Self-Calibration

K. S. Dwarakanath
(Formerly at Raman Research Institute)

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Calibration

Flux density calibration

visibilities --> flux density units

primary calibrator (~ once during the observing run)

Gain (bandshape) calibration of antennas

complex (amplitude and phase)

secondary calibrator (a few times during the observing run)

Why is this inadequate ?

approximate image ......

gain varies with time and direction

lack of strong unresolved calibrators near-by

electronics

atmosphere (high freq), ionosphere (low freq)

residual phase and amp errors limit

Phase and Amplitude Errors and Dynamic Range

N antennas – N(N-1) / 2 independent visibilities,
In an ideal situation, calibrated visibilities (vectors) add up in phase
Otherwise, there are residual phase (amp) errors

Phase error Φ on every baseline, the final image will be limited
to a dynamic range ~ N / Φ (single 'snapshot')
(peak ~ N**2, Φ uncorrelated, adds up to ~ $\varphi\sqrt{(N^{**}2)}$)

Amplitude error ε on each baseline, DR ~ N / ε

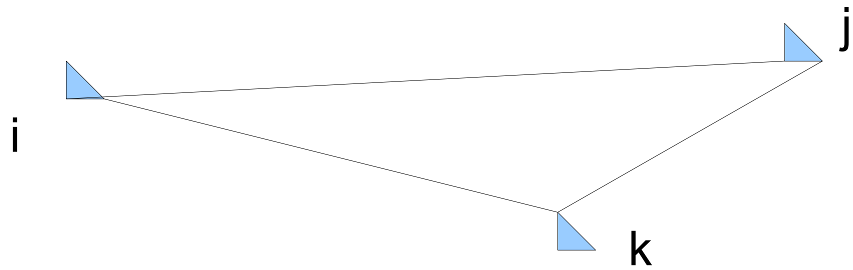
So, for a phase error ~ 10^0 , dynamic range (peak / rms) for VLA / GMRT is ~ 200
grossly inadequate (bright s

Phase error (Φ) results in asymmetric residuals, with an amplitude ~ Φ
~ $\Phi \sin(2\pi u_o l)$

(Perley 1985, for more details)

Closure Quantities

An appropriate sum of visibility phases around a closed loop of baselines Is free of antenna-based errors (Jennison 1958,



$V_{ij,\ obs}(t) = G_i(t)\, G_j^*(t)\, V_{ij,\ true}(t)$ + noise term; $G_i = A_i \exp(i\Theta_i)$

$\Phi_{ij,\ obs}(t) = \Phi_{ij,\ true}(t) + \Theta_i(t) - \Theta_j(t)$ + noise term

Antennas i, j, and k

$C_{ijk,\ obs}(t) = \Phi_{ij,\ obs}(t) + \Phi_{jk,\ obs}(t) + \Phi_{ki,\ obs}(t)$

$= \Phi_{ij,\ true}(t) + \Phi_{jk,\ true}(t) + \Phi_{ki,\ true}(t)$ + noise term

$= C_{ijk,\ true}(t)$ + noise term

Closure phases and amplitudes

For an array of N antennas, there are (N(N-1) / 2 – (N-1)) "good" phases
Similarly, there are (N(N-1) / 2 – N) "good" amplitudes

So, for every set of 4 baselines,

$$\Gamma_{ijkl}\ (t) = A_i(t)A_j(t)A_k(t)A_l(t)\ /\ A_i(t)A_k(t)\ A_j(t)A_l(t)$$

The A's are amplitudes of complex gains G's
The closure amplitude is free of antenna based errors

(Cornwell 1985, for more details)

Iterative Schemes

(Readhead & Wilkinson 1978, Cotton 1979, Cornwell & Wilkinson 1981)

Produce CLEAN images consistent with closure quantities

Iterative Scheme

$$V_{ij,\,obs}(t) = G_i(t)\, G_j^*(t)\, V_{ij,\,true}(t) + \varepsilon_{ij}(t)$$

Aim is to obtain a
Model sky, I ---> model visibilities, V_{mod} *---corrupted by gains -->* V_{obs}

Constraints on the model sky – positive brightness, confined structures

Minimize the following quantity (subject to closure) :

$$S = \sum \sum \left(V_{ij,\,obs}(t_k) - G_i(t_k)\, G_j^*(t_k)\, V_{mod,\,ij}(t_k) \right)^2$$

First sum is over k, the second sum is over i,j (i ≠ j)

Iterative Scheme (in practice)

Observed calibrated visibilities

FT, dirty image, CLEAN (not too deep !)

