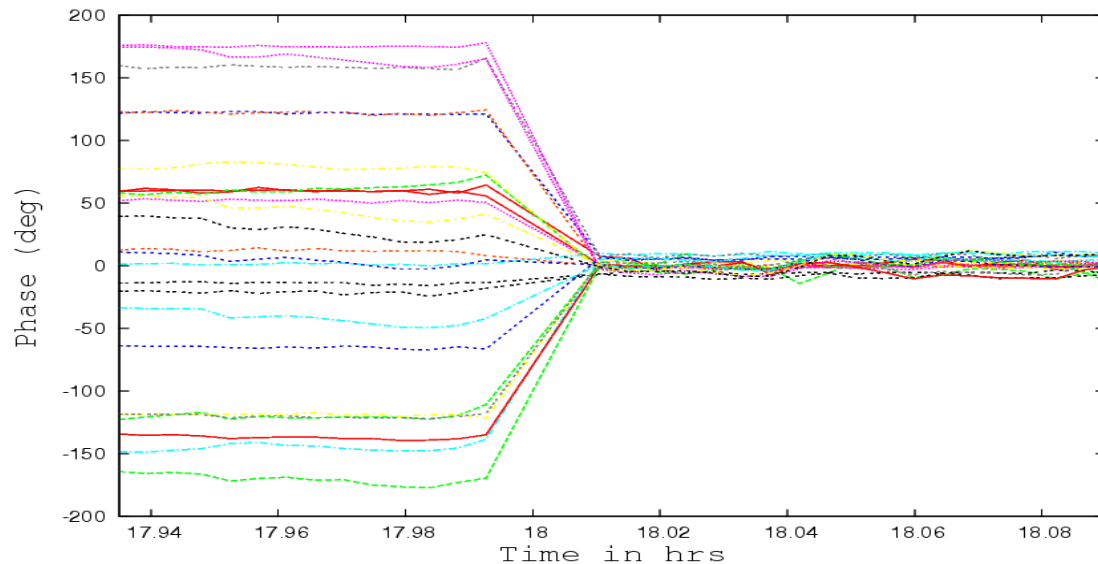
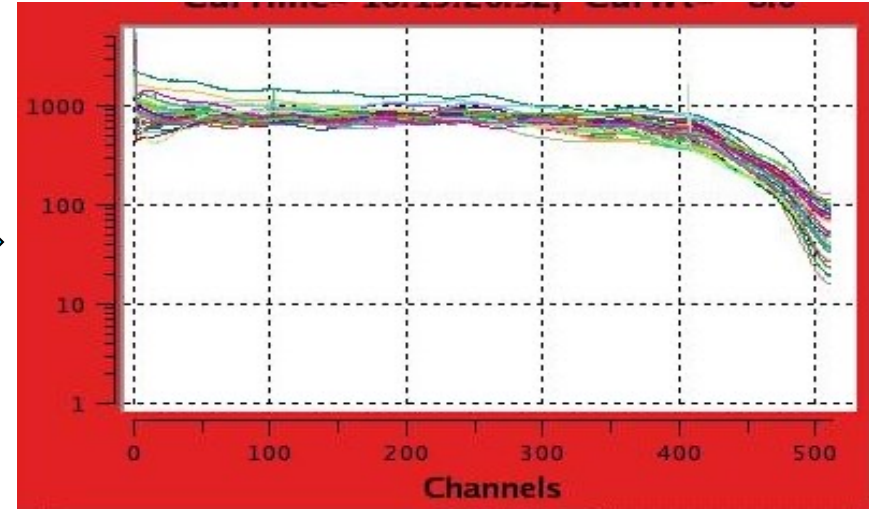
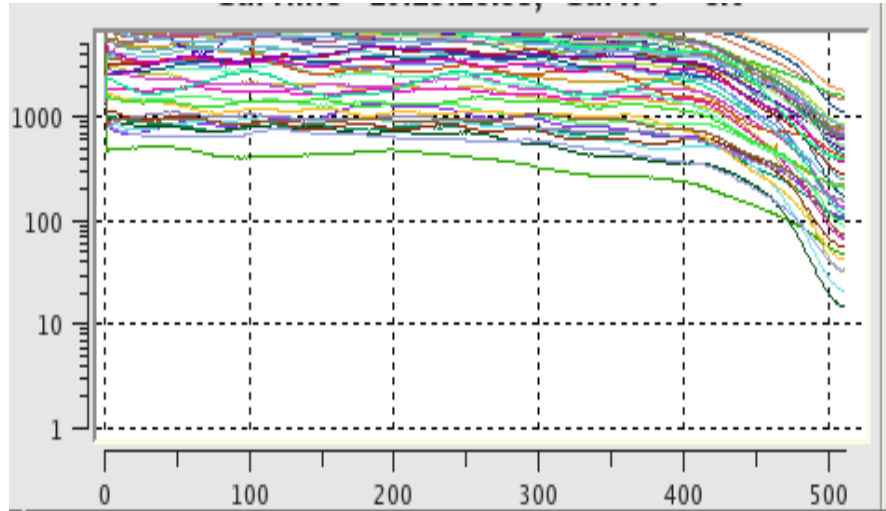


