


Sensitivity

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Radio Astronomy School (RAS-2023)

Main references, sources and credits:
- Low-frequency Radio Astronomy (primarily Chapters 3 and 5),
- Handbook of Pulsar Astronomy,
- Poonam Chandra and J. van Leeuwen's talks available online; and internet.

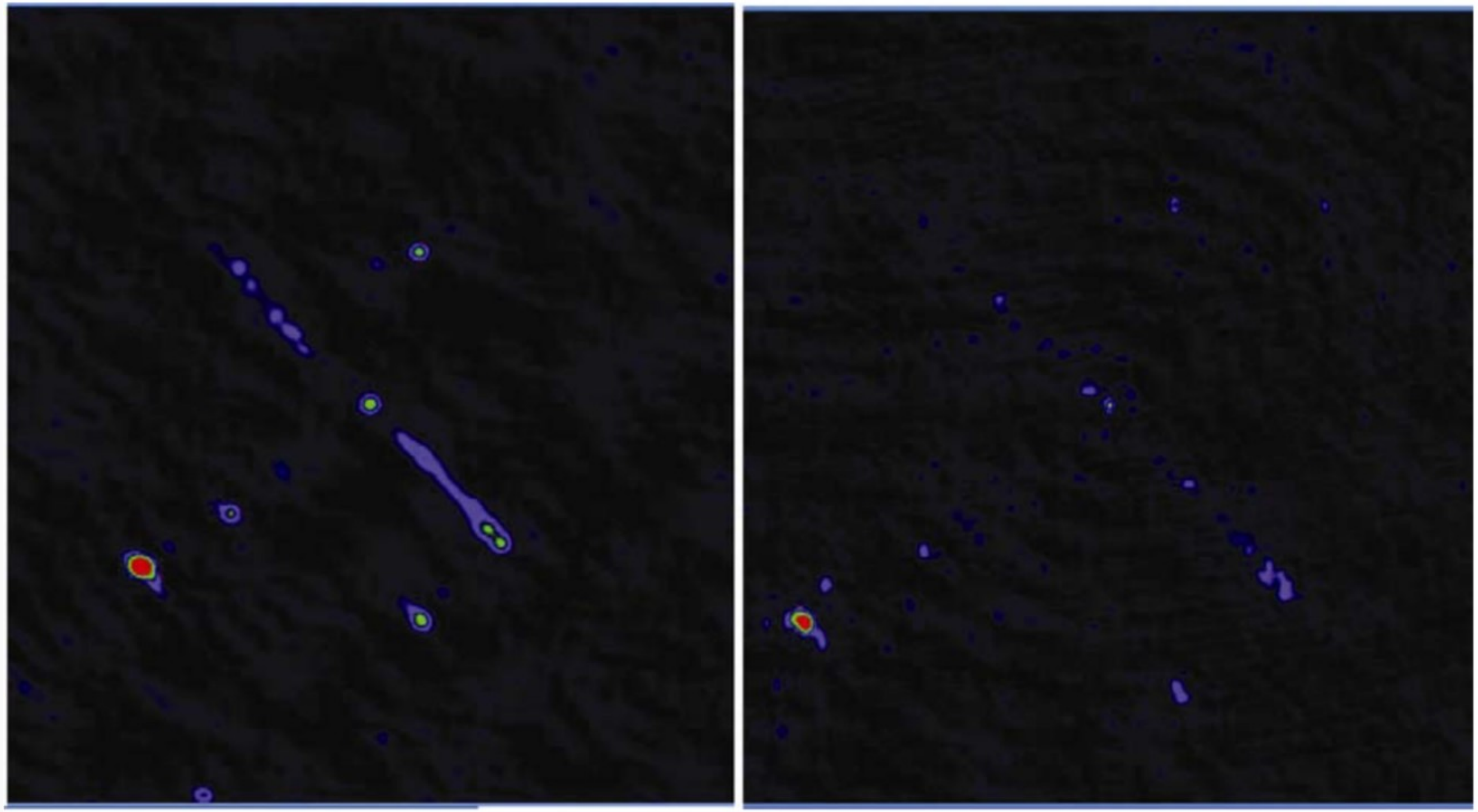
NCRA • TIFR

Giant Metrewave Radio Telescope
KHODAD

What is sensitivity?

What is sensitivity?