Initial Sky Model (approx, could also come from a different freq)

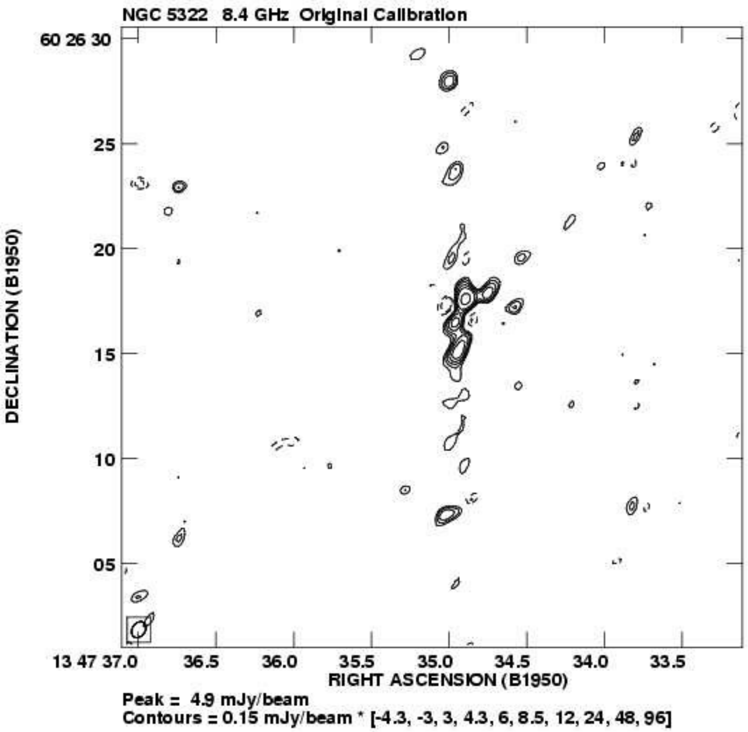
Estimate new antenna gains at shorter time intervals
with closure amp / phase constraints
and
Minimising the residual between model and observed visibilities

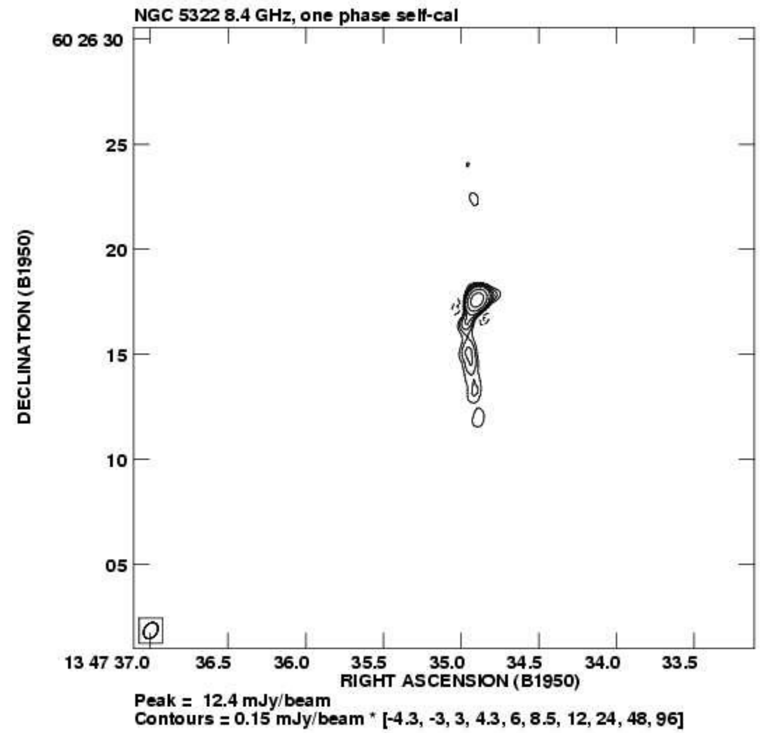
Apply the new gain solutions to obtain modified visibilities