# Cross spectrum in presence of correlated signals



# Amplitude and phase calibration for beamformer

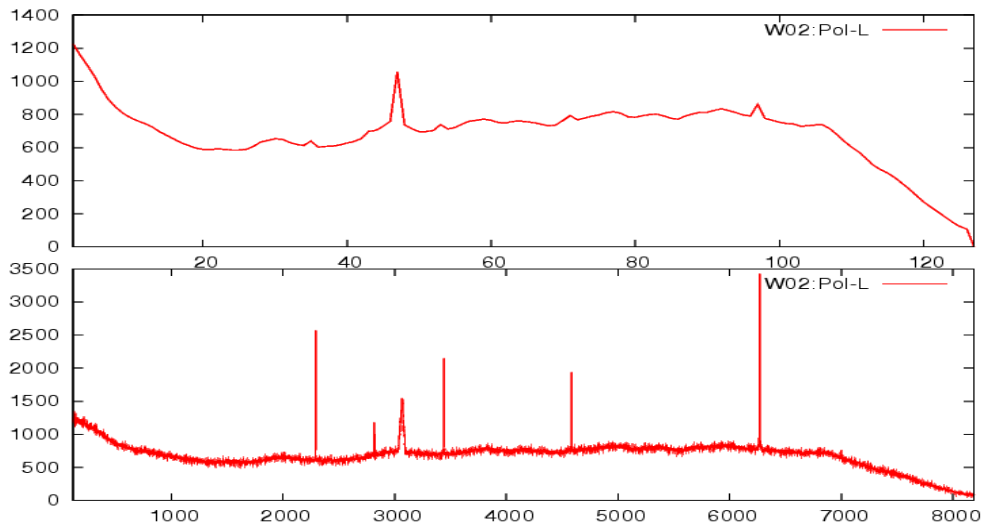
## Antenna based gain offset correction



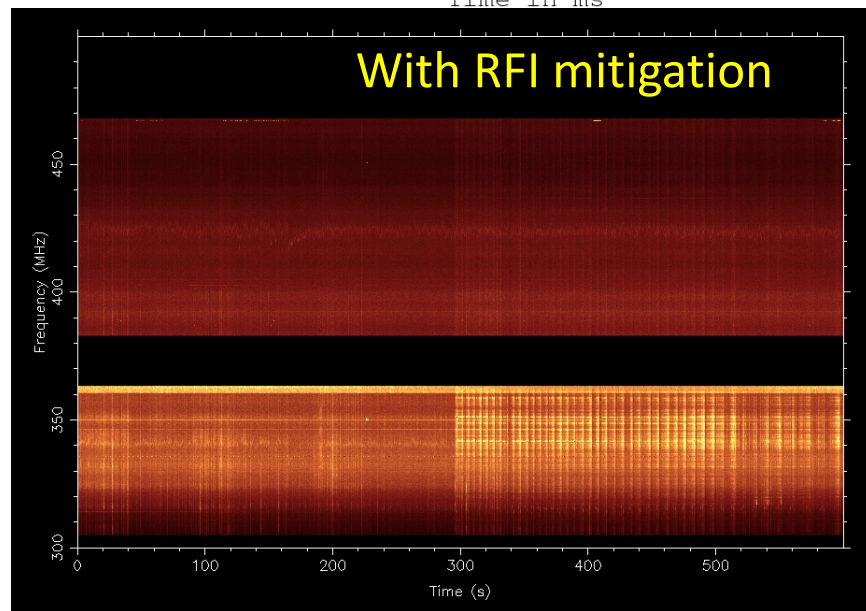
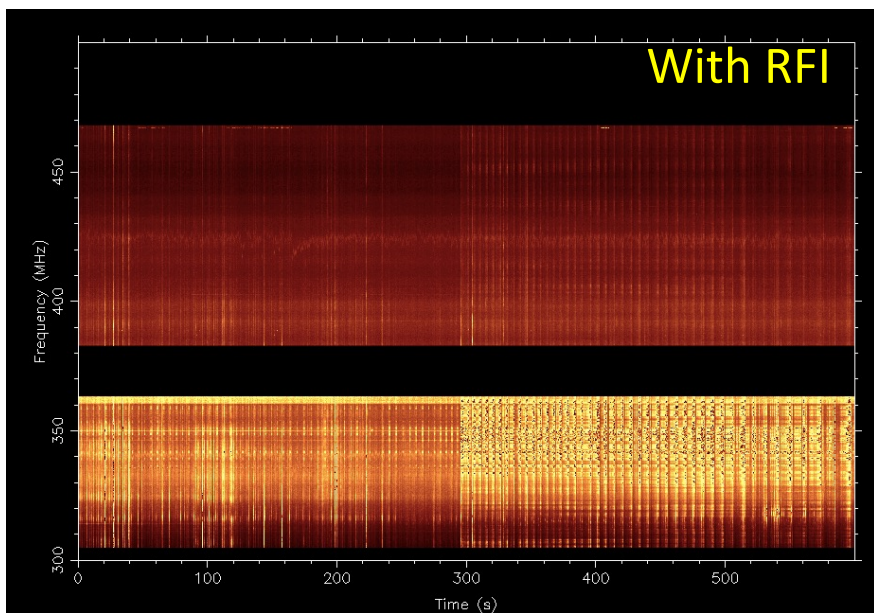
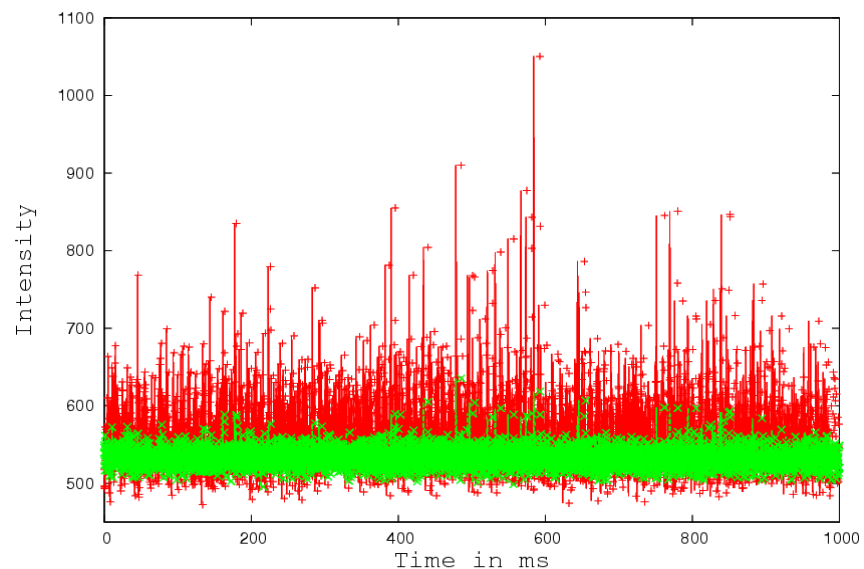
Antenna based phase  
offset correction

