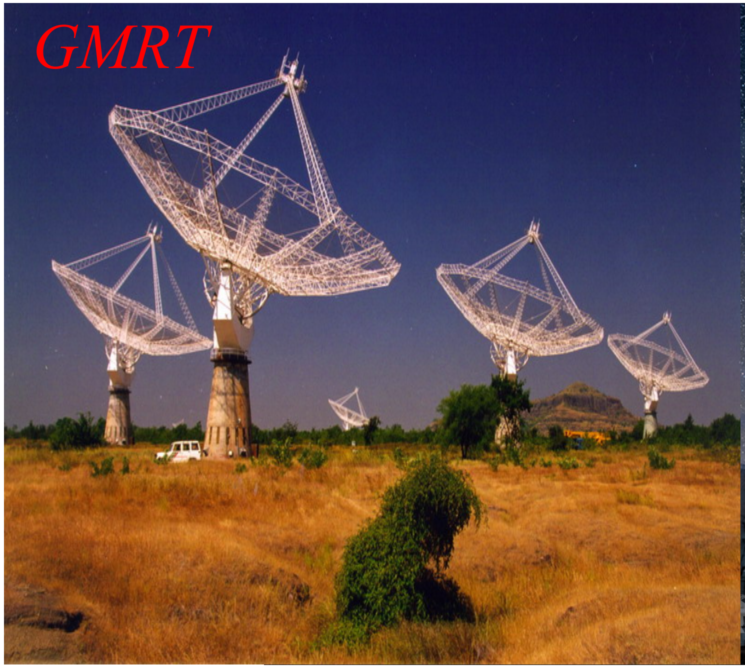
INTERFEROMETRY

Nissim Kanekar *(NCRA-TIFR)*


WSRT


VLA


ATCA


GMRT

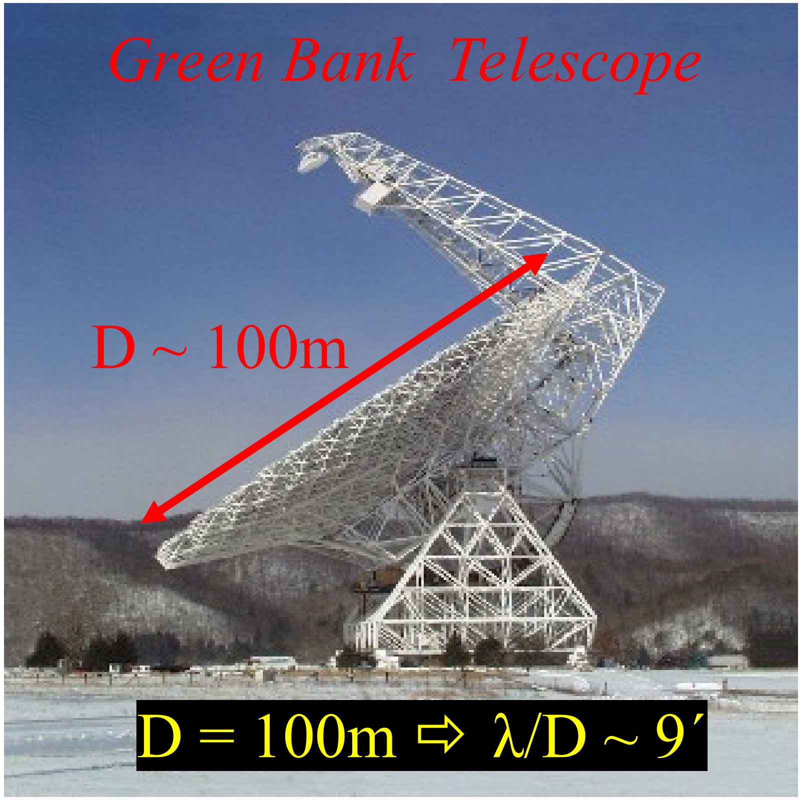

ALMA

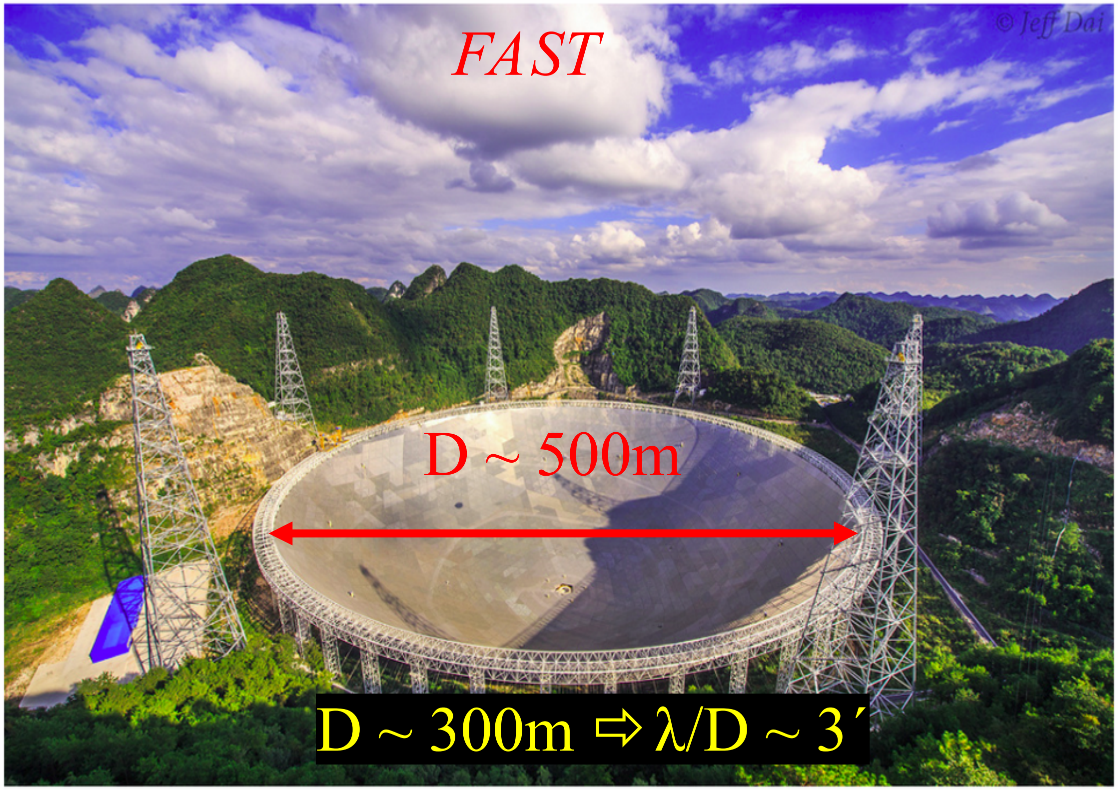

MeerKAT

(Courtesy: ASTRON, NRAO, ATNF, SARAO, GMRT) (Rick Perley, Frank Schinzel)

WHY INTERFEROMETRY?

- Angular resolution ~ λ/D ⇨ Need a *huge* single dish for high resolution.



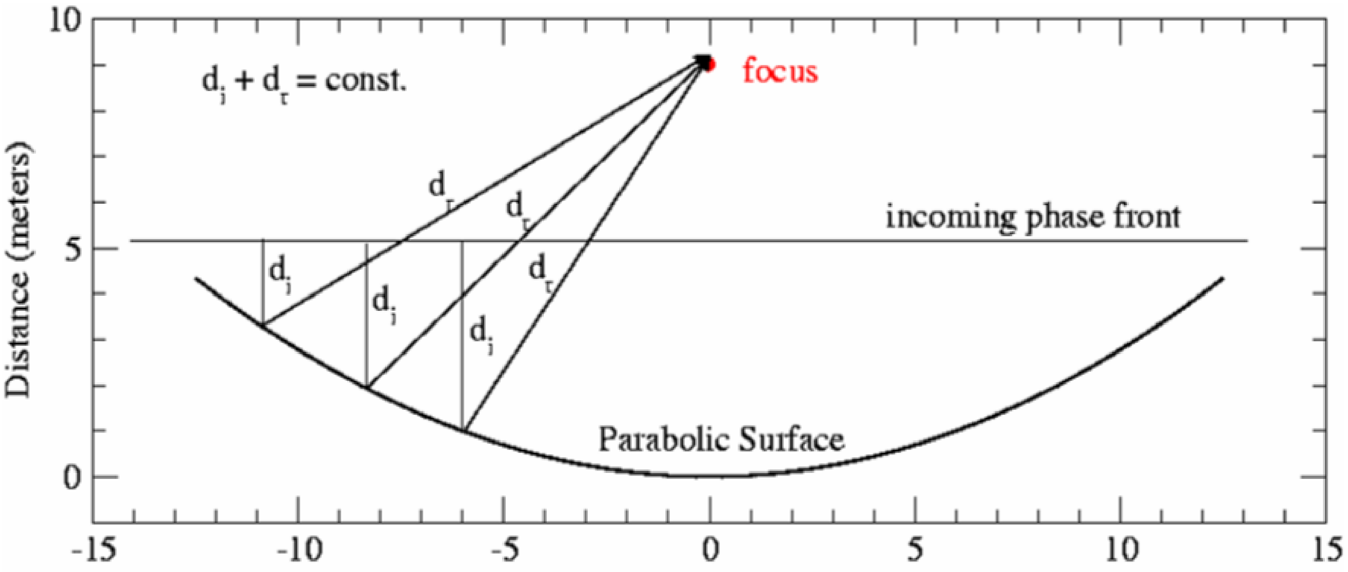


(Courtesy: NRAO, Jeff Dai)