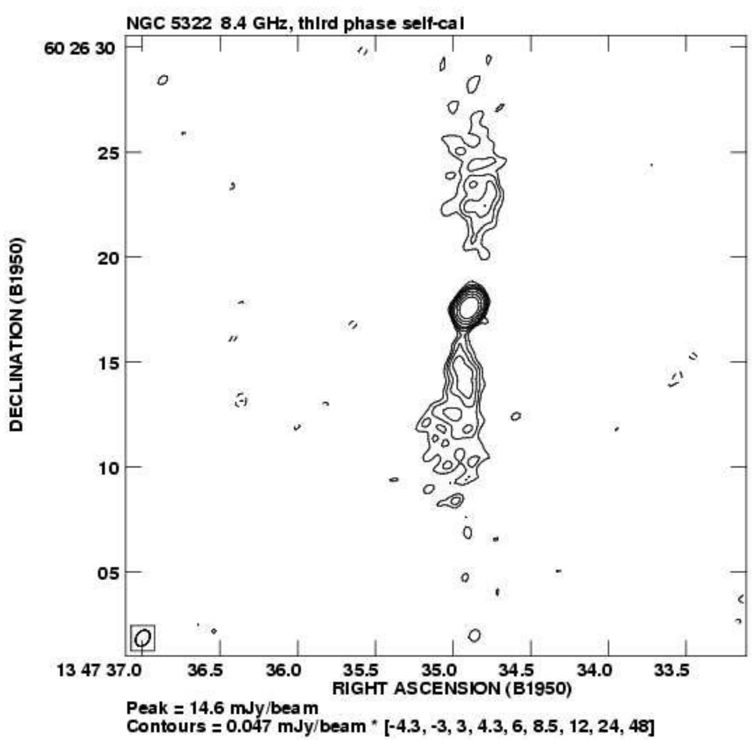
Make another Image

Stop, if satisfied; otherwise, go back to estimating gains
from this image

Can perform the self-cal + CLEAN loop until satisfied

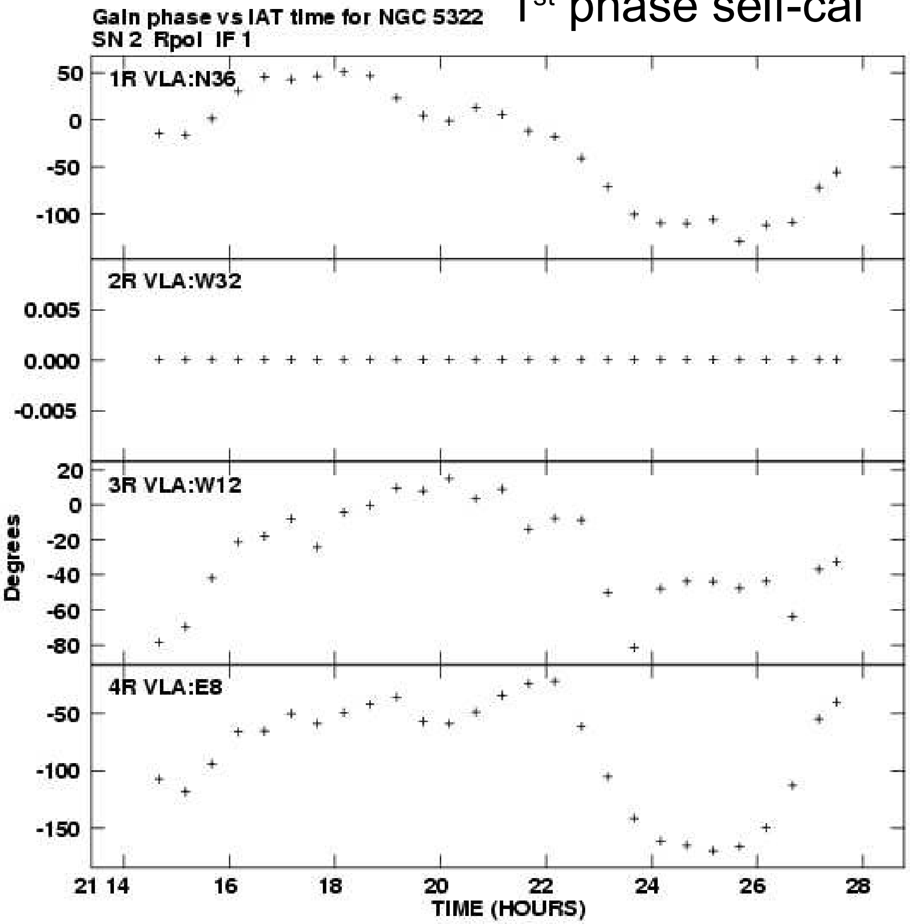


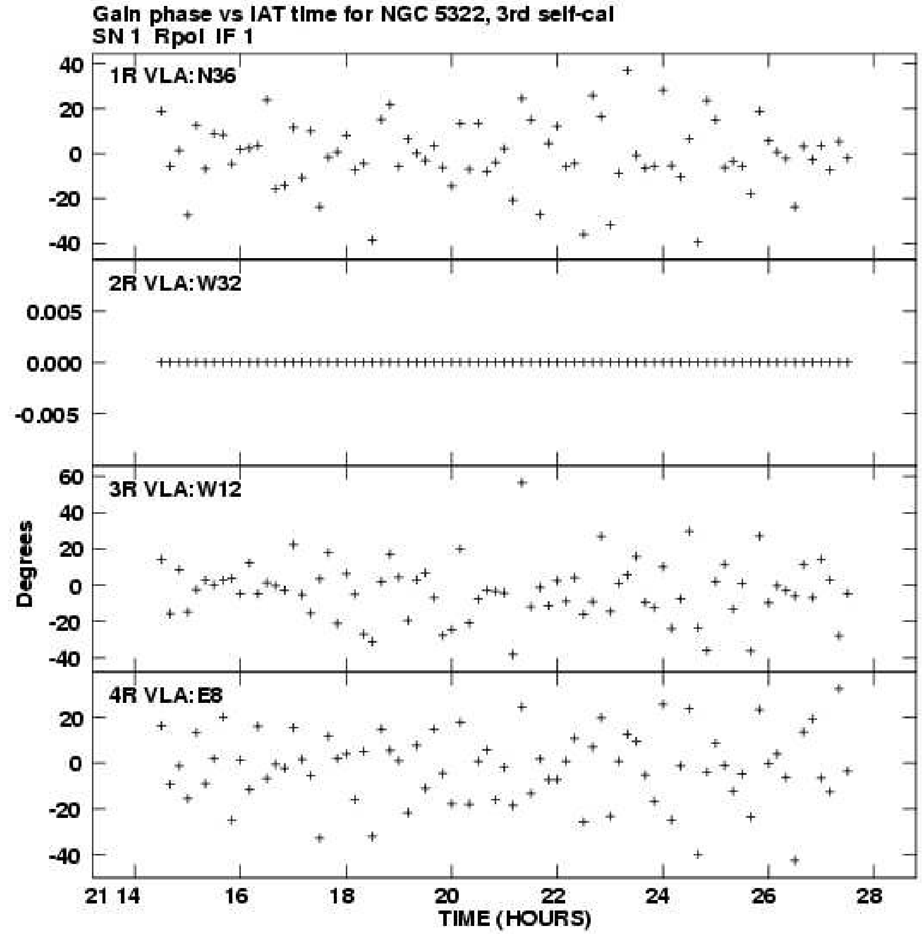




(Cornwell 2004)

1st phase self-cal





Self-cal works if the amplitude and phase errors are antenna based

- 2N unknowns, while there are ~ N**2 knowns
- For large N (like GMRT/VLA) this is a well constrained problem

Many parameters to play with

- No of CLEAN comps, type of solution,
- Integration time, uv range restriction, phase only, phase &amp

Self-cal will not work if the initial model is totally wrong, includes RFI,

- If there are non-isoplanicity conditions, or, baseline based errors

Free parameters in self-calibration

Number of CLEAN components passed in each iteration
(shallow CLEANing to start with)

uv range allowed for data to be used in solution
(extended emission, long baselines)

Averaging time
(signal to noise versus gain variation time scales)

Type of solution and weighting scheme
(L1 versus L2 solutions)

$$\mathcal{S} = \sum_{k} \sum_{\substack{i,j \\ i \neq i}} w_{ij}(t_k) \left| \widetilde{V}_{ij}(t_k) - g_i(t_k) g_j^*(t_k) \widehat{V}_{ij}(t_k) \right|$$