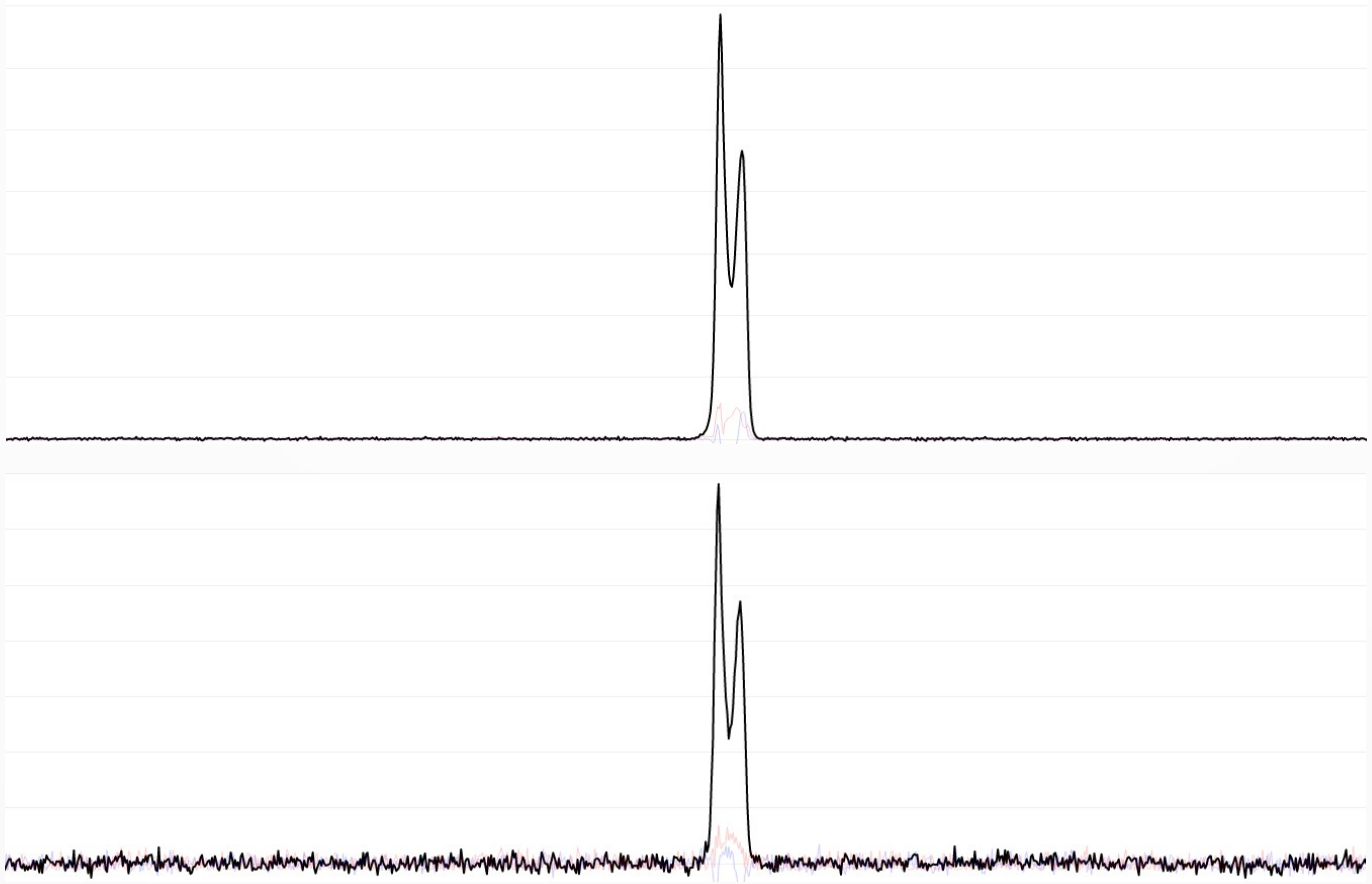
Which one has higher sensitivity?



IC: Poonam Chandra’s Lecture.

What is sensitivity?

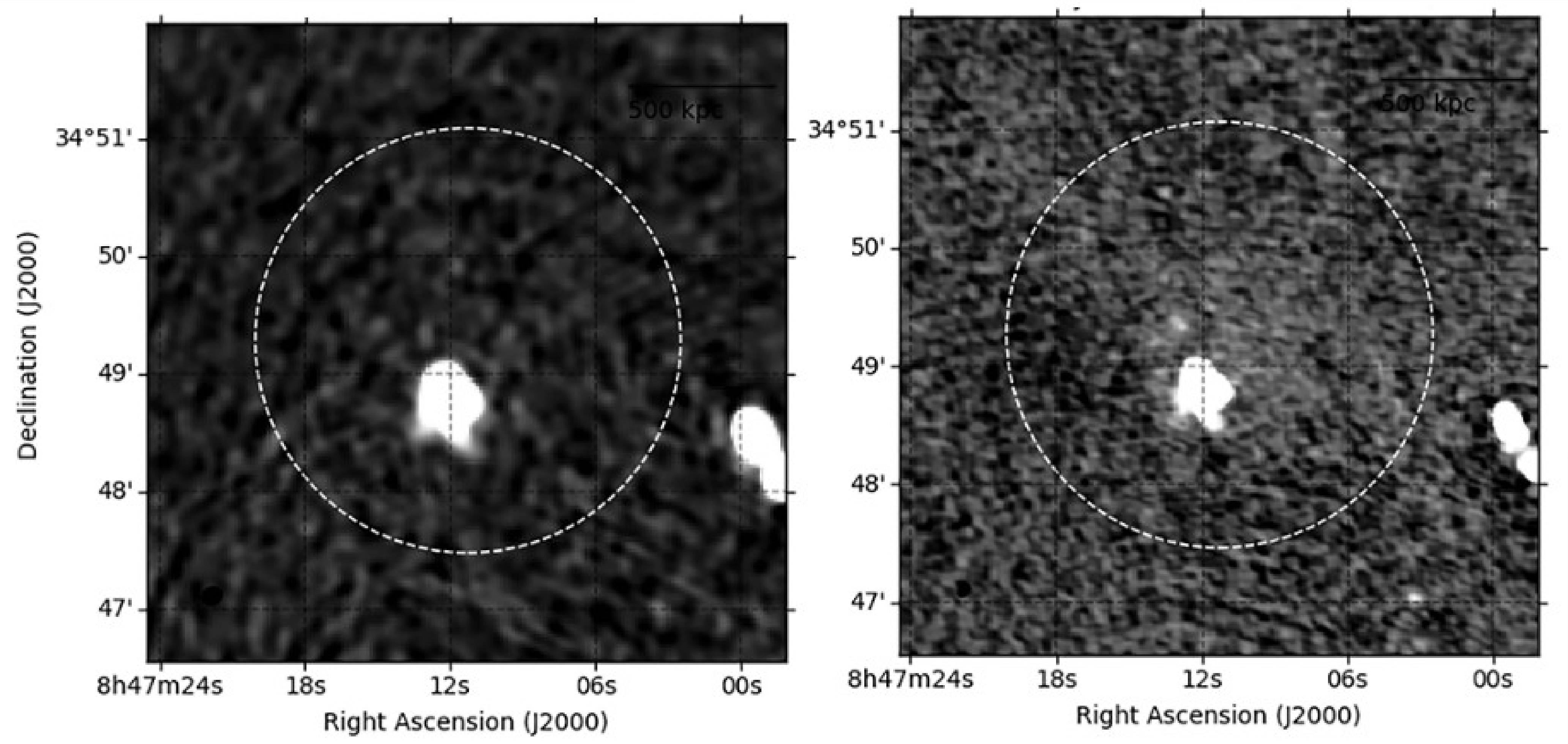
Which one has higher sensitivity?



