

Single dish radio astronomy

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Radio telescopes

Reflector + single feed/antenna

Can measure wave amplitude and phase (unlike optical devices)

Combine multiple antenna measurements together (interferometry)

References:

- 1) [NCRA Low Frequency Radio Astronomy Handbook](#)
- 2) [NRAO Essentials of Radio Astronomy](#)
- 3) [Tools of Radio Astronomy](#)



Measurement of the electric field

Feed/Antenna → couples electric field in space to voltage in wire (or vice versa)

→ Can also be just a long piece of wire (FM antenna, AM antenna)

$$E(t) = \sum E_i(t)$$

But: 1) Multiple independent radiators in each source, 2) The sources are independent

→ phases of $E_i(t)$ are random



Measurement of the electric field

Each source adds power = V^2/R to the feed via the E-field.

For single dish observations, we only measure noise power density (i.e. /Hz)

$$P_{\text{total}} = P_{\text{source}} + P_{\text{sky}} + P_{\text{Amplifier}} + P_{\text{ground}} \dots$$

On-source and off-source measurements

$$P_{\text{off-source}} = P_{\text{sky}} + P_{\text{Amplifier}} + P_{\text{ground}} \dots$$

$$P_{\text{source}} = P_{\text{total}} - P_{\text{off-source}}$$

Brightness temperature

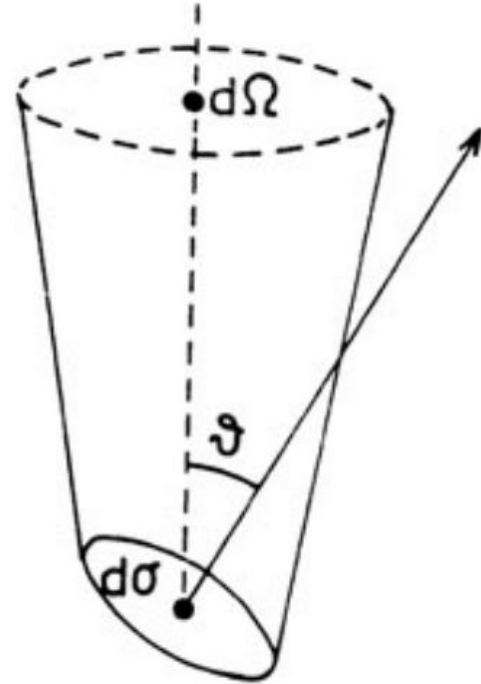
Specific intensity/Brightness \rightarrow power per unit time, per unit area, per unit freq, per unit solid angle

Total flux density from an astronomical source \rightarrow integrate over the solid angle

Given in units of Jansky (10^{-23} erg/cm²/s/Hz)

Brightness doesn't change with distance.

Flux density does



Brightness temperature

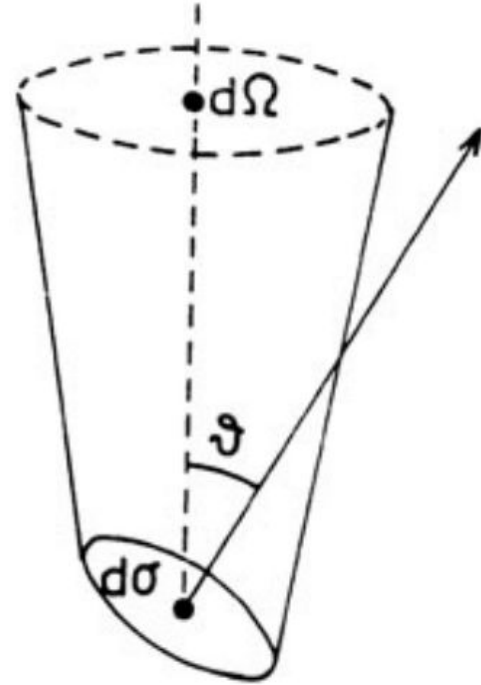
Blackbody radiation law

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

For radio, we're in the Rayleigh tail in most cases (except for $T < 100\text{K}$)

$$B_{\text{RJ}}(\nu, T) = \frac{2\nu^2}{c^2} kT$$

→ Temperature proxy for brightness (flux density per unit solid angle)



Johnson-Nyquist noise

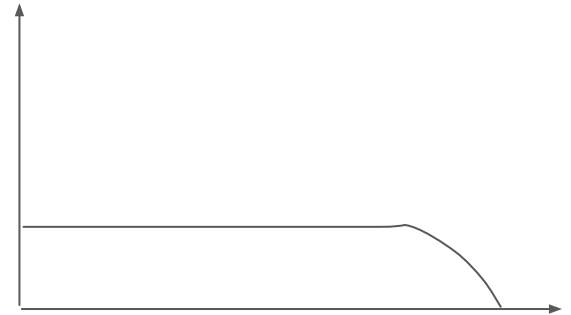
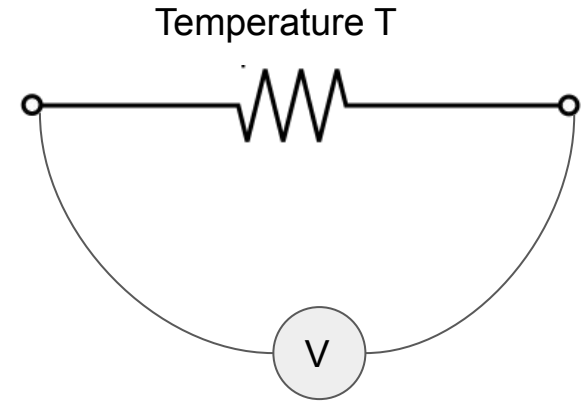
Random thermal motions of electrons cause noise

V will go +/- around zero (same for current i)