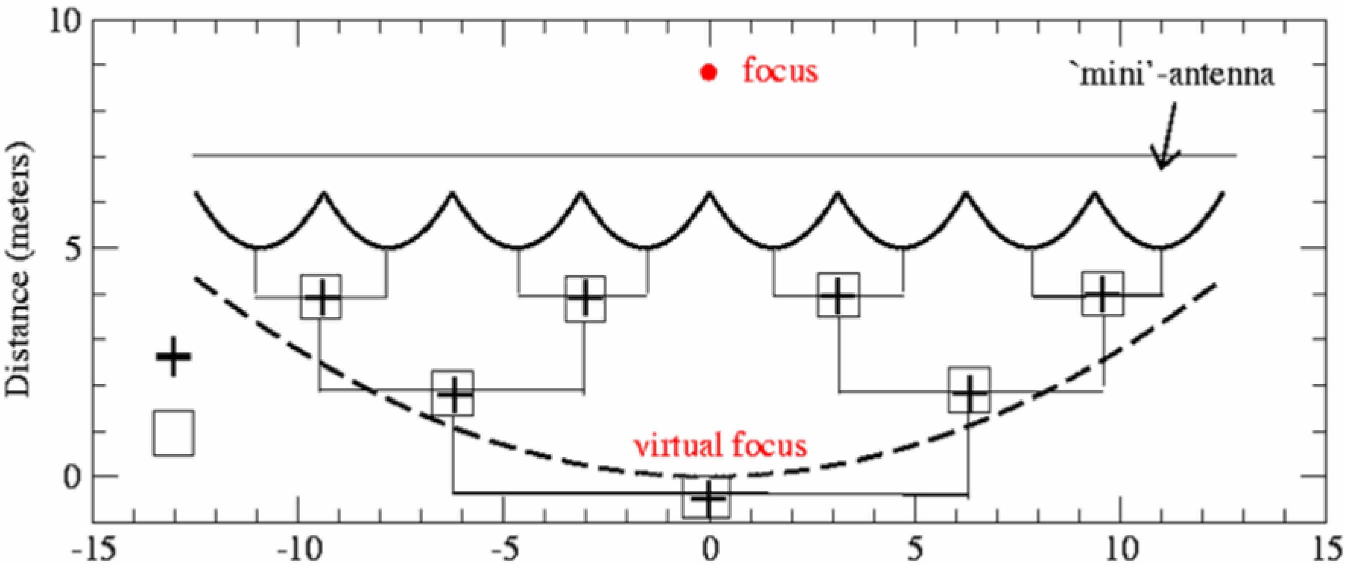
- Angular resolution of ~ 1′′ ⇨ Dish diameter ~ 42 km!!!

INTERFEROMETRY: PHILOSOPHY

- Single dish: Coherent sum of electromagnetic fields at the dish focus.

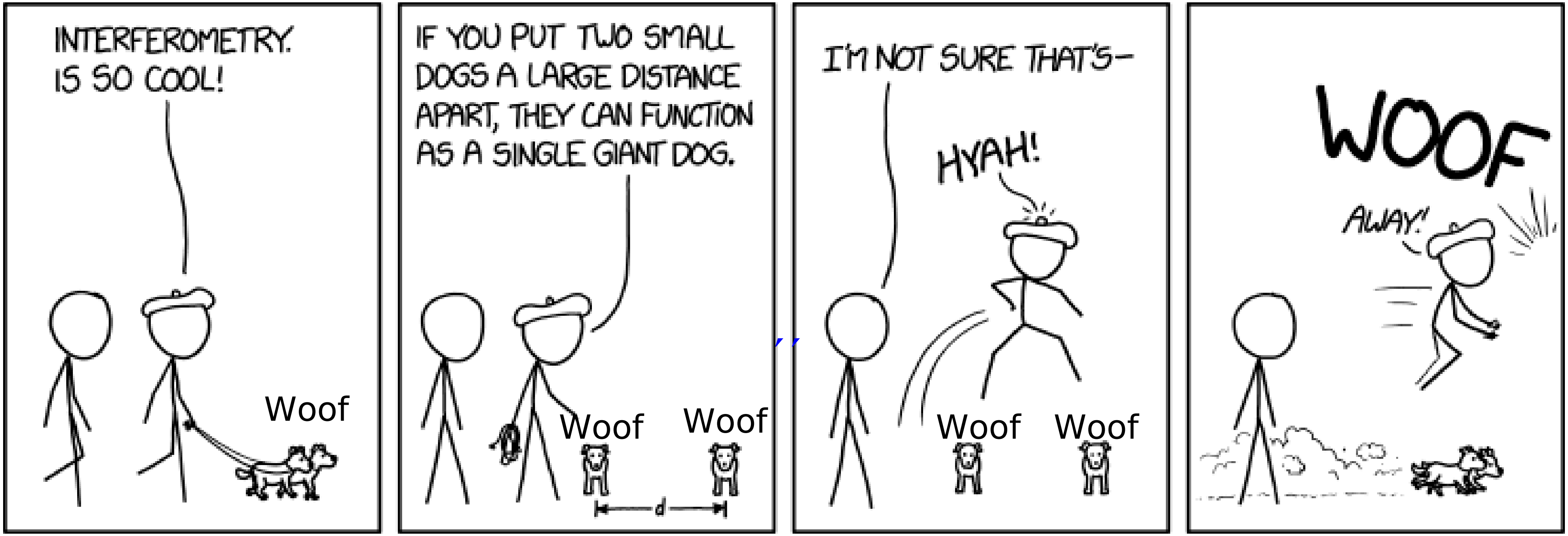


- Interferometer: Combine voltages of many small dishes, keeping phase information.
- Dishes do not need to be next to each other.



(Courtesy: Rick Perley)

INTERFEROMETRY: PHILOSOPHY



(Courtesy xkcd.com/1922)

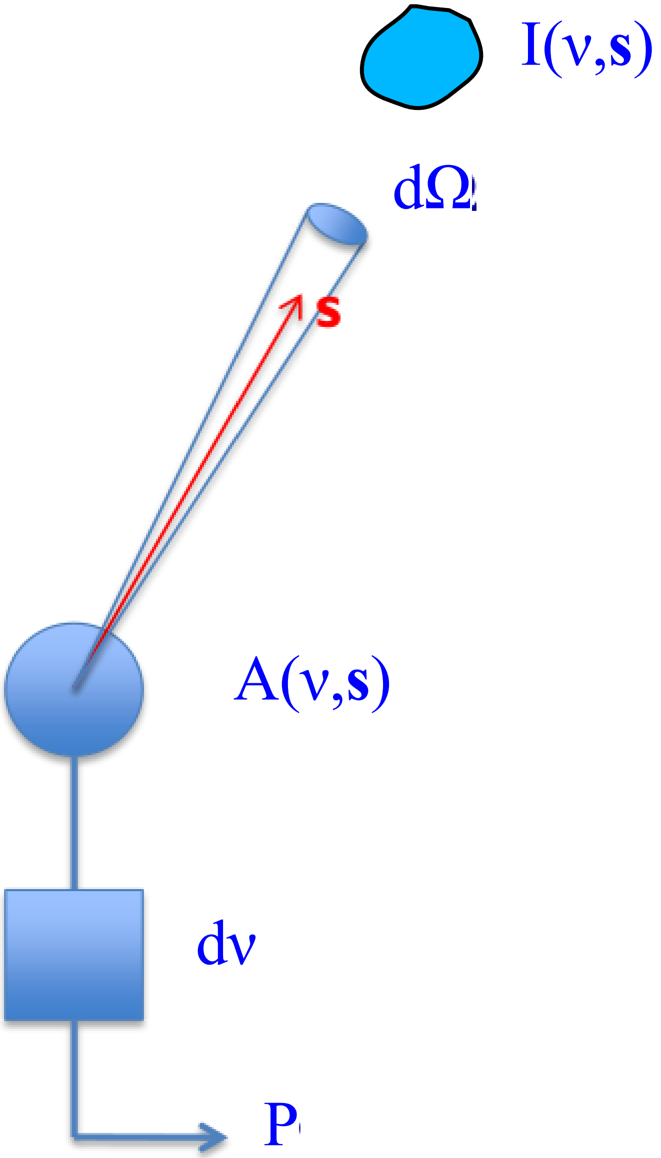
SIGNALS AND DETECTORS

- Distant radiation source of intensity $I(\nu,\mathbf{s})$.