Source: EPN pulsar database.

What is sensitivity?

Which one has higher sensitivity?



IC: George et al. 2021. (In reality, images are at different frequencies)

What is sensitivity?

- A measure of the “minimum detectable flux density”,
  - in an interferometric image,
  - in a time-series, or
  - by a telescope.
- A measure of the “noise”,
- Not a measure of the signal.

IC: GMRT

Why Sensitivity is Crucial?

- A building block of any radio telescope,
- Crucial to design observations, preparing technically sound observing proposals (e.g., 1 μJy in 1 hour with uGMRT band-4??),
- To accurately characterise observational data:
  - Estimates of the flux density,
  - Sensible upper limits, etc.
  - Error analysis of an image, a time-series, spectrum.

Radio Astronomy: Signals and Noise

- Flux density S(ν) units: Jansky (Jy); 1 Jy = 10^{-26} W/m^2/Hz
- More relevant for extended sources, Sky brightness (B): "W/m^2/Hz/sr" OR Brightness temperature: "K"; S(ν)$=\Omega$B(ν)
- Rayleigh-Jeans approximation to the Plank spectrum of a (thermal radiation from a) black-body:

$$T_B = \frac{\lambda^2}{2k} B(\nu)$$

- Brightness temperature is directly related to the physical temperature only for thermal emission and large optical depth.

Radio Astronomy: Signals and Noise

- In the radio regime, power per unit bandwidth: $P = k_B T$
- Equivalently, antenna temperature, $T_A = P/k_B$

- For effective collecting area A_e, power received per unit bandwidth from a source of flux density S:

  $P_{src} = S \times A_e / 2$; for each polarization

- $T_{src} = S \times A_e / 2k_B$ OR ***$T_{src} = GS$***; where $G = A_e/2k_B$ is the gain.

Radio Astronomy: Signals and Noise

- Considering the total system, “system temperature”

$$T_{sys} = \text{Total-power-received}/k_B$$

$$T_{sys} = T_{sky} + T_{spill} + T_{loss} + T_{rec}$$

- T_{sky}: the background sky brightness temperature,
- T_{spill}: the spillover noise temperature,
- T_{loss}: contribution from any lossy element in the path, and
- T_{rec}: the receiver temperature

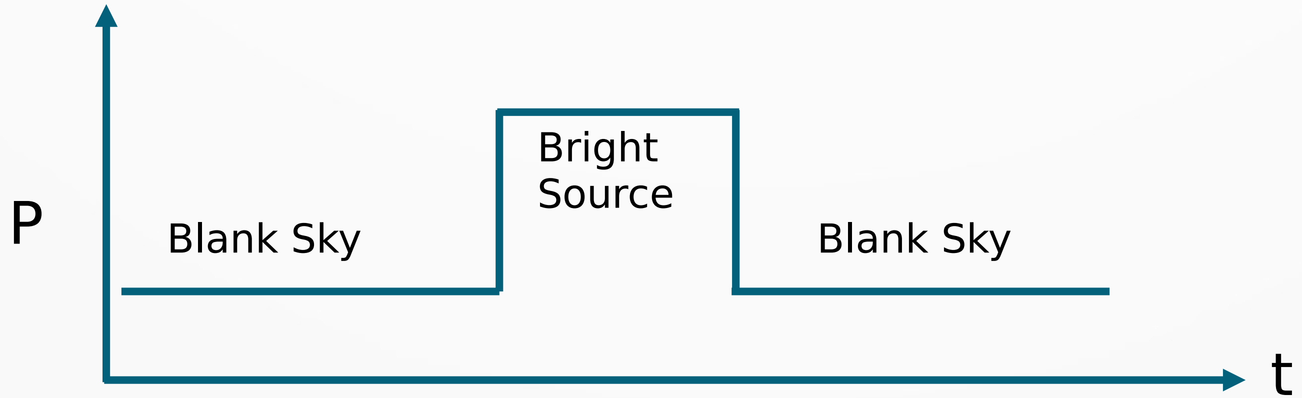
System Equivalent Flux Density (SEFD)

- Considering the total system, "system temperature"

  $$\boldsymbol{T_{sys} = GS_{sys}} \qquad (\text{recall } T_{src} = GS)$$

  $$\text{SEFD} = S_{sys} = T_{sys}/G$$

- SEFD takes into account the system temperature, efficiency and the collecting area --- a useful measure of the system performance.
- SEFD of a radio telescope can be measured by switching the telescope between blank sky and a bright source of known flux density.