→ V^2/R represents the power

Power per unit bandwidth = $k_B T$

Constant power per unit BW till high frequencies (quantum effects) → effectively white noise



Calibrated noise diode

Switchable source of noise

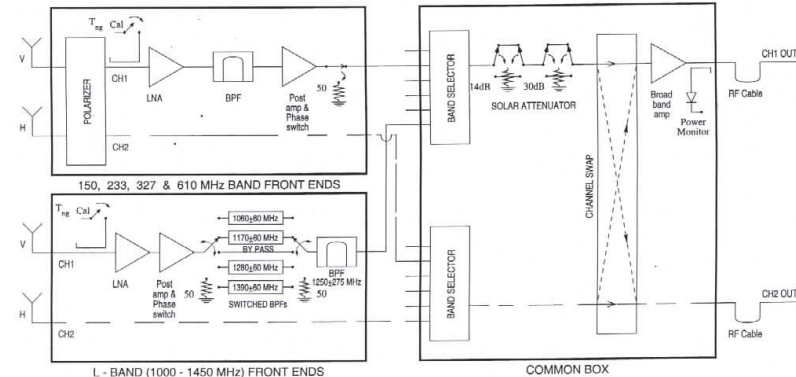
Calibrated in the factory

Known, small dependence on temperature, voltage

Allows the system noise/gain to be calculated



<https://www.keysight.com/sg/en/product/N4001A/sns-series-noise-source-10-mhz-18-ghz-enr-15-db.html>



Brightness temperature

But beam pattern makes a difference

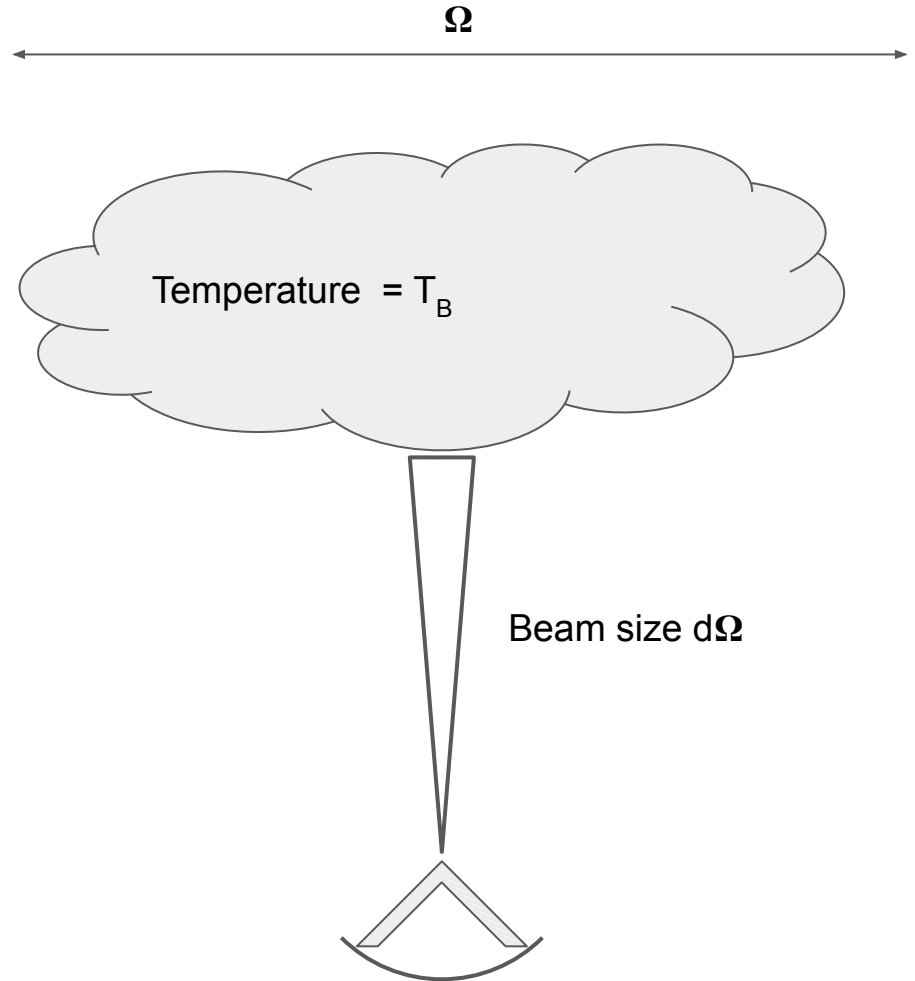
If beam size $d\Omega <$ angular size Ω

Power received = $d\Omega$ *Brightness

Effective brightness = Power/ $d\Omega$

== Brightness

Brightness temperature is correctly measured



Brightness temperature

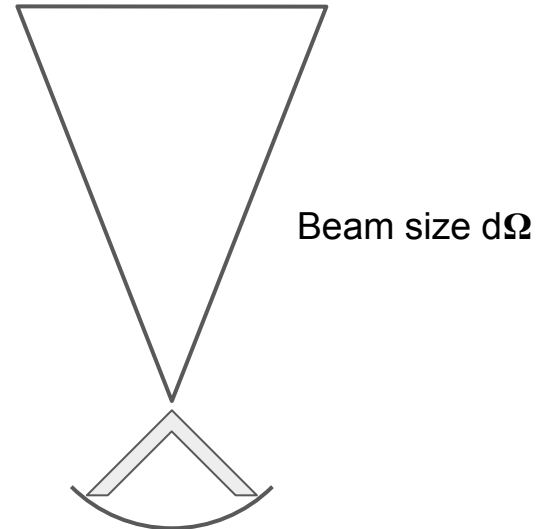
If beam size $d\Omega >$ angular size Ω

Power received = Ω *Brightness

Effective brightness = Power/ $d\Omega$

== Brightness* $\Omega/d\Omega$

Brightness temperature is diluted by a factor of $\Omega/d\Omega$ (solid angle ratio)