- Detector of area $A(\nu,\mathbf{s})$.

- Power dP from a small solid angle $d\Omega$ and a small frequency interval $d\nu$:
  $dP = A(\nu,\mathbf{s}).I(\nu,\mathbf{s}).d\nu.d\Omega.$

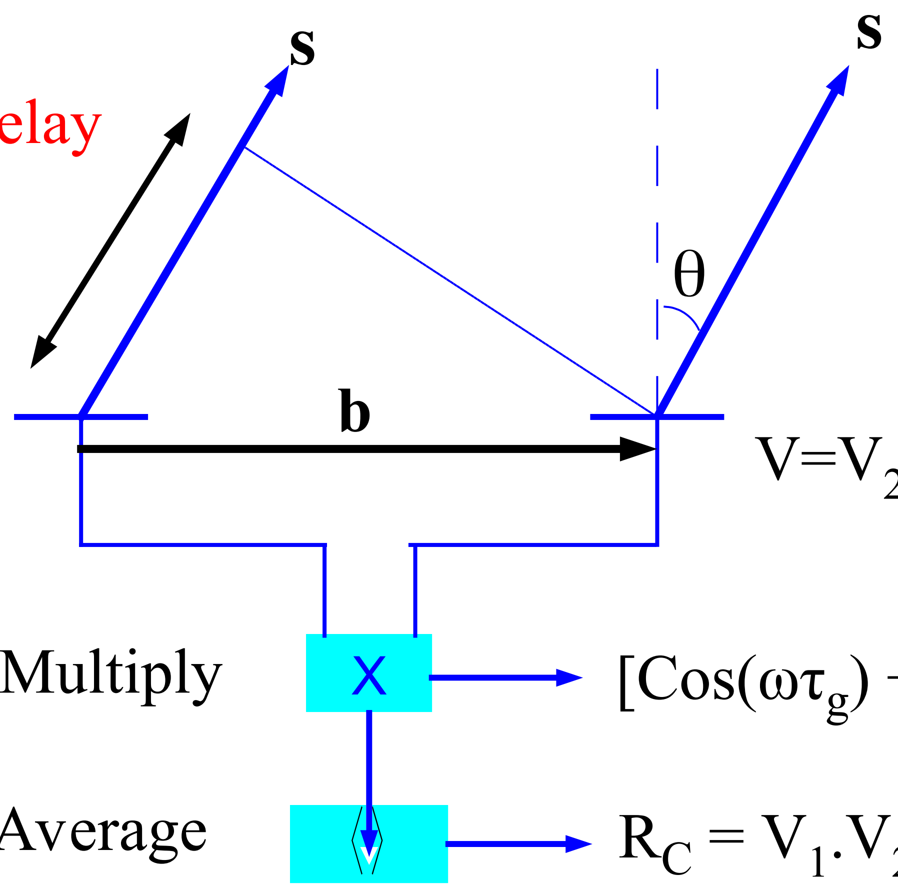
- Total power, $P = \iint A(\nu,\mathbf{s}).I(\nu,\mathbf{s}).d\nu.d\Omega.$



(Rick Perley, NRAO lectures)

QUASI-MONOCHROMATIC INTERFEROMETER

- Distant source of quasi-monochromatic radiation: $E_\nu(t) = E.\mathrm{Cos}[2\pi\nu t]$.



Geometric delay

$\tau_g = \mathbf{b}.\mathbf{s}/c$

$V = V_1\mathrm{Cos}[\omega(t-\tau_g)]$

$V = V_2.\mathrm{Cos}[\omega t]$

Multiply

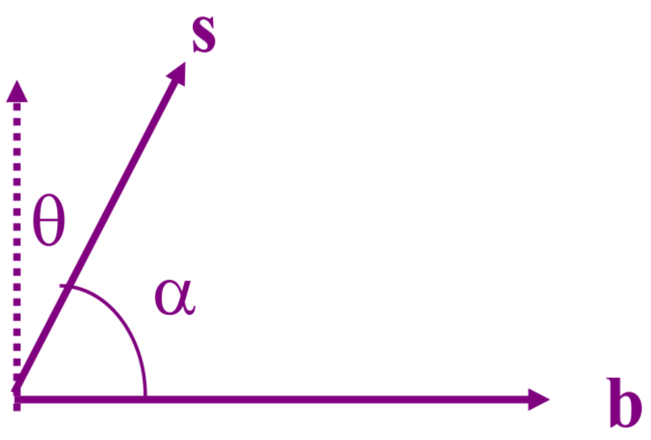
$[\mathrm{Cos}(\omega\tau_g) + \mathrm{Cos}(2\omega t - \omega\tau_g)]/2$

Average

$$R_C = V_1.V_2.\mathrm{Cos}(\omega\tau_g)/2$$
$$= V_1.V_2.\mathrm{Cos}(2\pi\mathbf{b}.\mathbf{s}/\lambda)/2$$

- “Cosine correlator”: Output depends on the baseline length, orientation. Does *not* depend on time, source distance, baseline location.

THE FRINGE PATTERN

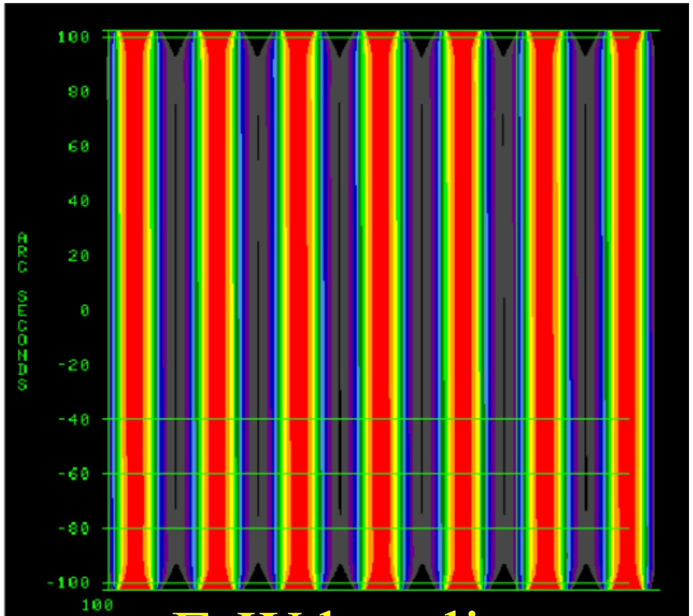


- $\mathbf{b}.\mathbf{s}/\lambda = u.\mathrm{Cos}\alpha = u.l$

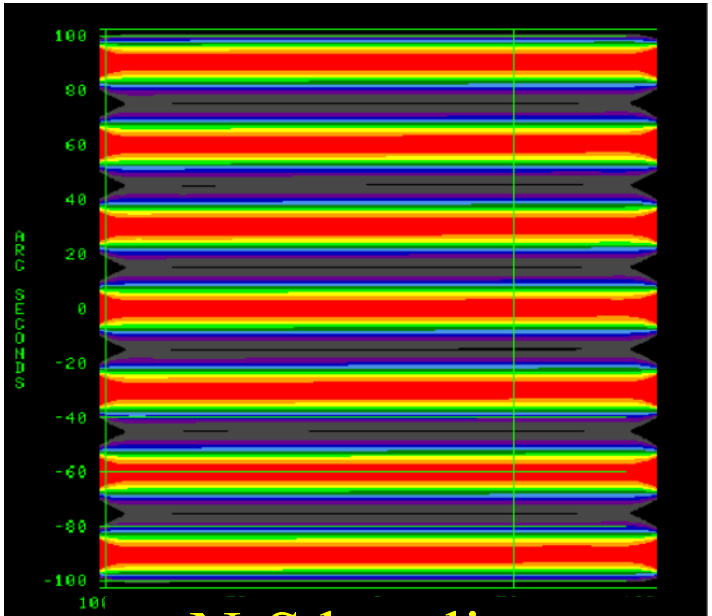
  $u \equiv$ Baseline length in wavelengths.