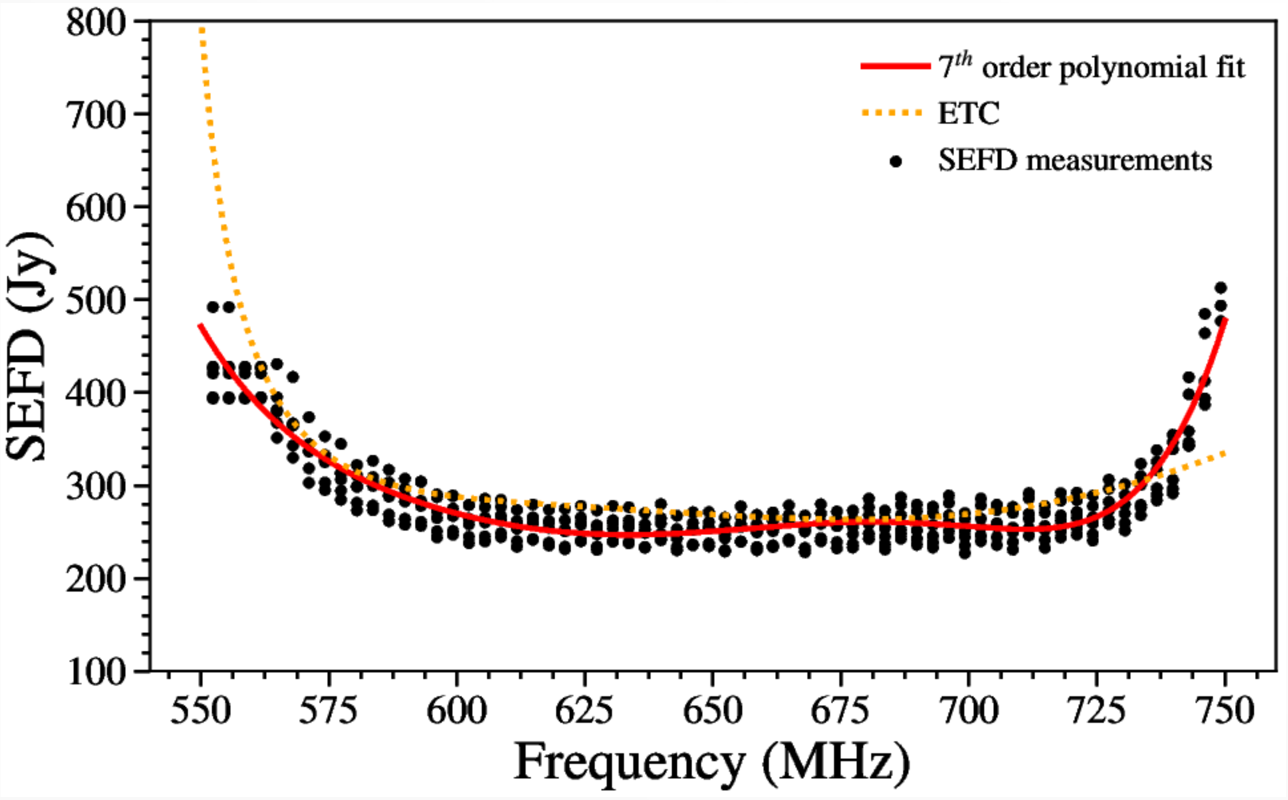
System Equivalent Flux Density (SEFD)