# Detecting and mitigating RFIs

## High spectral res

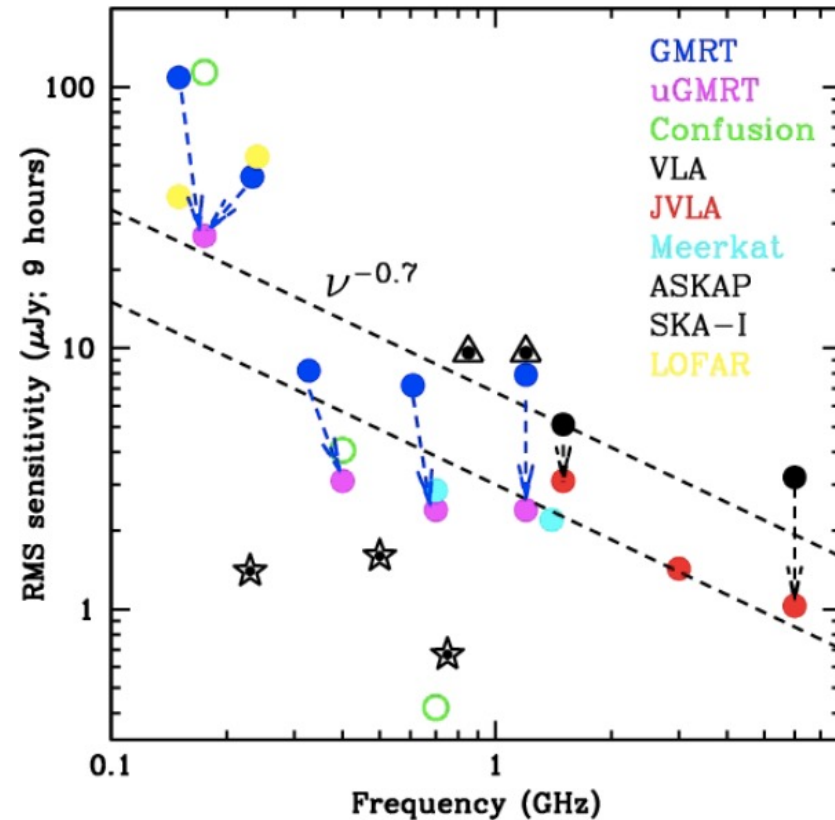


## Pre detection RFI removal



# The upgraded GMRT (uGMRT)

- ✓ Seamless frequency coverage from 150 to 1450 MHz
- ✓ Improved  $G/T_{\text{sys}}$
- ✓ Increase of instantaneous bandwidth from 32 MHz to 200 MHz/400 MHz
- ✓ Expected increase in sensitivity by 3x
- ✓ Time-domain study simultaneously in 3-4 frequencies



Credit: Nissim Kanekar





*Thank you*