  $l \equiv$ Direction cosine.

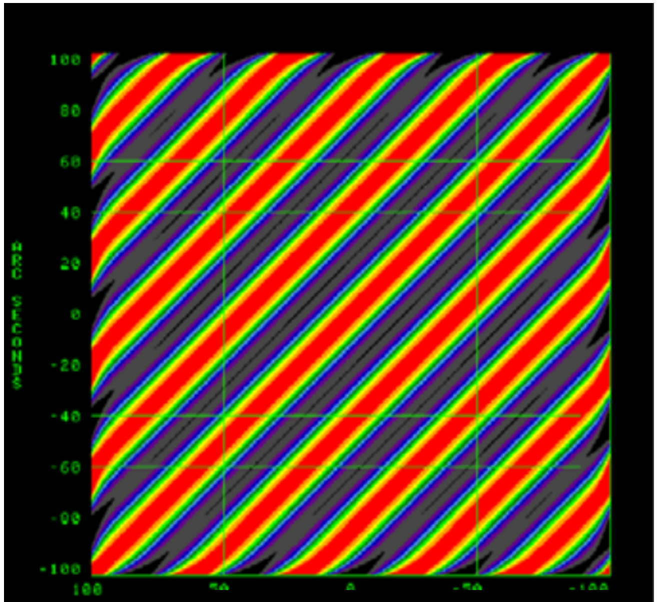
  ⇨ $R_C = P.\mathrm{Cos}(2\pi\, u.l)$



E-W baseline:
Vertical “fringes”.



N-S baseline:
Horizontal fringes.



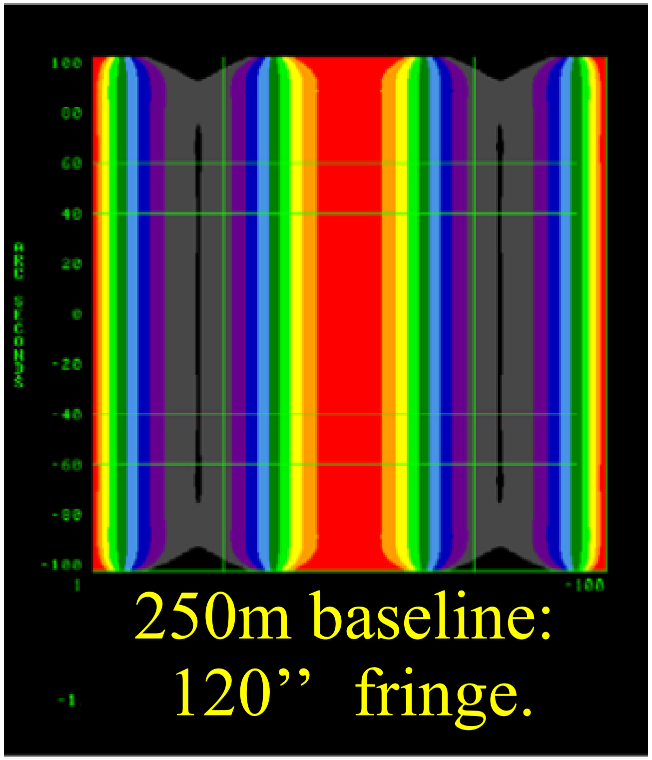
Rotated baseline:
Rotated fringes.

- Fringe angular spacing, $\Delta\theta \sim (\lambda/b)$.

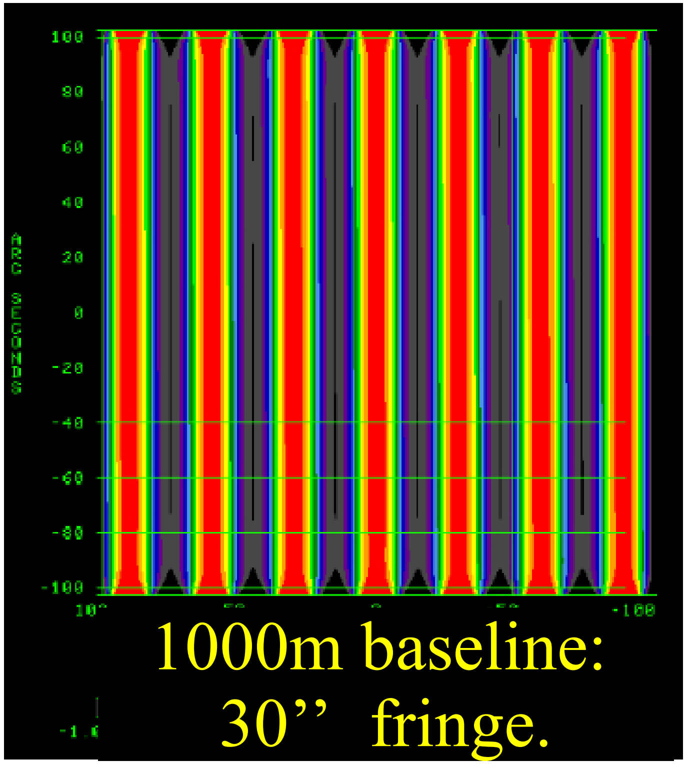
(Courtesy: Rick Perley)

THE FRINGE PATTERN

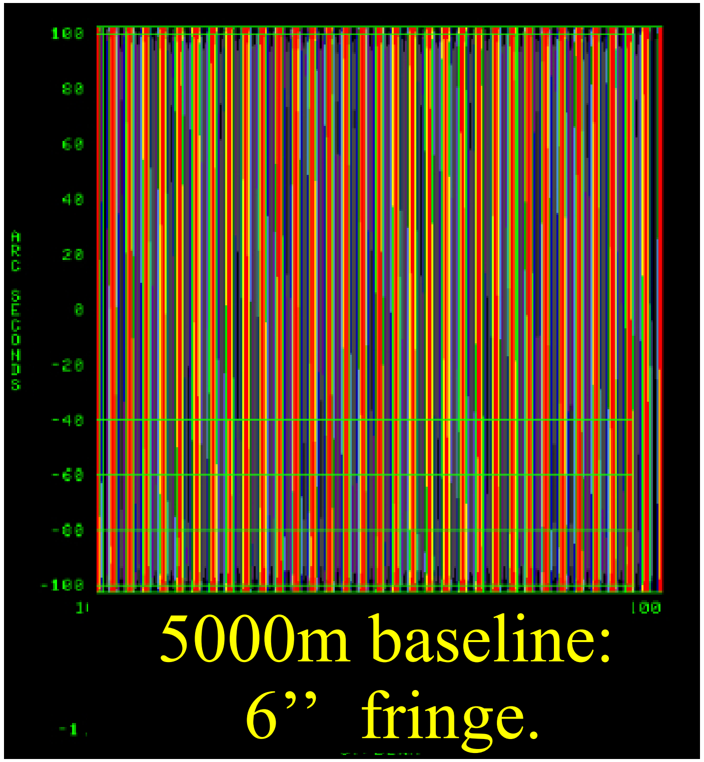
- Long baselines ⇨ Finer fringes.
  Short baselines ⇨ Wider fringes.



250m baseline:
120” fringe.



1000m baseline:
30” fringe.



5000m baseline:
6” fringe.