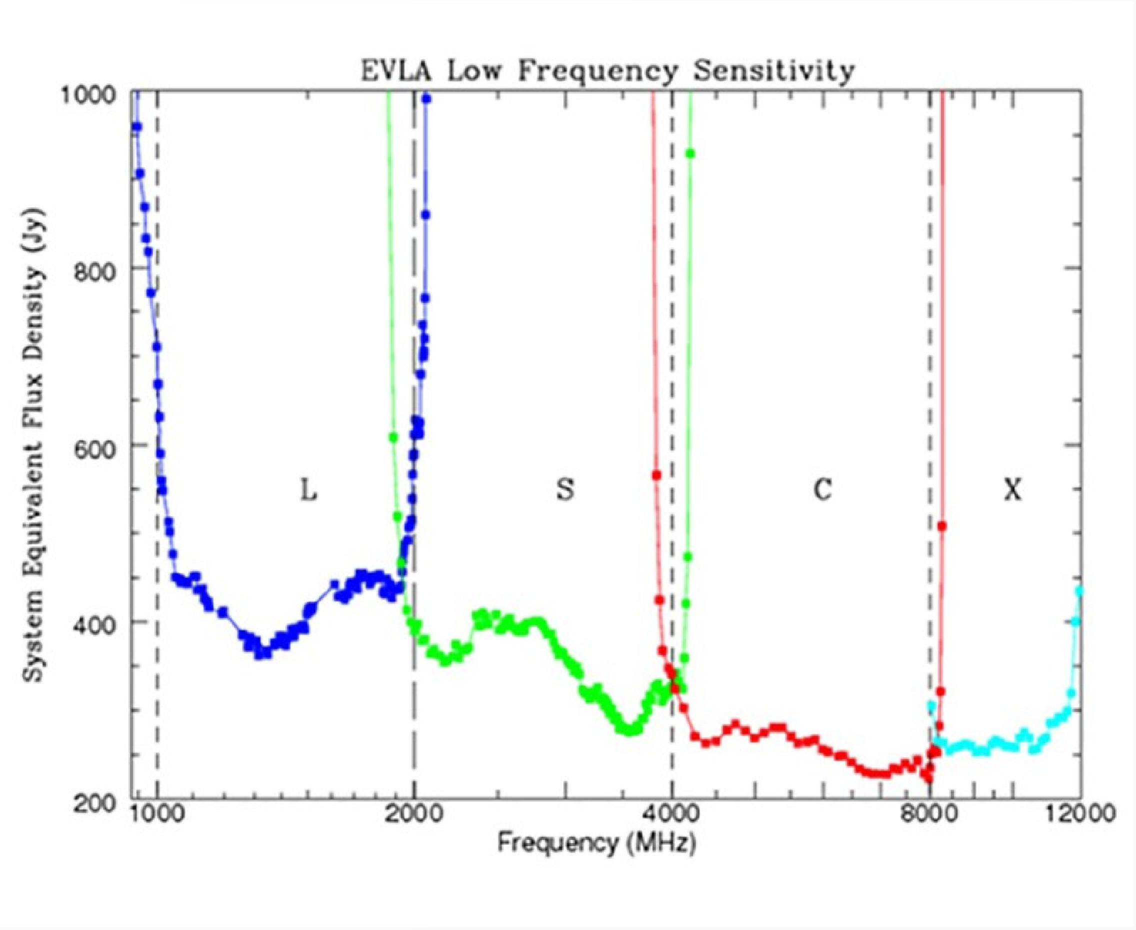
- SEFD is often measured in units of Jy --- flux density of a radio source that doubles the system temperature.

$$SEFD = S_{sys} = T_{sys}/G = 2k_B T_{sys}/A_e$$

- SEFD is a good way to compare the sensitivity of different telescopes.

System Equivalent Flux Density (SEFD)





Sensitivity: The Radiometer Equation

- Considering the total system, “system temperature”

$$T_{sys} = \text{Total-power-received}/k_B$$

$$T_{sys} = T_{sky} + T_{spill} + T_{loss} + T_{rec}$$

- T_{sky}: the background sky brightness temperature,
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- T_{loss}: contribution from any lossy element in the path, and
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Sensitivity: The Radiometer Equaiton

- The minimum temperature that a telescope can measure is limited by the noise (root mean square) fluctutions in the receiver system.

$$\Delta T_{\rm sys} = \frac{T_{\rm sys}}{\sqrt{n_{\rm p}\, t\, \Delta f}}.$$

- For a Δf bandwidth stochastic process, coherence time ~ 1/Δf
- => No. of independent samples in t seconds = t x Δf

Single Dish Sensitivity

- S/N = $T_{src} / \Delta T_{sys}$ (recall T_{src} = GS)

$$S_{\rm min} = \beta \frac{({\rm S/N_{min}}) T_{\rm sys}}{G \sqrt{n_{\rm p}\, t_{\rm int}\, \Delta f}}$$

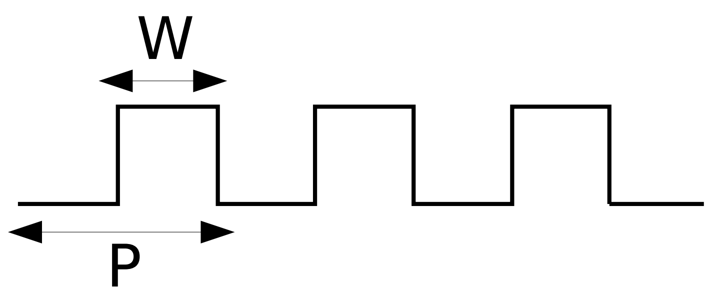
- β is the correction factor to account for imperfections (due to digitization and other factors).
- The minimum detectable "mean" flux density (corresponding to a S/N threshold S/N_{min}).

Single Dish Sensitivity for Pulsars

- S/N = T_{src} / ΔT_{sys} (recall T_{src} = GS)

$$S_{\rm min} = \beta \frac{({\rm S/N_{min}})T_{\rm sys}}{G\sqrt{n_{\rm p}\, t_{\rm int}\, \Delta f}} \sqrt{\frac{W}{P-W}}.$$

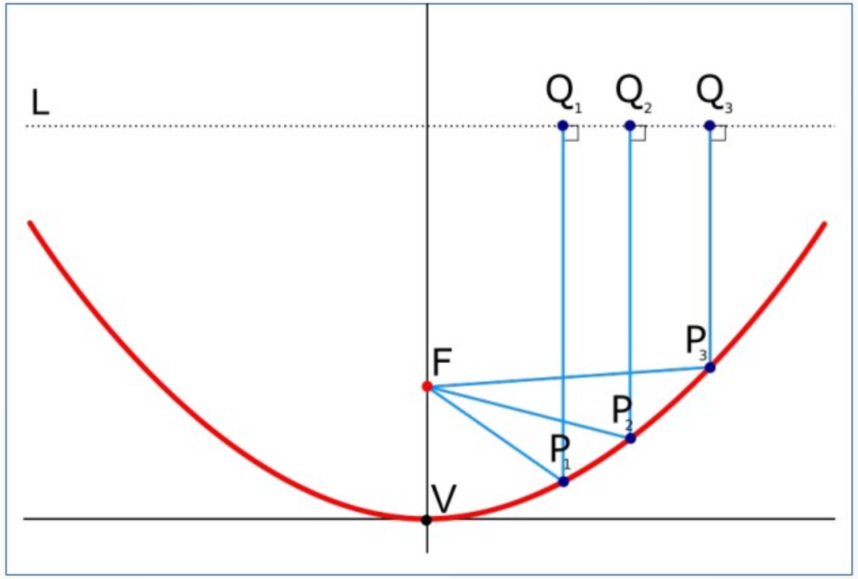
- The minimum detectable “mean” flux density (corresponding to a S/N threshold S/N_{min}) for a signal with period P and pulse-width W.