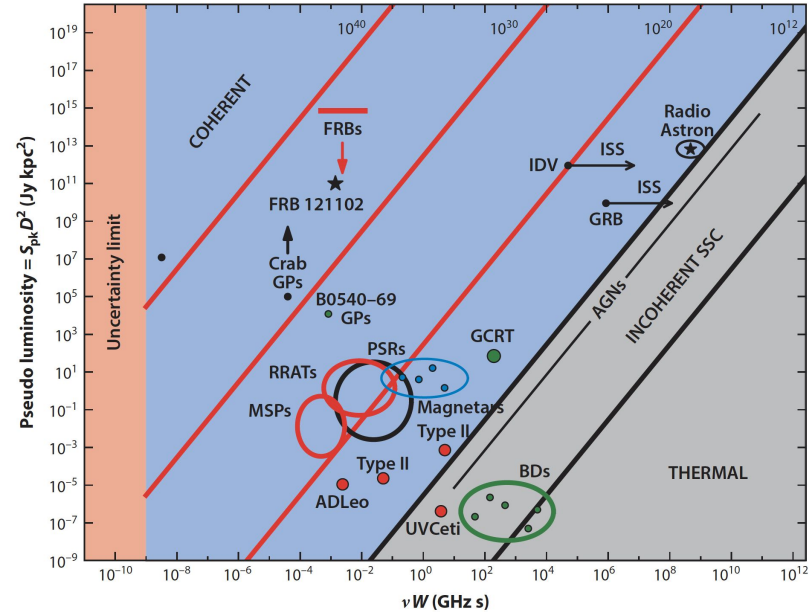


Brightness temperature

T_B == actual temperature only for blackbodies

If the radiation is non-thermal, T_B can be far higher than physical temperature

Brightness temperatures can be very high – 10^{40} K for FRBs



Cordes & Chatterjee
(2020)

Radiation Pattern

Each antenna has a radiation gain pattern

$$G(\theta, \varphi)$$

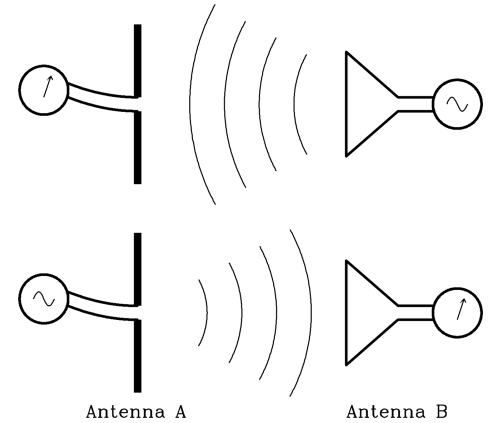
Power ratio transmitted per unit solid angle in direction (θ, φ)

(Reciprocity theorem \rightarrow also power recd)

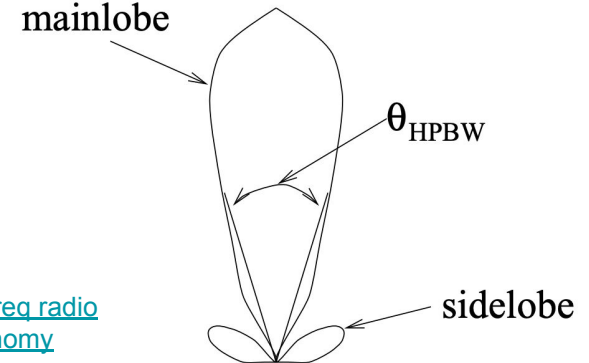
$$P_n(\vartheta, \varphi) = \frac{1}{P_{\max}} P(\vartheta, \varphi)$$

$$G(\vartheta, \varphi) = \frac{4\pi P(\vartheta, \varphi)}{\iint_{4\pi} P(\vartheta, \varphi) d\Omega}$$

Normalized to 4π
over the sphere



[Essentials of Radio Astronomy](#)



[Low freq radio astronomy](#)

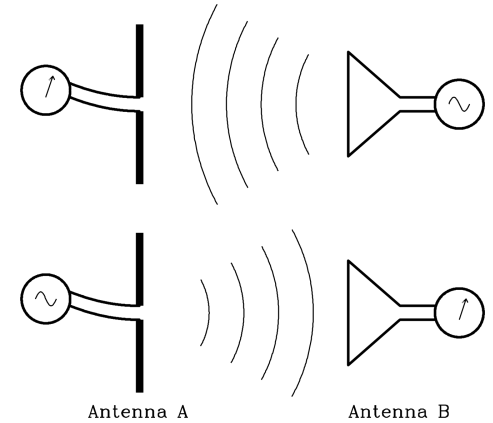
Radiation Pattern

$$\text{gain } G_{\text{dB}} = 10 \log_{10}(G)$$

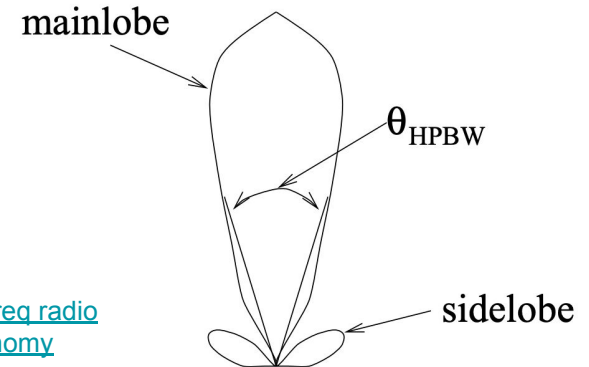
Gain/directivity is w.r.t. an isotropic lossless antenna (doesn't exist)

$$\text{Beam solid angle } \Omega_A = 4\pi/G_{\text{max}}$$

(Effectively the solid angle into which the light is transmitted or recd from)