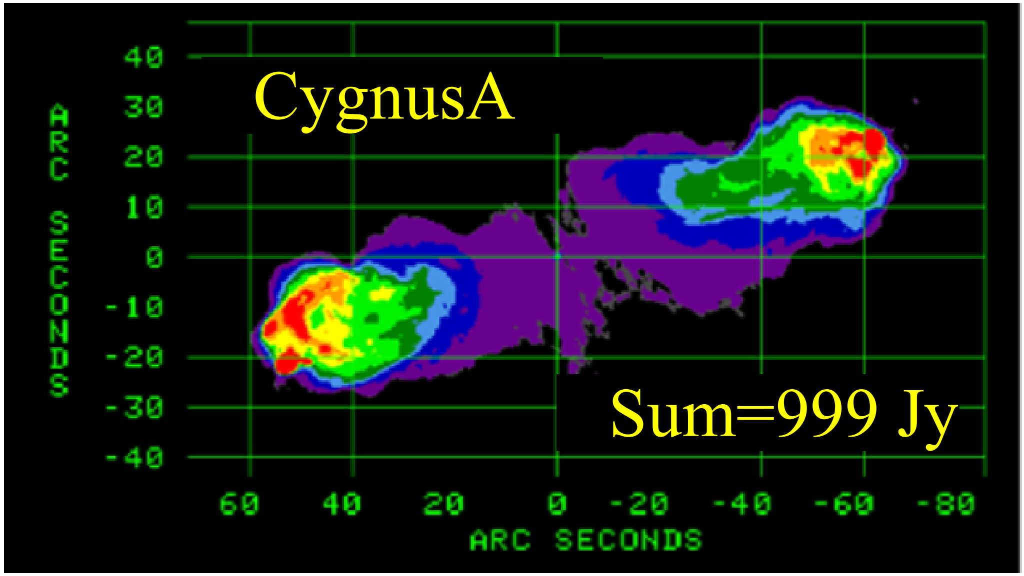
(Courtesy: Rick Perley)

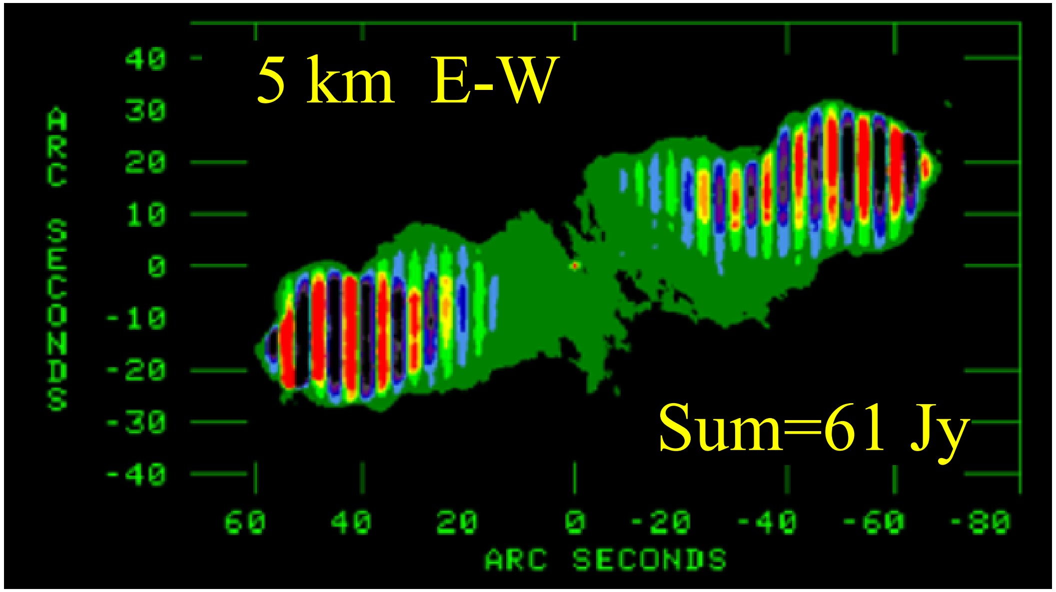
THE INTERFEROMETER RESPONSE

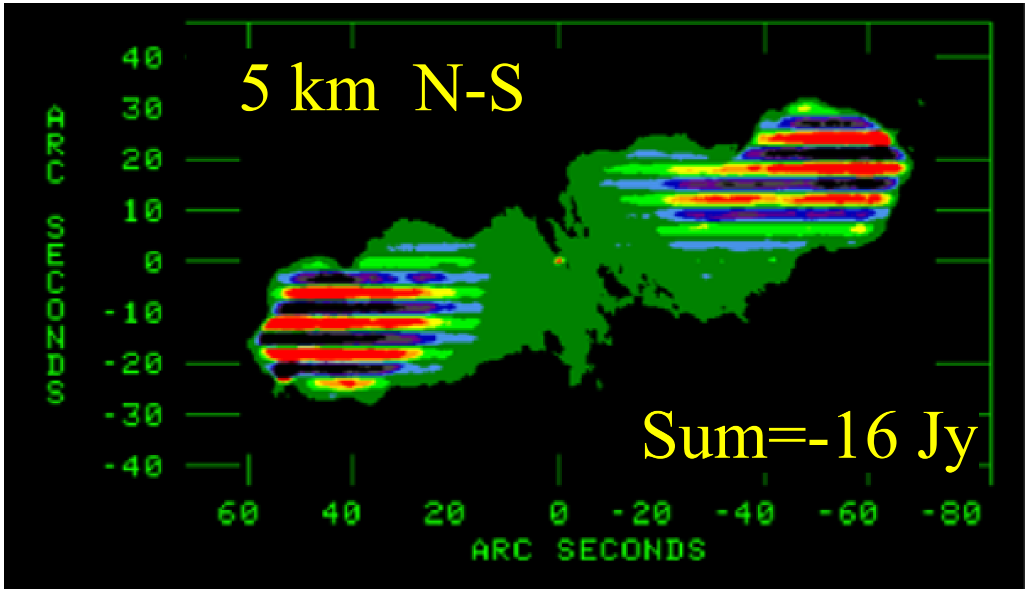
- For an extended source ⇨ $R_C = \int A(s) \,.\, I(s) \,.\, \mathrm{Cos}(2\pi\, \mathbf{b}.\mathbf{s}/\lambda)\ d\Omega$
- Throws a sinusoidal fringe pattern on the sky, multiples the sky intensity by the fringe pattern, and then integrates over the sky.

THE INTERFEROMETER RESPONSE

- CygnusA with a 5-km baseline at ~ 2 GHz.







(Courtesy: Rick Perley)

THE INTERFEROMETER RESPONSE

- For a “point source”, same interferometer response for all baselines.
- The interferometer response can be negative for a real source!
- The interferometer response tends to zero for the longer baselines: Sources get “resolved out”.
- The interferometer response tends to the total flux density for shorter baselines: The “zero spacing flux”.
- $R_C = \int A(s) \,.\, I(s) \,.\, \mathrm{Cos}(2\pi\, \mathbf{b}.\mathbf{s}/\lambda)\; d\Omega$

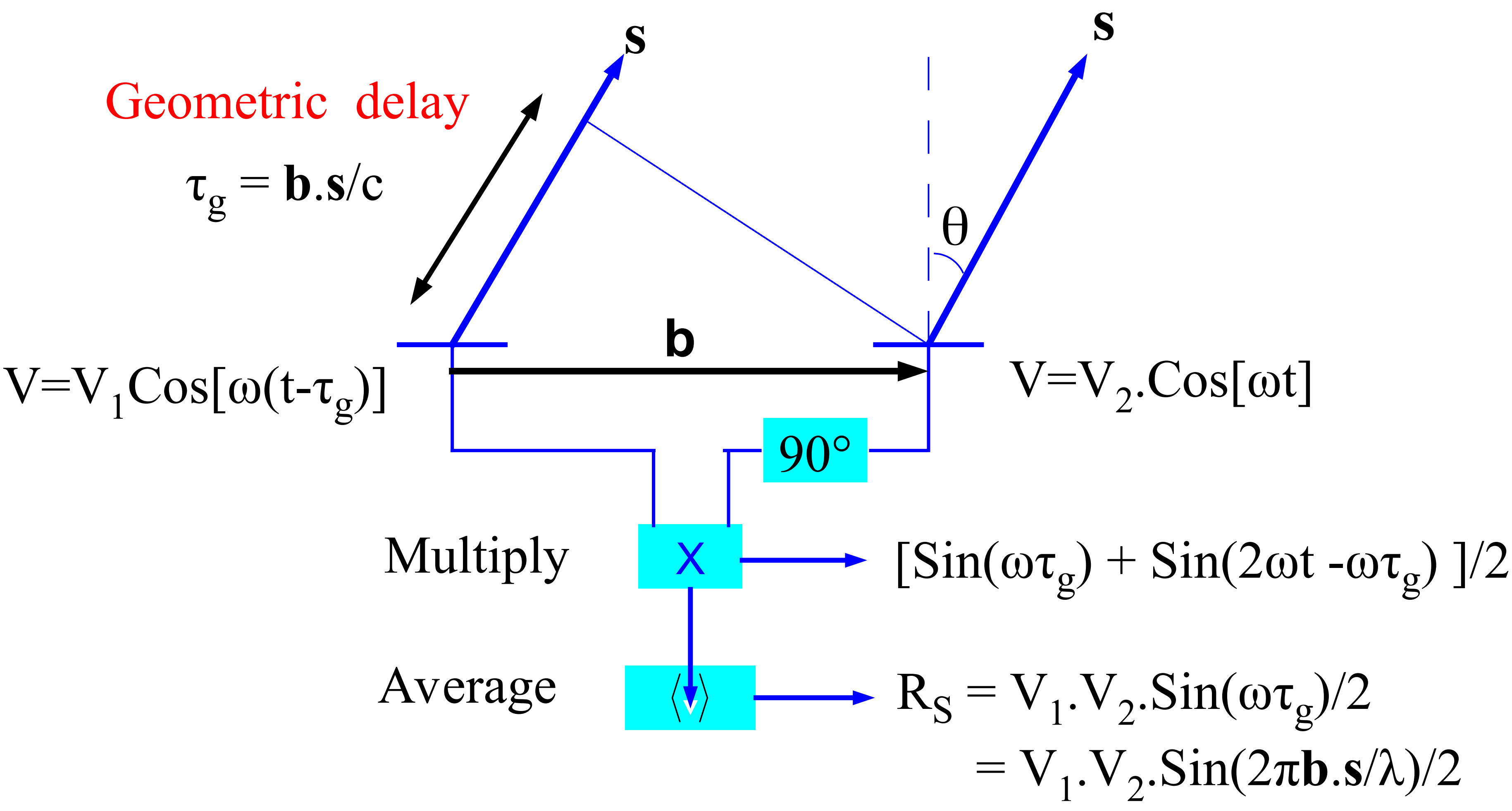
Sensitive to the “even” part of the sky intensity distribution.