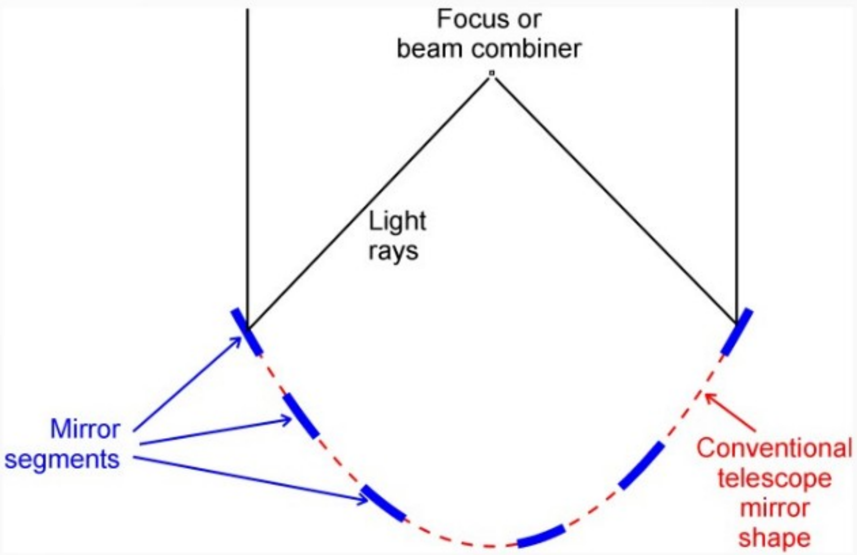


Phased-array Sensitivity

- For N "identical" elements, the effective collecting area increases by a factor "N".
- Hence the gain increases by the same factor.

$$S_{\rm min} = \beta \frac{({\rm S/N_{min}})T_{\rm sys}}{G\sqrt{n_{\rm p}\, t_{\rm int}\, \Delta f}} \; \frac{1}{\rm N}$$





IC: Divya Oberoi's lecture and wikipedia.

Sensitivity: 2-element interferometer

- Formal derivations in Wrobel and Walker 1999, and Ch. 5 of "Low frequency Radio Astronomy" book, a crude and heuristic way to think is the following:
- Effective collecting area is same as that of the individual elements.
- Correlation outputs from two-different channels of the complex-correlator => 2x independent samples

$$S_{\rm min} = \beta \frac{({\rm S/N}_{\rm min}) T_{\rm sys}}{G\sqrt{n_{\rm p}\, t_{\rm int}\, \Delta f}} \frac{1}{\sqrt{2}}$$

Sensitivity: N-element interferometer

- N-element interferometer (with 2-element correlations) is equivalent to ${}^{N}C_2$ independent 2-element interferometers, as it can be shown that:
  - Outputs from two 2-element interferometers with one common antenna are uncorrelated, and
  - Outputs from two 2-element interferometers with no common antenna are uncorrelated.

$$S_{\rm min} = \beta \frac{({\rm S/N_{min}})T_{\rm sys}}{G\sqrt{n_{\rm p}\, t_{\rm int}\, \Delta f}} \frac{1}{\sqrt{\rm N(N-1)}}$$

- For large values of N, N(N-1) -> N^2, and hence, the point-source sensitivity of an interferometer reaches that of a single antenna with area equal to the cumulative area of all the elements.

Image Sensitivity

Other factors that affect image-sensitivity:

- Sensitivity varies as a function of the offset from the pointing center.
- Radio frequency interference, T_{sys} variations, improper alignment of the dish and feed axis, elevation dependent pointing offsets, etc. degrades the sensitivity.
- Statistics in the image plane need to take into account like smoothing, deconvolution, etc.
- Phase irregularities caused by the ionosphere --- “seeing”, scintillation and position-offsets. Using a nearby calibrator with known/modelled visibility helps. Source not lying inside a iso-planatic patch introduces complexities.