[Essentials of Radio Astronomy](#)



[Low freq radio astronomy](#)

Radiation Pattern

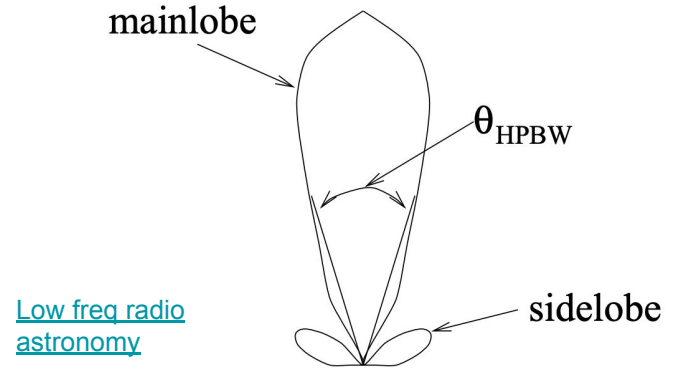
Same idea as the PSF in optical astronomy

Much larger angular sizes (for single dishes)

$\theta_{\text{HPBW}} = \lambda/D$ still works

Except λ comparable or slightly smaller than D .

D = diameter of the dish (or size of the last radiating element)



Effective Area

Effective area $A_e = \text{Power received}/\text{source flux density}$

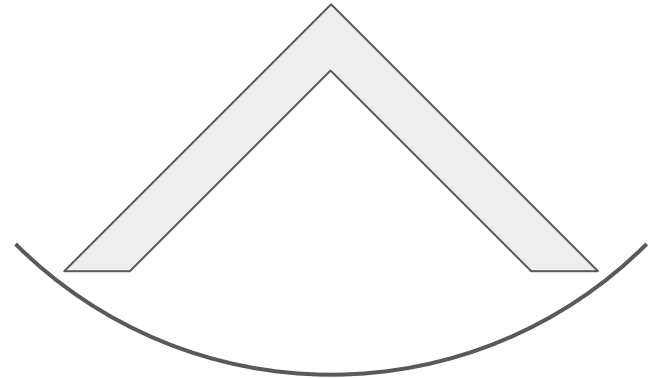
$$A_e = P_{rx}/S$$

Aperture efficiency $\eta = A_e/A_{\text{geometric}}$

A_e is direction dependent

Related to G_{max} as $G_{\text{max}} = 4\pi A_{e, \text{max}}/\lambda^2$

$\langle A_e \rangle = \lambda^2/4\pi$ (integrated over the sphere)



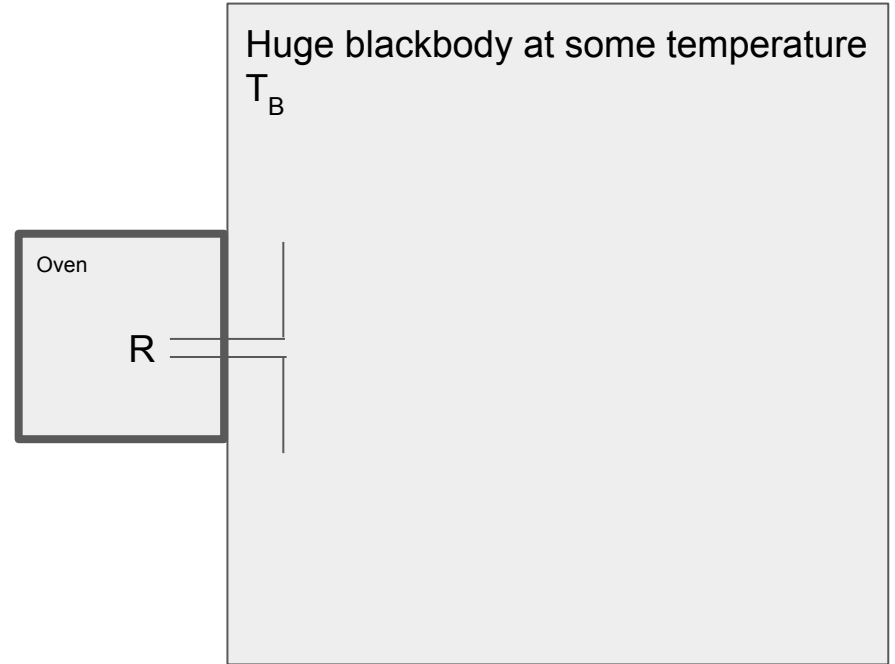
Antenna Temperature

Assuming perfect coupling with the antenna,

Power recd == power emitted (Johnson noise)

Load resistance R should equilibrate at T_B

Add directivity \rightarrow R should equilibrate at the average brightness temperature in the beam



Antenna Temperature

$$k_B T_A = \frac{1}{2} * A_e * \iint G(\theta, \varphi) B(\theta, \varphi) d\Omega$$

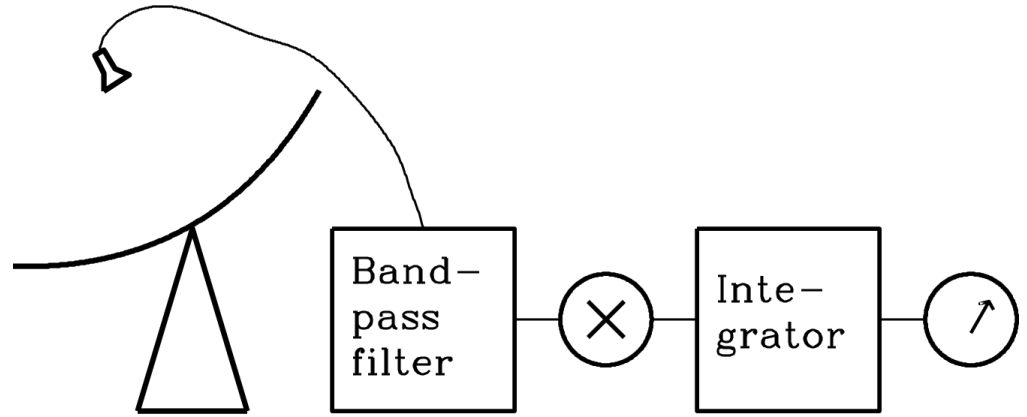
$B(\theta, \varphi)$ is the angular brightness distribution created by an extended object of brightness temperature T_B (or multiple different sources)