- Not sufficient to recover the “true” sky intensity distribution!

⇨ Need an “odd” fringe function.

THE SINE CORRELATOR

- 90-degree phase shift in one of the signal paths!



Geometric delay

$\tau_g = \mathbf{b}.\mathbf{s}/c$

$V=V_1\mathrm{Cos}[\omega(t-\tau_g)]$

$V=V_2.\mathrm{Cos}[\omega t]$

Multiply $[\mathrm{Sin}(\omega\tau_g) + \mathrm{Sin}(2\omega t - \omega\tau_g)\ ]/2$

Average $R_S = V_1.V_2.\mathrm{Sin}(\omega\tau_g)/2$

$= V_1.V_2.\mathrm{Sin}(2\pi\mathbf{b}.\mathbf{s}/\lambda)/2$

- For an extended source ⇨ $R_S = \int A(s)\ .\ I(s)\ .\ \mathrm{Sin}(2\pi\ \mathbf{b}.\mathbf{s}/\lambda)\ \ d\Omega$

THE COMPLEX CORRELATOR

- Define the complex visibility $V = R_C - i.R_S = A \, . \, e^{-i\varphi}$

  where $A = [R_C^2 + R_S^2]^{½}$ and $\varphi = \text{Tan}^{-1}[R_C/R_S]$

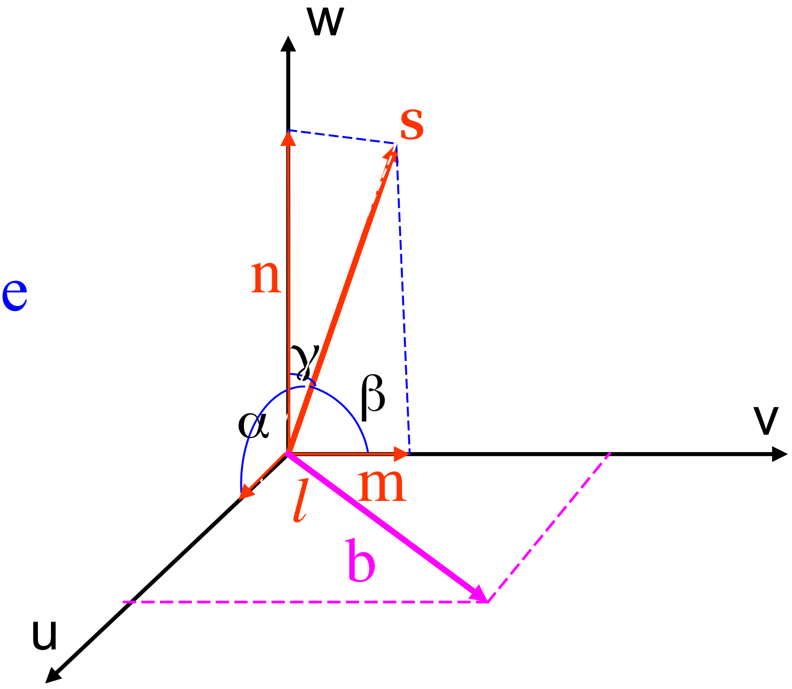
- $R_C = \int A(s) \, . \, I(s) \, . \, \text{Cos}(2\pi \, \mathbf{b.s}/\lambda) \; d\Omega$

  $R_S = \int A(s) \, . \, I(s) \, . \, \text{Sin}(2\pi \, \mathbf{b.s}/\lambda) \; d\Omega$

  ⇨ Visibility $V_\nu(\mathbf{b}) = R_C - i.R_S = \int A(\mathbf{s}) \, . \, I(\mathbf{s}) \, . \, e^{-2\pi i \, \mathbf{b.s}/\lambda} \; d\Omega$

- General relation between complex visibility and sky intensity!

COORDINATE SYSTEMS



- The baseline **b** has components $\lambda(u,v,w)$.

- For the unit vector **s**, the co-ordinates in the (u, v, w) system are the direction cosines $l = \mathrm{Cos}(\alpha)$, $m = \mathrm{Cos}(\beta)$, and $n = \mathrm{Cos}(\gamma)$.
  $l^2 + m^2 + n^2 = 1$

- $\mathbf{b}.\mathbf{s}/\lambda = (ul + vm + wn) = ul + vm + w\{(1 - l^2 - m^2)^{1/2} - 1\}$
  $d\Omega = dl.dm/n = (1 - l^2 - m^2)^{-1/2}\, dl.dm$

- $V_\nu(u,v,w) = \iint A_\nu(l,m)\, I_\nu(l,m)\, .\, e^{-2\pi i\,[ul+vm+w\{(1-l^2-m^2)^{1/2} - 1\}]}\, .\, (1 - l^2 - m^2)^{-1/2}\, dl.dm$

- Note: This is *not* a Fourier transform relation!!!

SPECIAL CASES

- All visibility measurements in a single plane, i.e. $w \approx 0$.
  - ⇨ $V_\nu(u,v,w) = \iint A_\nu(l,m).\ I_\nu(l,m)\ .\ e^{-2\pi i.[ul+vm]}.(1 - l^2 - m^2)^{-1/2}\ dl.dm$

  ⇨ 2-D Fourier transform between $V_\nu(u,v)$ and $I_\nu(l,m) \times (1 - l^2 - m^2)^{-1/2}$!

- All sources in a small sky region: i.e. $w.n \approx 0$ for non-zero $I_\nu(l,m)$.
  - ⇨ $V_\nu(u,v) = \iint A_\nu(l,m)\ .\ I_\nu(l,m)\ .\ e^{-2\pi i[ul+vm]}\ dl.dm$
- ⇨ 2-D Fourier transform between $V_\nu(u,v)$ and $A_\nu(l,m)\ .\ I_\nu(l,m)$!

- At low frequencies, cannot assume all sources in a small sky region. Must correct for 3-D effects: Faceting or w-projection.

(Cornwell et al. 2008, IEEE)

- One measures $V_\nu(u,v)$ and then obtains the sky intensity $I_\nu(l,m)$ by

$$A(l,m).I_\nu(l,m) = \iint V_\nu(u,v)\ .\ e^{2\pi i\,[ul+vm]}\ du.dv$$

RADIO INTERFEROMETRY

- The visibility $V_\nu(u,v)$ is related to the sky intensity distribution $I_\nu(l,m)$:

$$V_\nu(u,v) \approx \iint A_\nu(l,m) \,.\, I_\nu(l,m)\, e^{-2\pi i(ul + vm)}\, dl\, dm$$

(u,v,w) are components of the baseline; (l,m) are direction cosines.

- Measure *cross-correlations* of the voltages determined at different antennas, as a function of each antenna – antenna baseline. Then carry out a 2-D Fourier transform to infer the sky intensity distribution.

- As the Earth rotates, the separation of a pair of antennas relative to the source direction will change! Each antenna pair measures V(u,v) at a *changing* (u,v) location with time, yielding a curve in the u-v plane, and hence, better sampling of this plane!