Image Sensitivity

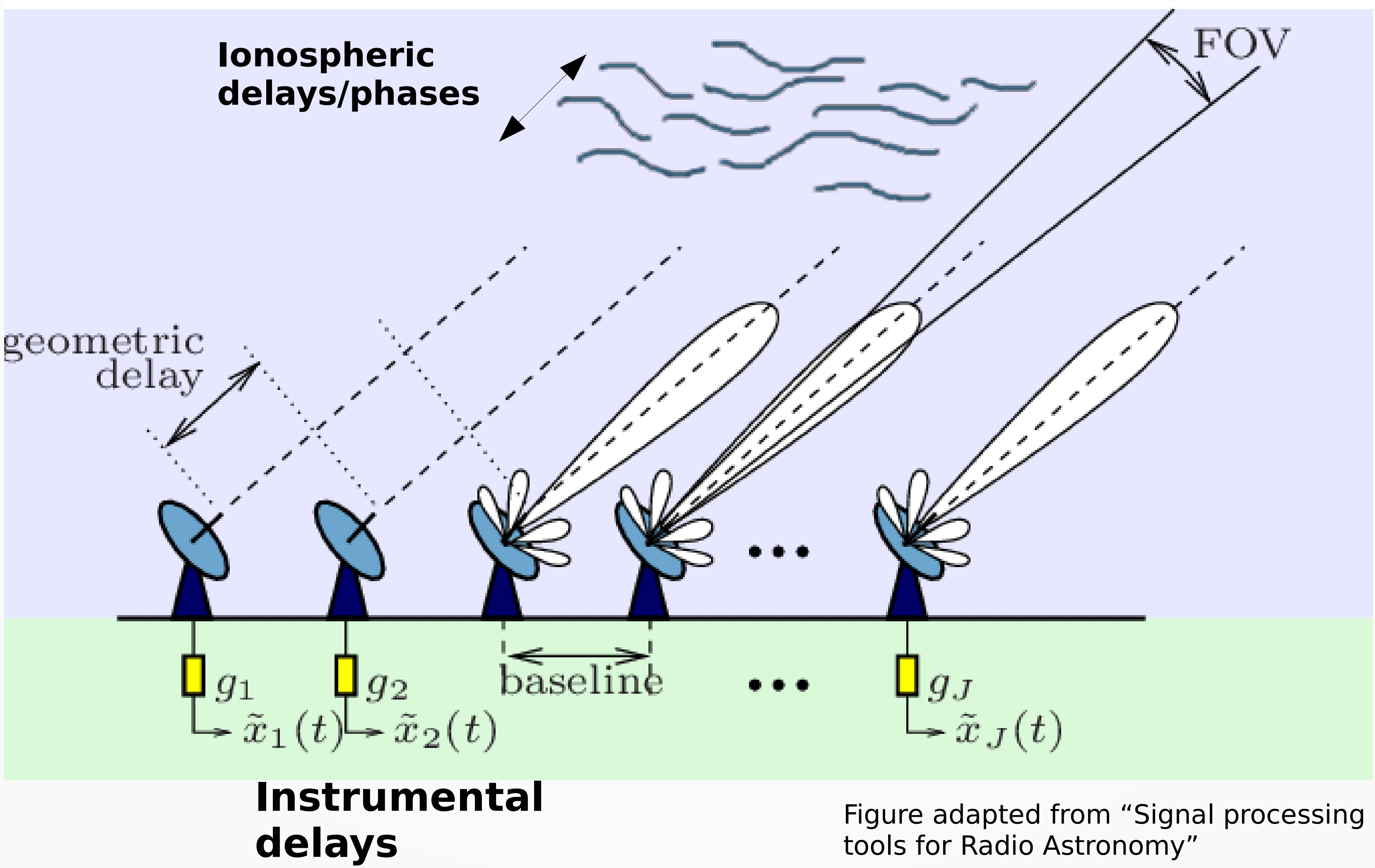


Figure adapted from “Signal processing tools for Radio Astronomy”

Image Sensitivity

Other factors that affect image-sensitivity:

- Errors in determining antenna calibration parameters,
- Choice of weighting --- natural weighting and no tapering has highest sensitivity, -> robust -> uniform,
- Confusion limit due to the sources within the synthesized beams,
- Confusion from the uncleaned sources lying in the sidelobes of the primary beam.