Antenna temperature \rightarrow measures the contribution of the source to the antenna power

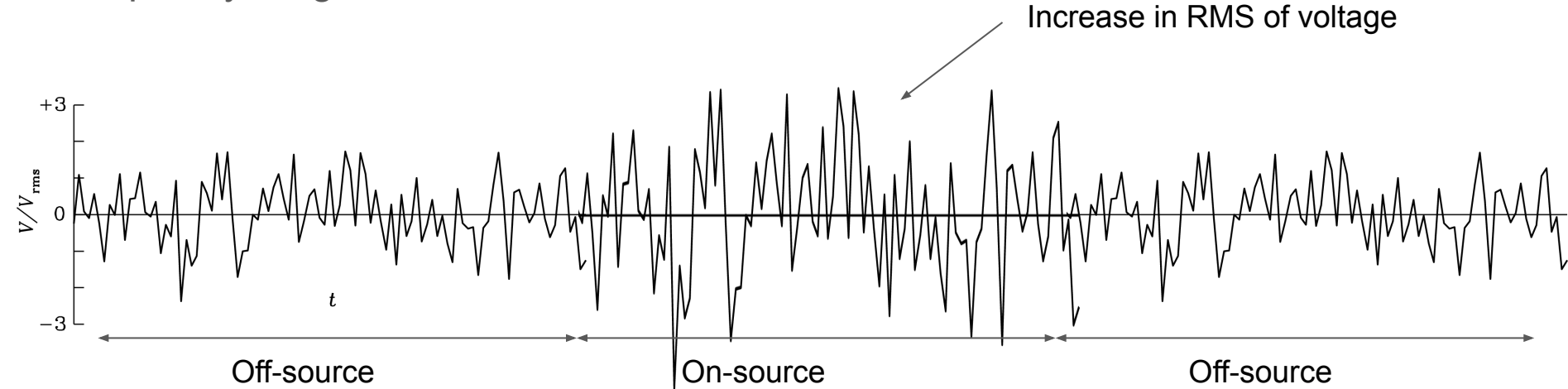
Signal chain

Simple non-heterodyne signal chain

Measures power over a small frequency range



Increase in RMS of voltage



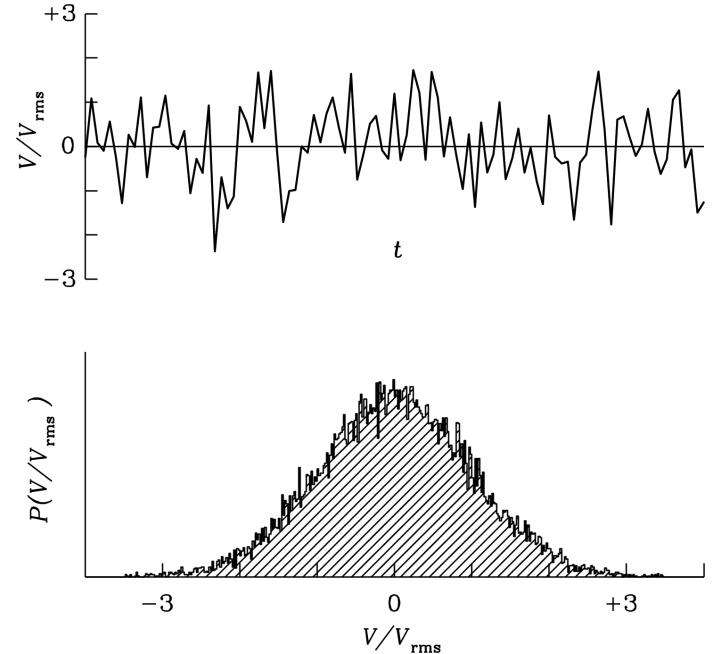
Radiometer equation

How do we measure power?

Square the voltage and average over some bandwidth and time

How many independent measurements?

→ Shannon-Nyquist sampling theorem



Shannon-Nyquist Sampling Theorem

Any function having finite bandwidth Δv and duration τ can be represented by $2\Delta v\tau$ independent samples spaced in time by $(2\Delta v)^{-1}$

→ Having more samples than this will not give more information about your measurement. They will not be independent.

We measure voltage with $2\Delta v\tau$ samples.

Error in power measurement = 2*error in voltage measurement

→ If we average the power over a bandwidth Δv and time τ , we get $\Delta v\tau$ independent measurements → error in average power measurement reduces by $\text{sqrt}(\Delta v\tau)$