⇨ Earth-rotation Aperture Synthesis

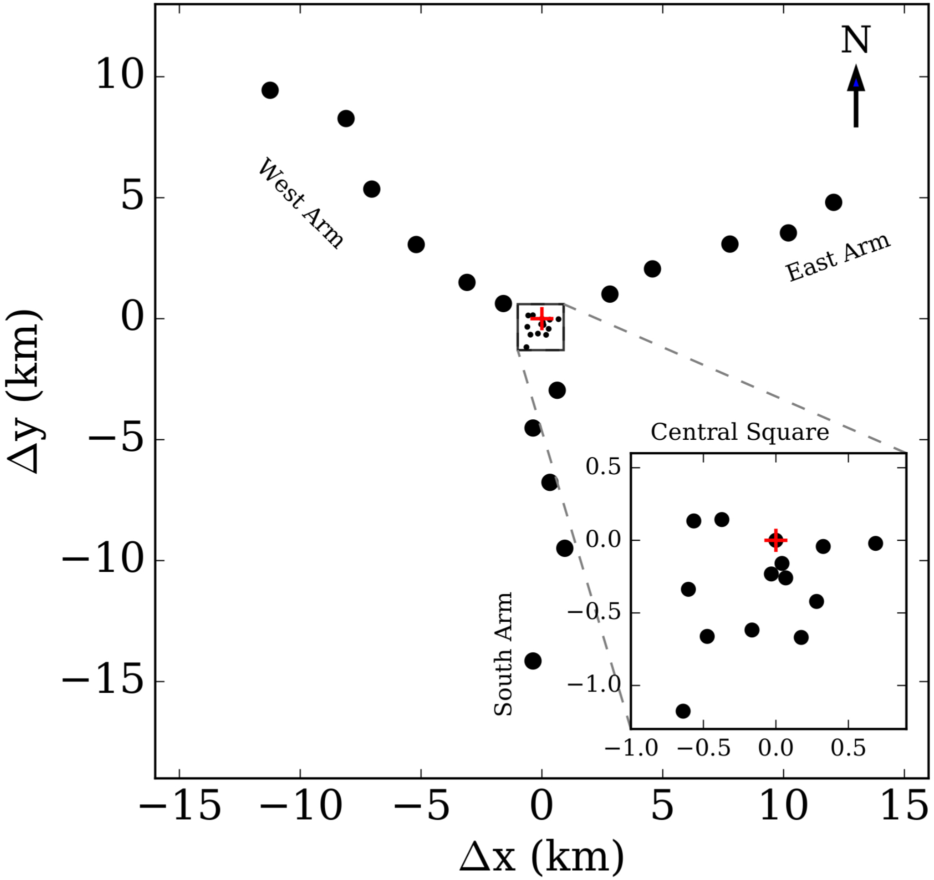
- ⇨ Distribute your antennas so as to obtain the best (most uniform) coverage of the 2-D u-v plane, for *all* directions.

ARRAY CONFIGURATIONS

- Y-shaped array optimal for $\sim 20 - 40$ antennas (VLA, GMRT). Random or spiral array for more than ~ 50 antennas (ALMA, LOFAR).

(Courtesy: NRAO)

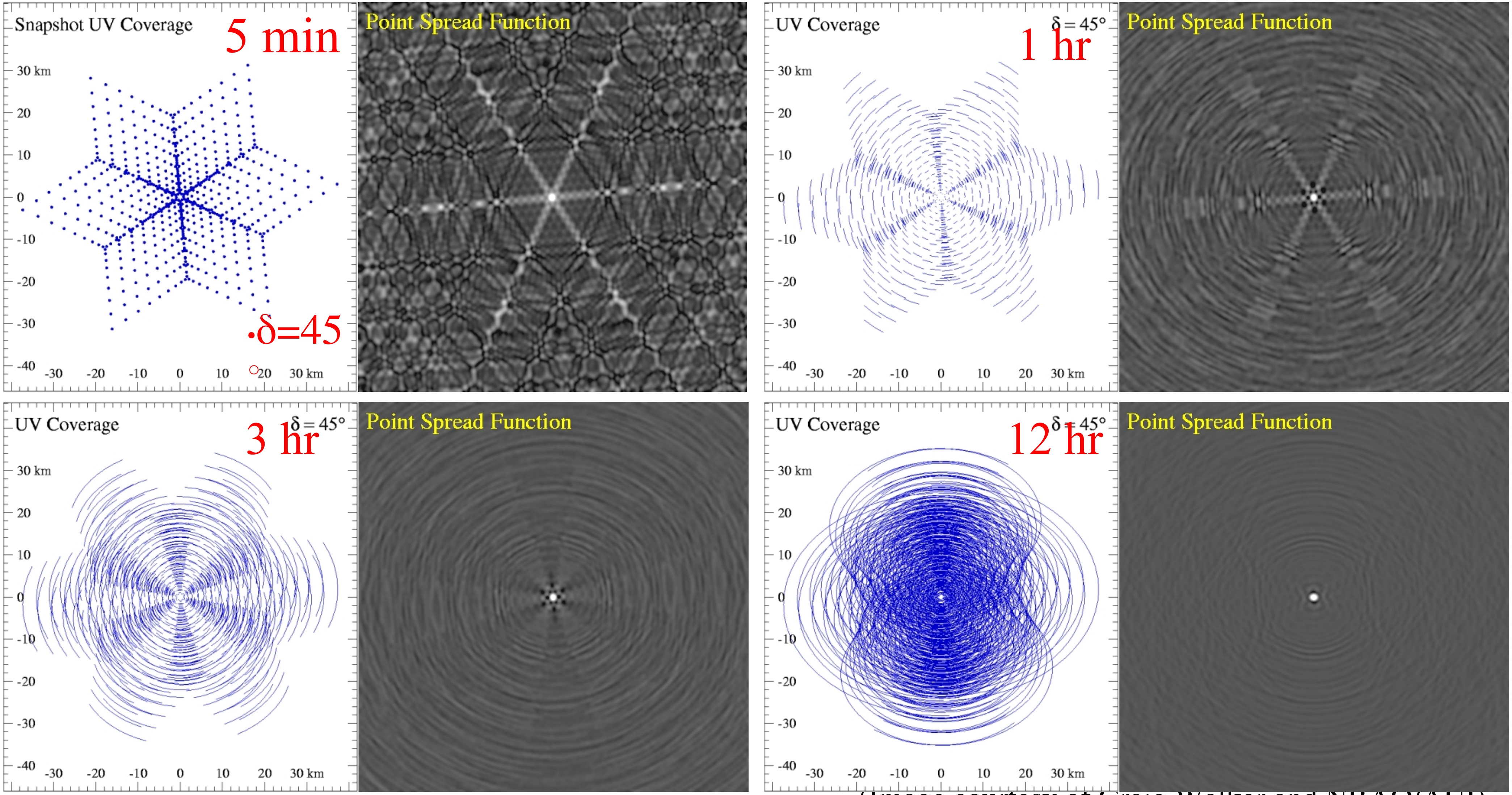
(Patra et al. 2018, MNRAS)



- VLA: 27 antennas on rails in a Y-array. Moved every 4 months to get different u-v coverage. Longest baseline $\sim$ 1 km, 3.3 km, 11 km, 35 km.
- GMRT: 30 fixed antennas, 14 in a 1-km central square, 16 in Y-array.