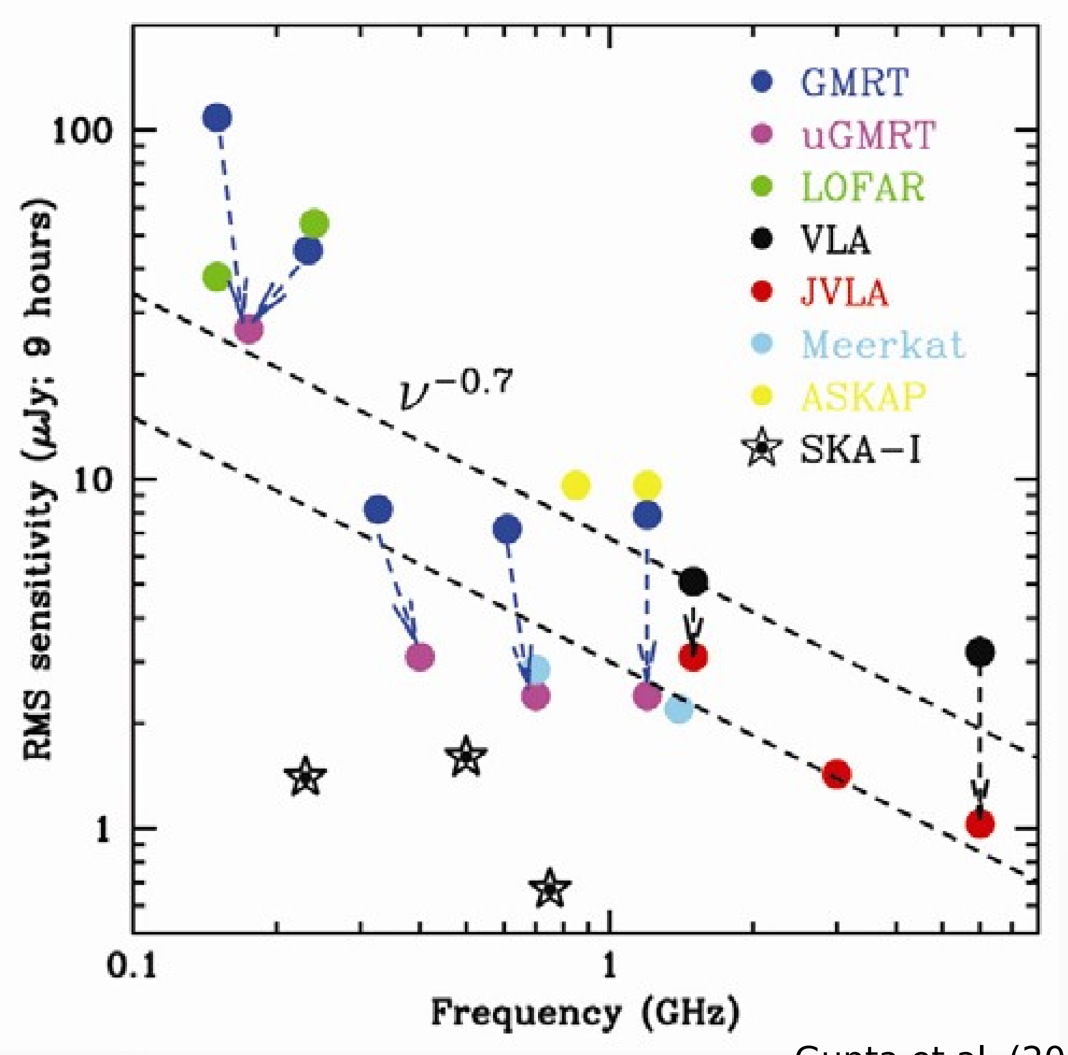
uGMRT Sensitivity Parameters

Table 1: uGMRT System Parameters

Band	Frequency range MHz	Gain (K/Jy)	T_{sys} K	RMS noise† μJy/Bm	Primary Beam$^{\$}$ $'$	Synth. beam$^{\$}$ $''$
Band-2	(125 – 250)	0.33	760 – 240	500	120	17.3
Band-3	(250 – 500)	0.38	165 – 100	15	75	8.3
Band-4	(550 – 850)	0.35	≈ 100	6	38	4.3
Band-5	(1000 – 1460)	0.22–0.28	≈ 75	2.5	23	2.3

Table from GMRT status document

uGMRT Sensitivity



Gupta et al. (2017)

uGMRT Sensitivity/Exposure-time Calculator

GMRT Exposure Time Calculator (Continuum/Spectral Line)

Help In case of queries, please write to gmrtcalc[at]ncra.tifr.res.in.

Users are advised to run the ETC on the Firefox or Chrome browsers.
Problems have been noticed in some versions of Safari.

1	Observation Type	?	Continuum	
2	Observing Band	?	Band-2 (125-250 MHz)	
3	Representative Frequency	?	200	MHz
4	Number of antennas	?	26	
5	Bandwidth	?	200	MHz
6	Usable Bandwidth	?	50	MHz
7	Number of Polarizations	?	2	
8	Image weighting	?	Natural	
9	Source co-ordinates(J2000)	?	RA 00h 00m 00.00s	Dec 00d 00' 00.00"
10	Sky temperature (T_sky, K)	?	0	auto calculate
11	Calculation Type	?	On-Source Time	
12	RMS noise	?	100	μJy/Bm
13	On-source Time	?	00h 00m 00s	
14	Fudge Factor	?	1	
15	On-source Time including Fudge Factor	?	00h 00m 00s	
16	Overheads	?	00h 00m 00s	auto calculate
17	Extra Bandpass/Polarization Time	?	00h 00m 00s	
18	Total Time (15+16+17)	?	00h 00m 00s	
19	Confusion Limit (σ_c*)	?		μJy/Bm

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http://www.ncra.tifr.res.in:8081/~secr-ops/etc/etc.html

GMRT Exposure Time Calculator (Pulsars/Transients)

Help In case of queries, please write to gmrtcalc[at]ncra.tifr.res.in.

Users are advised to run the ETC on the Firefox or Chrome browsers.
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1	Observation Type	?	Pulsars/Transients	
2	Observing Mode	?	Folded Profile	
3	Observing Band	?	Band-2 (125-250 MHz)	
4	Representative Frequency (ν)	?	200	MHz
5	Beam mode	?	IA	
6	Number of antennas	?	26	
7	Bandwidth	?	200	MHz
8	Usable bandwidth	?	50	MHz
9	Channel resolution	?	0.1	MHz
10	Number of polarizations	?	2	
11	Source co-ordinates(J2000)	?	RA 00h 00m 00.00s	Dec 00d 00' 00.00"
12	Period	?	1000	ms
13	Pulse Width	?	100	ms
14	Dispersion measure	?	1	pc cm^-3
15	Sky temperature (T_sky, K)	?	0	auto calculate
16	Calculation Type	?	On-Source Time	
17	RMS noise	?	100	μJy
18	On-source Time	?	00h 00m 00s	
19	Fudge Factor	?	1	
20	On-source Time including Fudge Factor	?	00h 00m 00s	
21	Overheads	?	00h 00m 00s	auto calculate
22	Total Time (20+21)	?	00h 00m 00s	

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