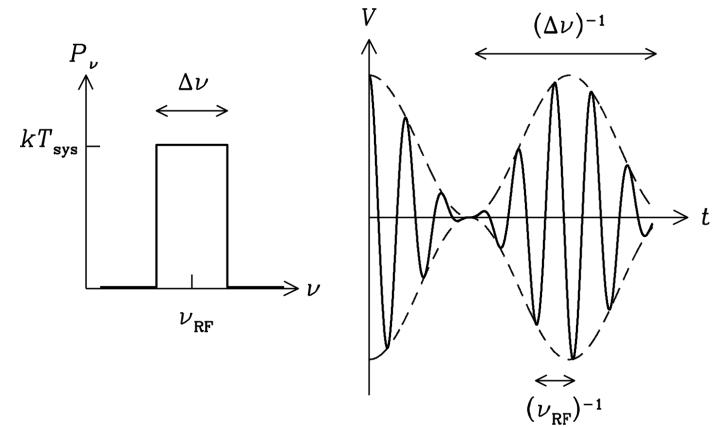
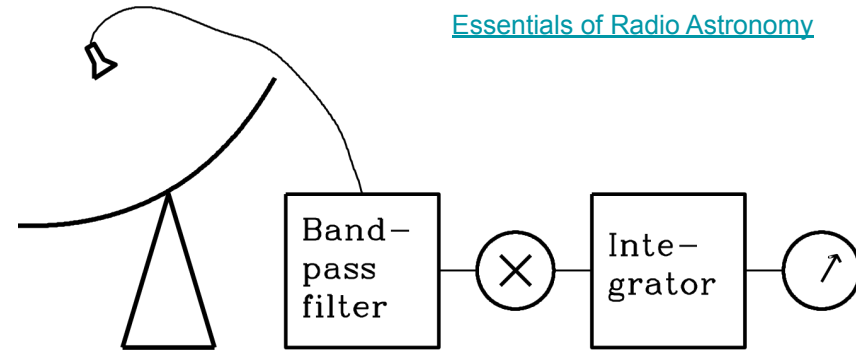
Radiometer equation

How do we measure power?

Square the voltage and average over some bandwidth and time

Bandpass filter $\rightarrow \nu_{\text{RF}} - \Delta\nu/2$ to $\nu_{\text{RF}} + \Delta\nu/2$,

Integrator \rightarrow output voltage V_o proportional to V_i^2 over timescale $\tau \gg 1/\Delta\nu$



For a detailed derivations see [ERA](#)

Radiometer equation

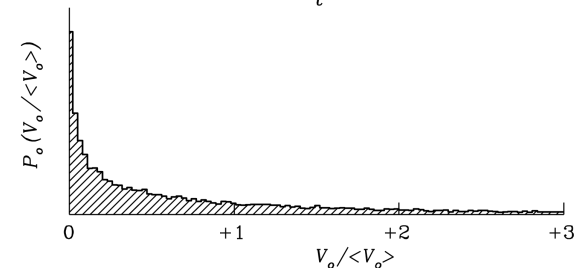
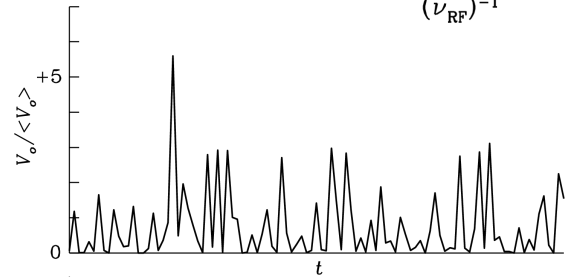
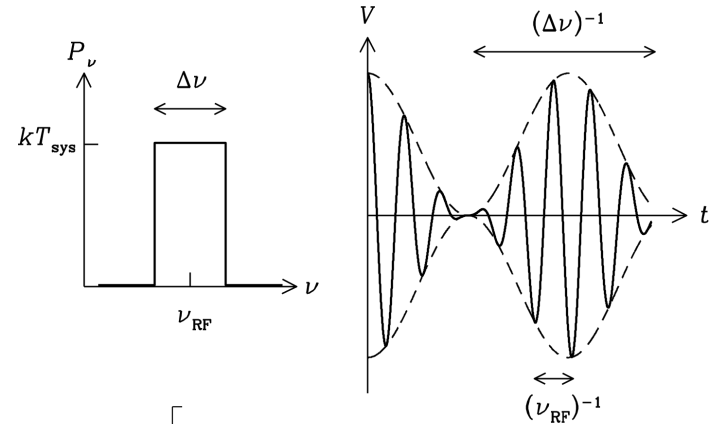
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Integrator \rightarrow output voltage V_o proportional to V_i^2 over timescale $\tau \gg 1/\Delta\nu$

If total noise power is T_s , the error in measurement is $T_s/\sqrt{\Delta\nu\tau}$



Noise sources

$$T_s = T_{\text{CMB}} + T_{\text{sky}} + \Delta T_{\text{source}} + T_{\text{atm}} + T_{\text{Ground}} + T_{\text{Amplifier}} \dots$$