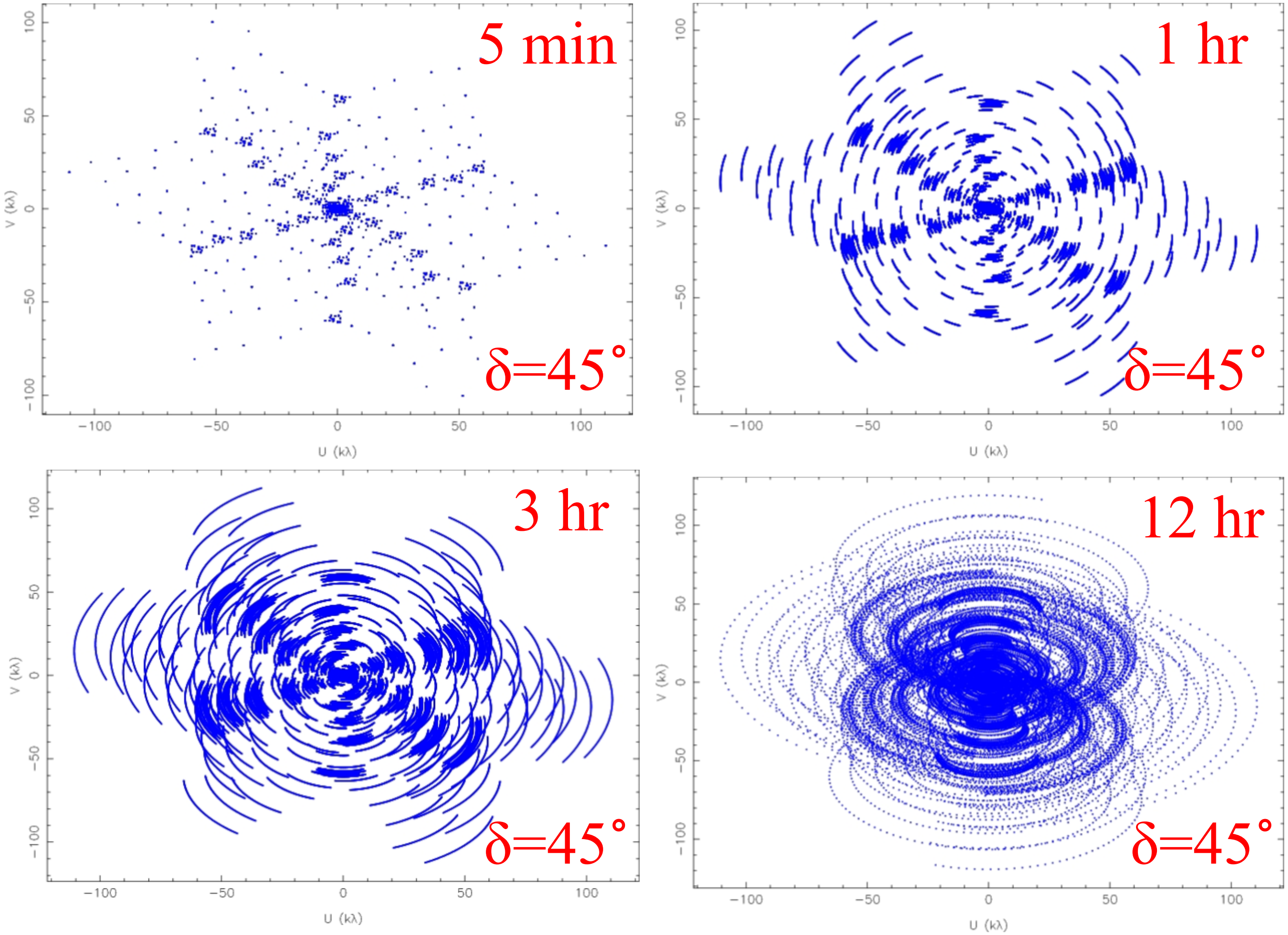
ARRAY U-V COVERAGE: THE VLA

- The set of all baseline vectors during the observations. Its 2-D Fourier transform is the synthesized beam. Observing a source for even a few hours gives a much better u-v coverage and a "cleaner" beam.



(Image courtesy of Craig Walker and NRAO/AUI)

Array U-V Coverage: The GMRT

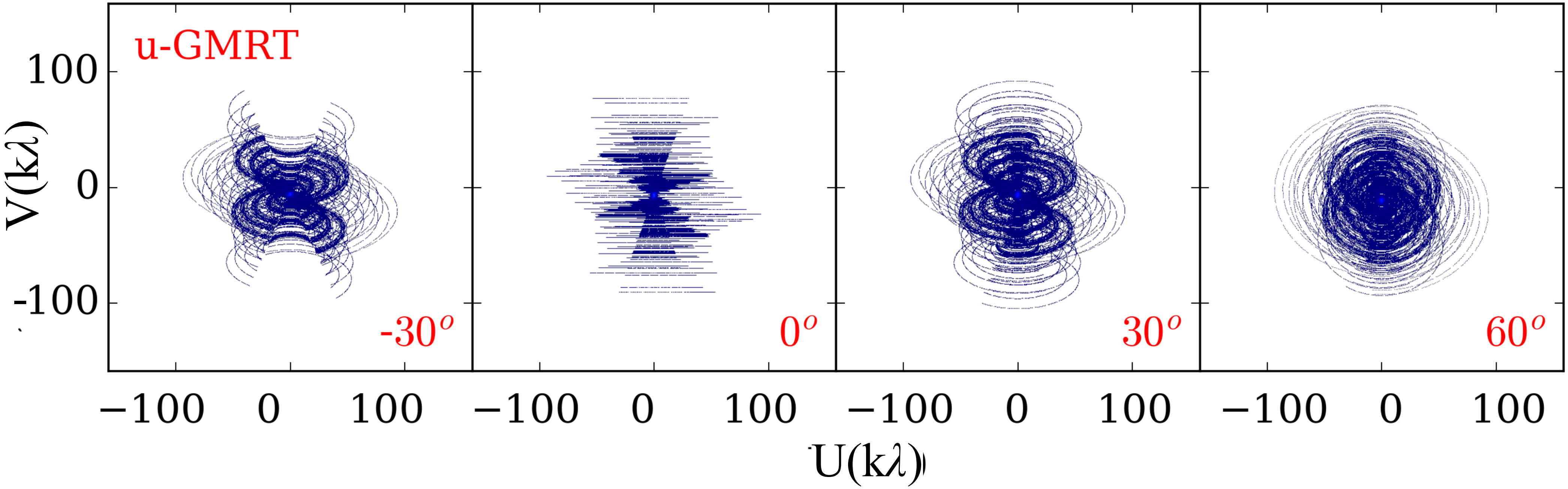


(Courtesy: Jayaram Chengalur)

- Dense patches due to the large number of central square antennas.

ARRAY U-V COVERAGE: THE GMRT

(Patra et al. 2018, MNRAS)



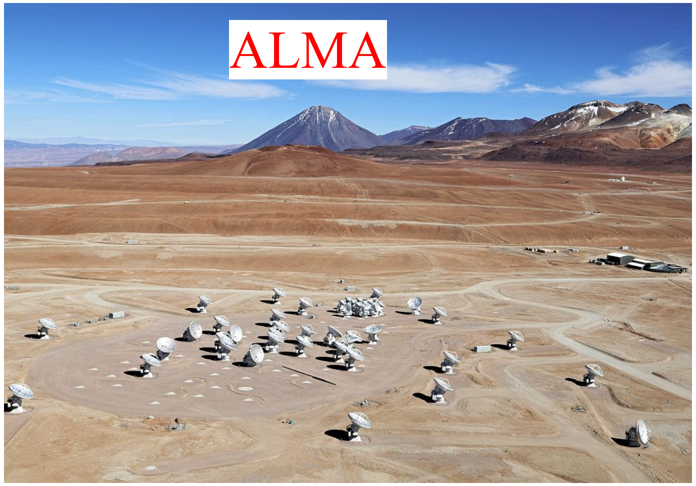
- "Reasonable" u-v coverage at all declinations in a 12-hr track.

RESOLUTION

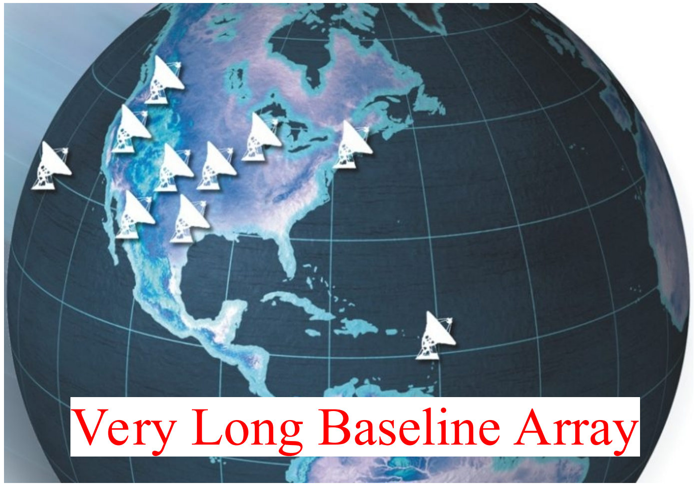
- Radio interferometry: Far better angular resolution than in the optical!

$\lambda \sim 6000$Å, $D = 2.4$m
⇨ R ~ 0.05 arcsec

$\lambda \sim 7$mm, $D \sim 35$ km
⇨ R ~ 0.04 arcsec



$\lambda \sim 0.3$mm, $D \sim 16$ km
⇨ R ~ 4 milli-arcsec



$\lambda \sim 7$mm,
$D \sim 10{,}000$ km
⇨ R ~ 0.15 milli-arcsec

(Courtesy: NRAO, NASA)