$$\mathcal{S} = \sum_{k} \sum_{\substack{i,j \\ i \neq j}} w_{ij}(t_k) \left| \widetilde{V}_{ij}(t_k) - g_i(t_k) g_j^*(t_k) \widehat{V}_{ij}(t_k) \right|^2$$

(Cornwell and Fomalont, 1989)

References

Cornwell 1985, NRAO Synthesis Imaging Summer School

Cornwell 2004, NRAO Synthesis Imaging Summer School

Cornwell & Fomalont, 1989, ASPC, 6, 185

Cornwell & Wilkinson 1981, MNRAS, 196, 1067

Cotton 1979, AJ, 84, 1122

Jennison 1958, MNRAS, 118, 276

Perley 1985, NRAO Synthesis Imaging Summer School

Readhead & Wilkinson 1978, ApJ, 223, 25

Consider observing a source of unit amplitude strength at the phase center

N(N-1) / 2 complex visibilities are recorded at any given instant of time

Ideal case : $V(u) = \delta(u-u_k)$

Discrepant : $V(u) = \delta(u-u_o)\, e^{-i\varphi}$

Image : $I(l) = \int V(u)\, e^{i2\pi ul}\, du$

Each 'good' baseline : $2\cos(2\pi u_k l)$

'bad' baseline : $2\cos(2\pi u_o l - \varphi)$ --> $2[\cos(2\pi u_o l) + \varphi \sin(2\pi u_o l)]$ (small φ)

Image : $I(l) = 2\varphi\sin(2\pi u_o l) + 2\sum \cos(2\pi u_k l)$
(summation over k = 1 to N(N-1) / 2)

Synth beam is just the second term, leaving (an odd) residual ~ φ

Hence, DR ~ N^2 / φ, but if all baselines have random errors, DR ~ N / φ

Amplitude error, discrepant : $V(u) = (1+\varepsilon)\delta(u-u_o)$, obtain the same result
But, with φ --> ε and sin --> cos

(Perley 1985)