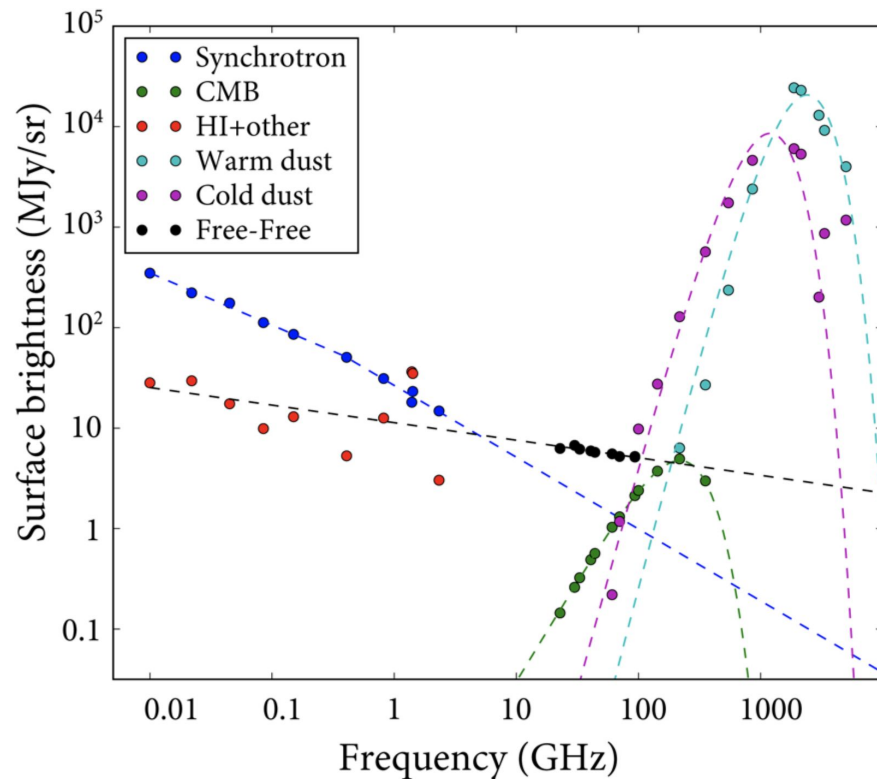
$$T_{\text{CMB}} = 2.73 \text{ K}$$

T_{sky} depends on frequency and position on the sky – different components

T_{atm} – frequency dependent and opacity dependent

T_{ground} – spill over from the ground at 300K

$T_{\text{amplifier}}$ – noise added by the first amplifier (mainly) and rest of the electronics



[Zheng et al \(2017\)](#)

Noise sources

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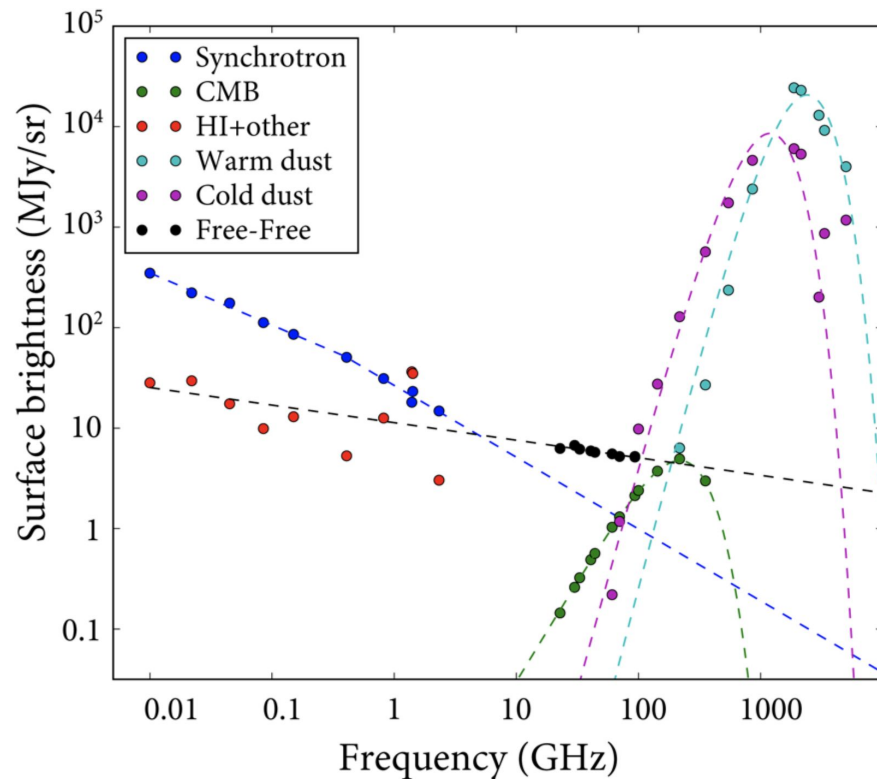
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[Zheng et al \(2017\)](#)

Heterodyne systems

Developing electronics for each frequency is hard

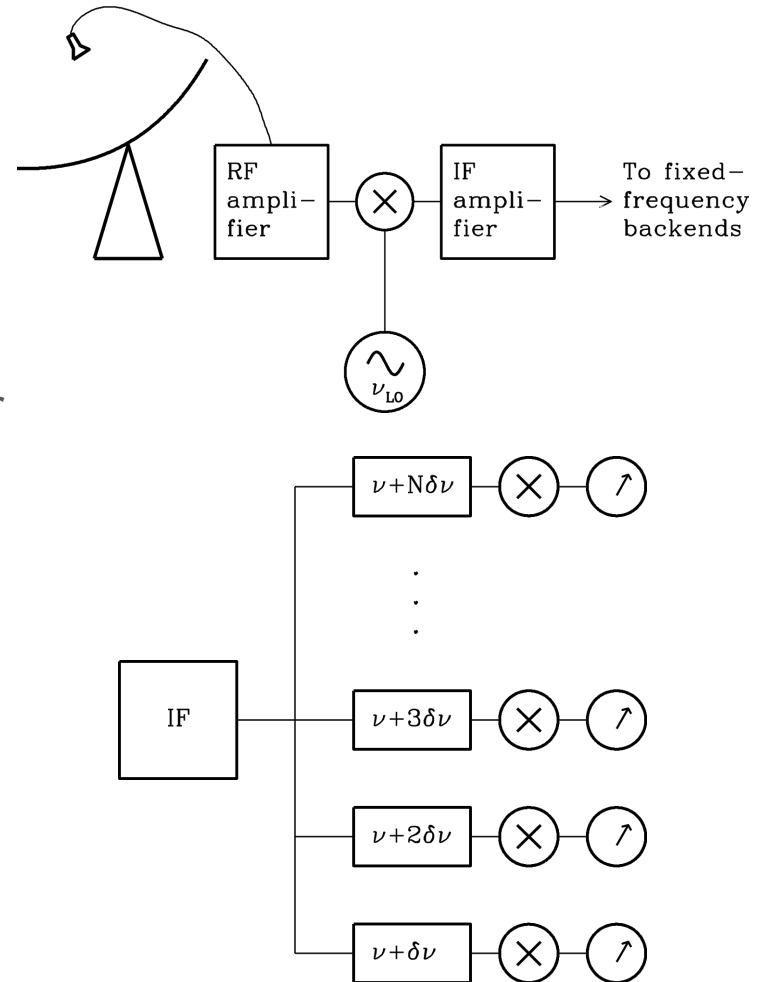
Mix (multiply) incoming RF with a local oscillator

$$\sin(\omega_{\text{RF}}t) * \sin(\omega_{\text{LO}}t) = \frac{1}{2}\cos((\omega_{\text{RF}} - \omega_{\text{LO}})t) - \frac{1}{2}\cos((\omega_{\text{RF}} + \omega_{\text{LO}})t)$$

Discard high frequency part ($\omega_{\text{RF}} + \omega_{\text{LO}}$)

Design everything else for $\omega_{\text{IF}} = \omega_{\text{RF}} - \omega_{\text{LO}}$

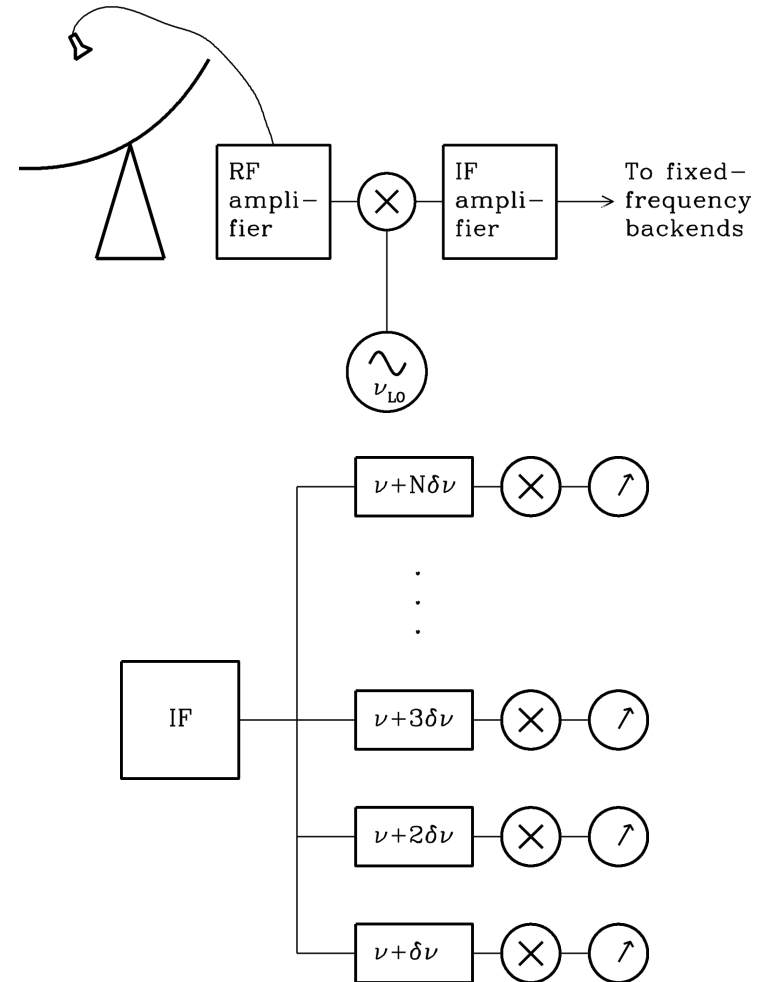
Tune the RF observed by changing LO



Heterodyne systems

Spectroscopy is done in 2 ways

- 1) Hardware filterbanks – separate electronic filters for each IF
- 2) Digital filterbanks – FFT the timeseries, get power in separate channels (more in next lecture)



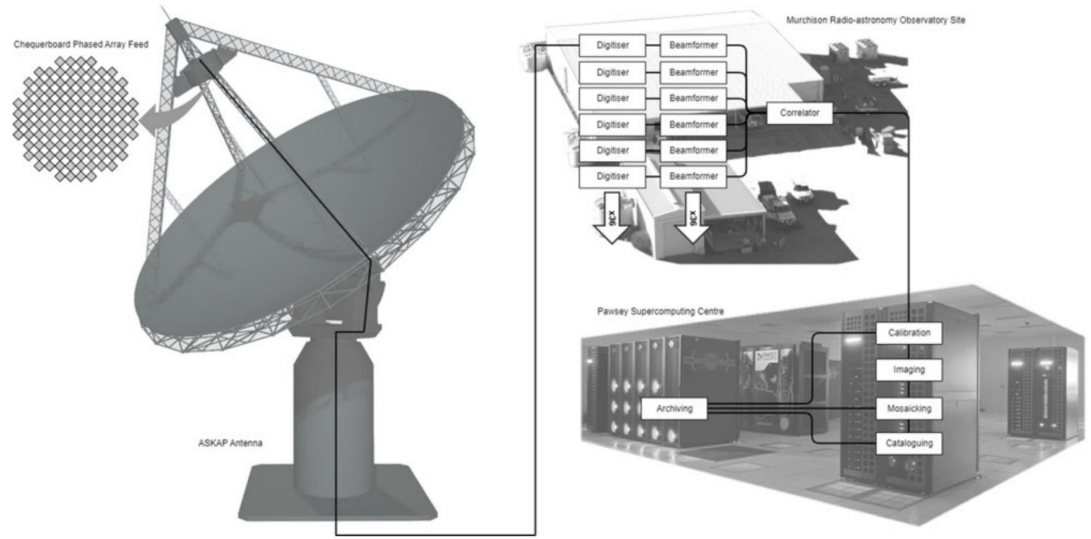
Focal plane array

Most telescopes have a single feed at the focus → single pixel camera FoV == primary beam size

Phased array feed/Focal plane array → Multiple feeds

Complex response to electric fields from different directions

Needs beamforming



Allows for large survey speeds

ASKAP, Westerbork